WRITTEN PRELIM EXAM – FIRST DAY

Fall 2010

September 28, 2010
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

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$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

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$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

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$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

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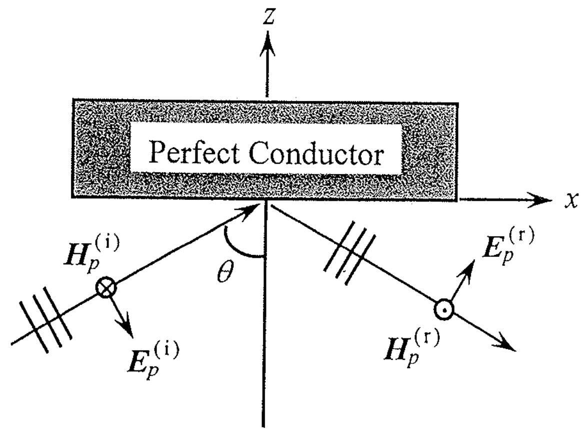
Question 1

System of units: MKSA

A monochromatic plane-wave of frequency ω arrives from free space at an angle of incidence θ at the flat surface of a perfect conductor. The plane-wave is linearly polarized in the p-direction, as shown in the figure.



(1 pt) a) Write the expression for the incident plane-wave, identifying its k-vector, its E-field, and its H-field. (The E-field is assumed to be known, but the H-field must be related to the E-field.)

(2 pts) b) Write the expression for the reflected plane-wave, identifying its k-vector, its E-field, and its H-field. (The reflected E-field must be related to the incident E-field via the boundary conditions at the surface of the perfect conductor.)

(2 pts) c) Verify conservation of energy by checking that the incident and reflected beams have the same (time-averaged) rate of flow of energy per unit cross-sectional area per unit time.

(2 pts) d) Find the surface-current density $J_s(x,y,t)$ at the surface of the perfect conductor.

(2 pts) e) Find the surface-charge density $\sigma_s(x,y,t)$ at the surface of the perfect conductor.

(1 pt) f) Verify that the surface charge- and current-densities obtained in parts (d) and (e) above satisfy the charge-current continuity equation, namely, $\nabla \cdot J_s(x,y,t) + \partial\sigma_s(x,y,t)/\partial t = 0$.

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Question 2

A lens system focuses in air a collimated beam of light. Assume that the focused beam has a full angle of convergence of 90 degrees (the marginal ray angle is 45 degrees with respect to the optical axis of the lens). The collimated beam of light overfills the lens aperture.

1) What is the numerical aperture of the focused beam? (1 point)
2) If a parallel plate of thickness 1 mm and index 1.5 mm is inserted between the lens and the focused spot, how much does the focal point move along the optical axis? The plate is perpendicular to the optical axis of the lens. (3 points)
3) If the index of refraction of the plate changes to 1.51 how much does the focus change in axial position? (1 points)
4) If the plate in part 2 moves 1 mm closer to the lens and along the optical axis, how much does the focal point move? (1 point)
5) The plane parallel plate is moved from the converging beam side of the lens to the collimated beam side of the lens. Because of a misalignment, the plate was mounted with a tilt of 10 degrees with respect to the lens optical axis. What happens to the lateral and longitudinal position of the focused spot? (2 points)
6) The lens system of part 1 now focuses the spot into a thick block of glass of index 1.5. What is the numerical aperture of the focused beam inside the glass? (2 points)

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Question 3

Consider a two-level atom in a resonant cavity, $\omega = \omega_{21}$. The interaction Hamiltonian for the system is of the form $\hat{H}_I = \hbar g(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^+)$. The bare atom-cavity states are of the type $|j,n\rangle$, where $j = 1,2$ designate the atomic states and $n \geq 0$ is the photon number. In the following we focus on the states $|1,1\rangle$ and $|2,0\rangle$, which contain exactly one excitation.

(a) [3 pts] Let $|\psi(t)\rangle = c_{11}(t)|1,1\rangle + c_{20}(t)|2,0\rangle$. Find the equations for $dc_{11}(t)/dt$ and $dc_{20}(t)/dt$, and write down solutions $c_{11}(t)$ and $c_{20}(t)$ for which there is one photon in the cavity at $t = 0$. What is the characteristic oscillation frequency of the probability amplitudes, and what is this quantity called in quantum optics?

Note: This problem should be very familiar - we have solved variations of it several times in OPTI 544, for both classical and quantum driving fields.

We now repeat the analysis with two identical atoms in the cavity. In that case the interaction Hamiltonian is $\hat{H}_I = \hbar g(\hat{\sigma}_+^{(A)}\hat{a} + \hat{\sigma}_+^{(B)}\hat{a} + \hat{\sigma}_-^{(A)}\hat{a}^+ + \hat{\sigma}_-^{(B)}\hat{a}^+)$, where the indices A, B indicate one or the other atom. The bare atom-cavity states are now of the type $|j,k,n\rangle$, where $j,k = 1,2$ designate the atomic states, and we focus on the states $|1,1,1\rangle$, $|2,1,0\rangle$ and $|1,2,0\rangle$ which contain exactly one excitation.

(b) [4 pts] Let $|\psi(t)\rangle = c_{111}(t)|1,1,1\rangle + c_{210}(t)|2,1,0\rangle + c_{120}(t)|1,2,0\rangle$. Find the equations for $dc_{111}(t)/dt$, $dc_{210}(t)/dt$ and $dc_{120}(t)/dt$, write down a solution for which there is one photon in the cavity at $t = 0$. What is the characteristic oscillation frequency of the probability amplitudes?

(c) [3 pts] Based on the results in (a) and (c), what characteristic oscillation frequency would you expect with N identical atoms in the cavity?

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Question 3

Consider a 2 mirror laser cavity 60 cm in length with an inhomogeneously broadened gain medium that occupies the entire region between the 2 mirrors. The lasing transition at 632 nm is Doppler