Optical Sciences PhD Qualifying Exam:
Instructions and Equation Sheet

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The following are some helpful items:

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$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

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$\oint_S (\mathbf{F} \cdot \mathbf{n}) \, da = \int_V (\nabla \cdot \mathbf{F}) \, d^3x$

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$\oint_S (\mathbf{n} \times \mathbf{F}) \, da = \int_V (\nabla \times \mathbf{F}) \, d^3x$

System of units: SI (or MKSA)

1) A homogeneous plane-wave undergoes total internal reflection within a glass prism of refractive index $n = n_0$, as shown. As usual, it is assumed that $\mu(\omega) = 1.0$ at optical frequencies, and that $\boldsymbol{\sigma} = \boldsymbol{k}/k_0$ is the normalized k-vector. The resulting evanescent plane-wave in the free-space region beyond the prism (i.e., $x \geq 0$) has the following E- and H-fields:

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp[\mathrm{i}(k_0\boldsymbol{\sigma}\cdot\boldsymbol{r} - \omega t)],$$

$$\boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{H}_0 \exp[\mathrm{i}(k_0\boldsymbol{\sigma}\cdot\boldsymbol{r} - \omega t)].$$

Let $\boldsymbol{\sigma} = \mathrm{i}\sigma_x\hat{\boldsymbol{x}} + \sigma_z\hat{\boldsymbol{z}}$, while $\boldsymbol{E}_0 = E_{x0}\hat{\boldsymbol{x}} + E_{y0}\hat{\boldsymbol{y}} + E_{z0}\hat{\boldsymbol{z}}$ and $\boldsymbol{H}_0 = H_{x0}\hat{\boldsymbol{x}} + H_{y0}\hat{\boldsymbol{y}} + H_{z0}\hat{\boldsymbol{z}}$. In general, σ_x and σ_z are real-valued, while $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ are complex.
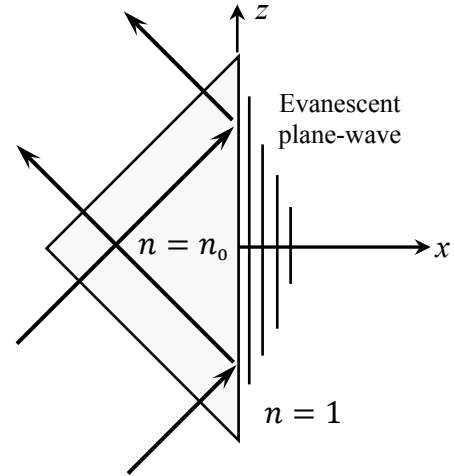
a) What is the relationship between σ_x and σ_z?

b) What does Maxwell's first equation say about the relation between E_{x0} and E_{z0}?

c) What does Maxwell's 4th equation say about the relation between H_{x0} and H_{z0}?

d) What is the relation between (E_{x0}, E_{y0}, E_{z0}) and (H_{x0}, H_{y0}, H_{z0}) based on Maxwell's 3rd equation?

e) If $E_{z0} = 0$, which components of $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ will vanish as well?

f) If $H_{z0} = 0$, which components of $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ will vanish as well?

An optical system in air is comprised of two thin lenses. The lens focal lengths are f_1=20mm and f_2=80mm. An aperture stop is placed between the two lenses so that the system is telecentric in both the object space and image space. A circular object with a radius of 2mm is placed at a distance of 20mm to the left of the first lens. The slope of the marginal ray for the on-axis object point is u_0=0.2 in object space. The system is unvignetted over the entire object field of view. A circular detector is placed at the image plane to capture the image.

(a)   Determine the transverse or lateral magnification of the system. (1 point)

(b)   Determine the distance, Z'_I, of the image plane relative to lens 2 and the minimum diameter of the image detector. (2 points)

(c)   Draw a neat diagram showing the optical axis, the two lenses, the object, the stop, the image and the detector. On the same diagram, trace the chief ray and marginal ray through the system. Show all axial distances (3 points)

(d)   Determine the slope, u'_i, of the marginal ray in the image space. (1 point)

(e)   Determine the diameter, D_s, of the aperture stop, and the minimum lens diameters D_1 for the lens 1 and D_2 for the lens 2 for the system to be unvignetted. (2 points)

(f)   If the object is displaced from its initial location to the left by a distance of Δz=2mm, determine the direction and the amount you need to move the detector to stay in focus. (1 point)

Consider the figure below. Three quasi-monochromatic plane waves are incident on an image sensor. Beams 1 and 3 come from the *same* source and have the same path lengths. Beam 2 comes from a *different* source than the other beams. Beam 1 is incident on the sensor at an angle of $-30°$. Beam 2 is normally incident to the sensor. Beam 3 is incident on the sensor at an angle of $+30°$. Beams 1 and 3 have the same amplitude A_1 and Beam 2 has an amplitude A_2. All three beams have a wavelength λ and their propagation vectors all lie in the y-z plane. You can assume the beams are all linearly polarized in the same direction. Answer the questions below.
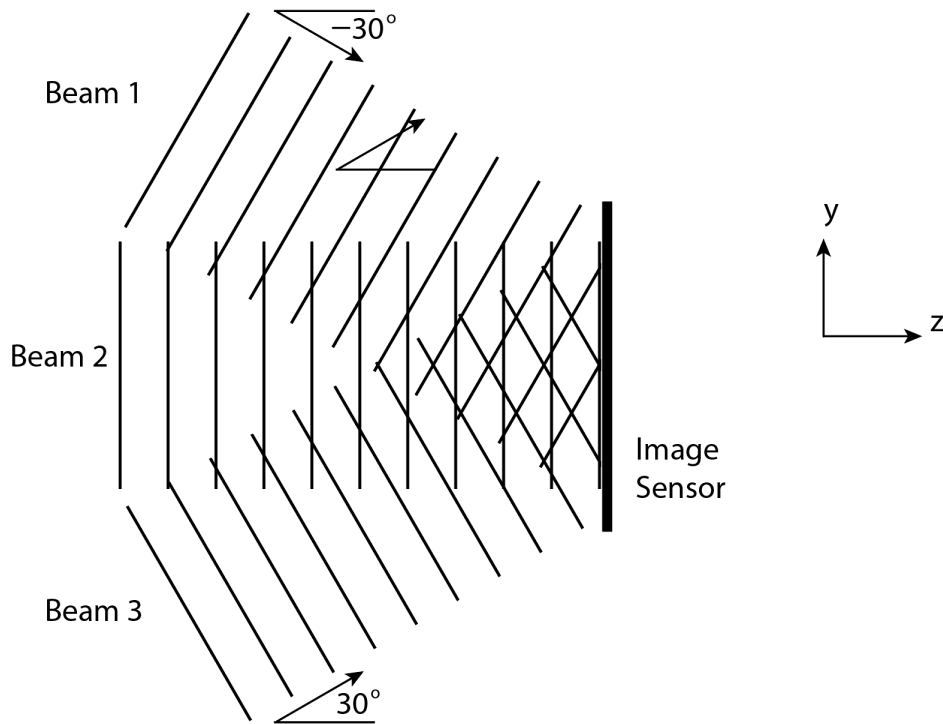
(a) Write expressions for the electric fields associated with each of the three beams.

Now consider Beam 2 to be turned off:

(b) If Beam 2 is shut off (i.e. $A_2 = 0$), derive an expression for the irradiance pattern on the sensor.

(c) What is the spacing between the fringes on the sensor?

(d) What is the visibility of the fringes on the sensor?

When Beam 2 is turned on:

(e) If Beam 2 is turned on (i.e. $A_2 > 0$), how does the irradiance pattern on the sensor change?

(f) What is the spacing between the fringes on the sensor?

(g) What is the visibility of the fringes on the sensor?

An electron is in the spin state:

$$|\chi\rangle = a_1 \begin{pmatrix} i \\ 2 \end{pmatrix}.$$

This spinor is written in the usual basis of eigenstates of the S_z spin operator. At time $t = 0$, a uniform magnetic field $\vec{B}$ is turned on, where $\vec{B} = B_o\hat{z}$, and B_o is a constant. The Hamiltonian describing the energy of the particle in the field given in terms of its gyromagnetic ratio γ is:

$$\hat{H} = -\gamma\vec{B} \cdot \hat{\vec{S}}$$

You may make use of the spin-1/2 operators:

$$\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

(a) Find the normalization coefficient a_1.

(b) If a measurement of the angular momentum along the z axis is made, what are the possible values that would be measured and the probability for each?

(c) Write an expression for the time dependent spinor state $|\chi(t)\rangle$ for times $t > 0$. Be sure to express your answer using quantities defined in this problem (for example B_o and γ).

(d) Calculate the expectation value, $\langle S_z \rangle$, of the electron spin angular momentum along the z axis.

(e) Calculate the expectation value, $\langle S_x \rangle$, of the electron spin angular momentum along the x axis as a function of time. Express your answer as a real sinusoidal function.

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PhD Qualifying Exam, Fall 2021

Opti 501

System of units: SI (or MKSA)

2) Two counter-propagating, linearly-polarized, homogeneous plane waves, both having frequency ω and wavelength λ_0, are trapped in the free-space region between a pair of perfectly conducting mirrors, as shown. The mirrors are parallel to the xy-plane, their separation d being an integer-multiple of $\lambda_0/2$. The propagation directions are $\boldsymbol{\sigma} = \hat{\boldsymbol{z}}$ and $\boldsymbol{\sigma}' = -\hat{\boldsymbol{z}}$. The (complex) E-field amplitudes $E_{x0} = |E_{x0}| \exp(i\varphi_0)$ and $E'_{x0} = |E'_{x0}| \exp(i\varphi'_0)$ have equal magnitudes; that is, $|E_{x0}| = |E'_{x0}|$.
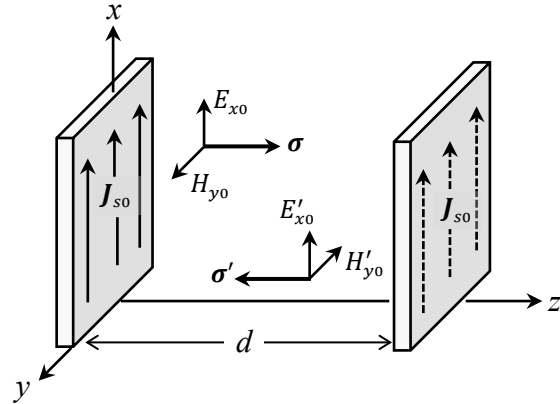
a) Write expressions for the total E- and H-fields in the cavity between the mirrors. (These expressions can be further simplified by taking into account the conditions at the mirrors.)

b) In terms of $|E_{x0}|$, find the magnitude of the surface-current-density $J_{s0}\hat{\boldsymbol{x}}$ on the mirror surfaces.

c) Determine the total E- and H-field energies trapped within the cavity. Show that, at those instants of time when the E-field energy is zero, the H-field energy is at a maximum, and vice-versa.

d) Write an expression for the Poynting vector $\boldsymbol{S}(z, t)$ inside the cavity. Interpret the oscillations of $\boldsymbol{S}(z, t)$ at a fixed location in space as time varies during each oscillation period.

Hint: $\cos a + \cos b = 2 \cos[\frac{1}{2}(a + b)] \cos[\frac{1}{2}(a - b)]$;

$\cos a - \cos b = -2 \sin[\frac{1}{2}(a + b)] \sin[\frac{1}{2}(a - b)]$.

PhD Qualifying Exam, Fall 2021
OPTI 502. Day 2

1. Let s and s' be the conjugate object and image distances of a lens that produces a real image. The distance between the conjugate points along the optical axis is L > 4F where F is the lens focal length. There are two sets of conjugate distances that have the same L.   The distance the lens must be shifted between these two sets of conjugates is D.

    Show that the lens focal length is given by

$$F = \frac{L^2 - D^2}{4L}$$

D

Lens is moved

L
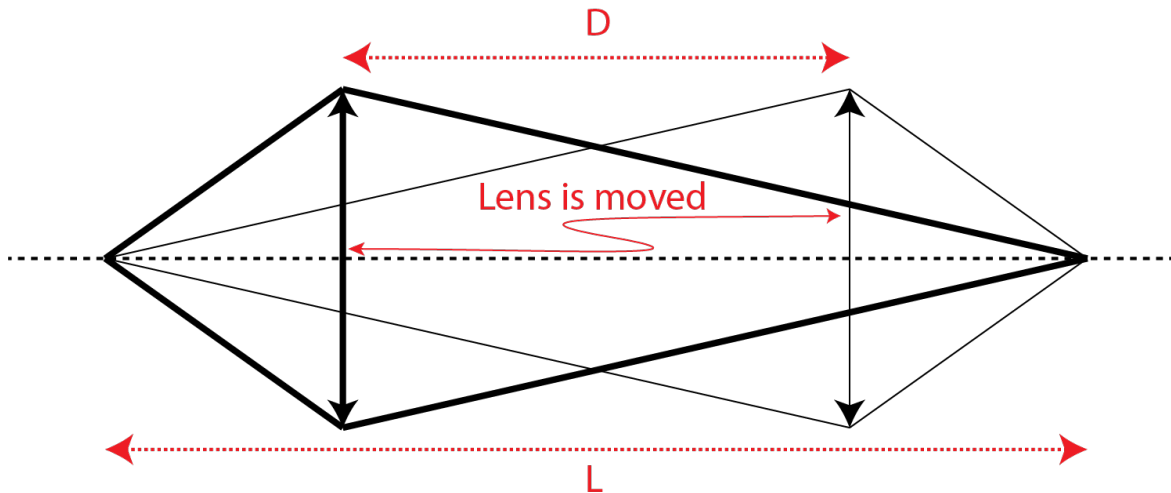
2. A collimated beam at a wavelength of 632.8 nm is incident on a transmissive grating of period 0.1 mm. The grating is at the front focal point of a lens. The diffracted beams produced by the grating are focused by the lens to produce an array of focused spots at its focal plane. Each spot is laterally separated by 1 mm from its neighbors. Determine the focal length of the lens. The grating equation is

$$n'\sin(\theta') - n\sin(\theta) = \frac{m\lambda}{d}.$$

    where m is the diffracted order, λ is the wavelength, Θ and Θ' are the angles of incidence and diffraction, n and n' are the indices of refraction in object and image spaces, and *d* is the grating period. (In this case n=n'=1). Assume that the lens image height is given by h=F sin(θ'). In the figure below only the order m=1 is shown.

Thin Lens

ϴ'

Axis

Grating
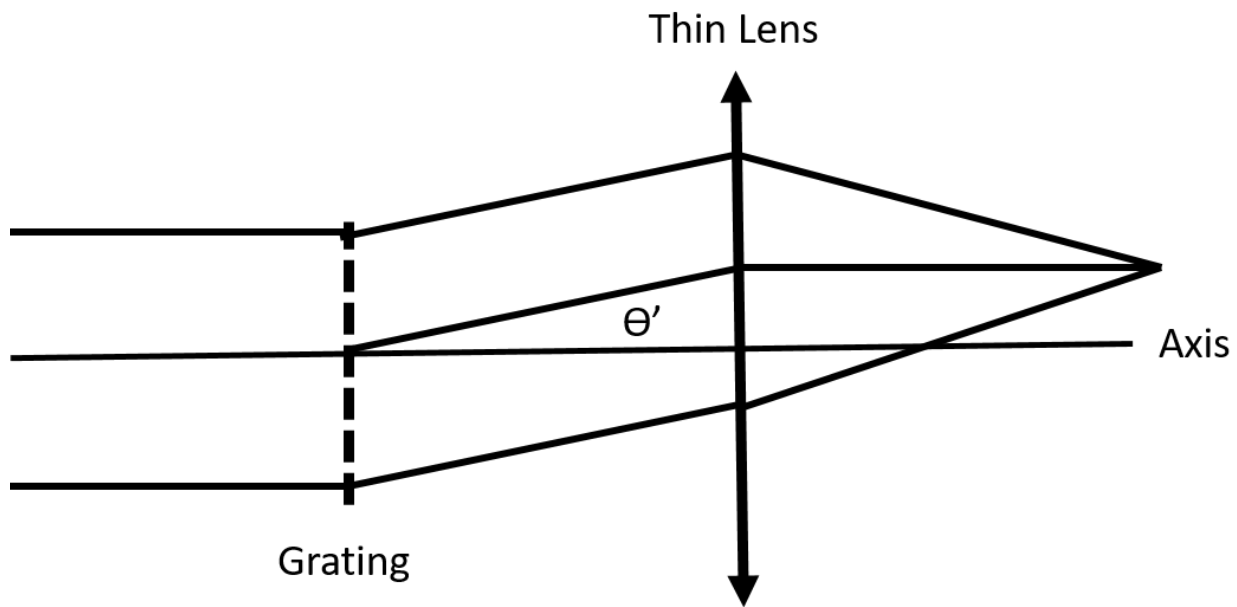
3. Discuss which method would be more precise in practice to measure focal lengths. Why?

PhD Qualifying Exam

Fall 2021

OPTI 505 Day 2

The Fresnel diffraction integral for the field passing through an aperture in the plane $z_s = 0$ can be written as

$$U_o(x_o, y_o) = \frac{-j\exp(jkz_o)}{\lambda z_o} \int_{ap} U_s(x_s, y_s)\exp\left\{\frac{jk}{2z_o}[(x_o - x_s)^2 + (y_o - y_s)^2]\right\} dx_s dy_s$$

(a) Show that this integral can be written as

$$U_o(x_o, y_o) = \frac{-j\exp(jkz_o)}{\lambda z_o}\exp\left[\frac{jk}{2z_o}(x_o^2 + y_o^2)\right]$$

$$\times \int_{ap} U_s(x_s, y_s)\exp\left[\frac{jk}{2z_o}(x_s^2 + y_s^2)\right]\exp\left[\frac{-jk}{z_o}(x_o x_s + y_o y_s)\right] dx_s dy_s$$

(b) Suppose a normally incident plane wave with wavelength $\lambda = 0.5\mu m$ illuminates a circular aperture with a diameter of 1mm. What is the Fresnel number N_f as a function of z_o for this aperture?

(c) What is the approximate minimum distance z_o the observation plane needs to be from this aperture in order for the Fresnel approximation to hold? You can assume the observation point is on axis.

(d) What approximation is made to the expression in part (a) to go from the Fresnel region to the Fraunhofer region?

(e) What is the approximate minimum distance z_o the observation plane needs to be from the circular aperture for the Fraunhofer approximation to hold?

(f) What is the value of the phase term $\exp\left[\frac{jk}{2z_o}(x_s^2 + y_s^2)\right]$ at the distance found in part (e)?

(g) Is the Fresnel approximation valid in the Fraunhofer region?
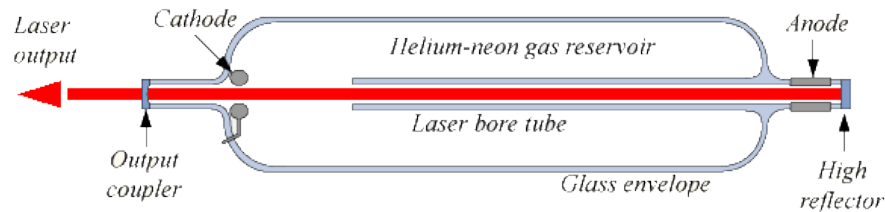
Figure 1: HeNe laser.

Consider the schematic in Fig. 1 of a standard commercial He-Ne laser, formed by embedding a He-Ne gas discharge tube into a two-mirror Gaussian beam resonator. The output coupling mirror has radius of curvature R and the opposite mirror is planar. The distance between the mirrors is $L = 10$ cm. The mirrors are coated to be reflective near the fundamental 633 nm lasing transition, which due to Doppler broadening exhibits a gain bandwidth of 1.5 GHz.

1. Suppose $R = \infty$. Roughly how many different wavelengths will the laser emit simultaneously?

2. What is the smallest R that will support a stable resonator mode?

3. In practice scenarios (1) and (2) are seldom used for laser resonators. Explain why.

4. Suppose $R = 40$ cm. What is the waist size of the laser beam at the output coupler?

5. Suppose the output power of the laser in (4) is 10 mW. At what distance will its irradiance fall below the FAA "Distraction" threshold of 50 nW/cm^2? (Note: You may ignore refraction through the output coupler.)

Useful formulae:
$R(z) = (z^2 + z_0^2)/z$
$w^2(z) = w_0^2(1 + z^2/z_0^2)$
$z_0 = \pi w_0^2/\lambda$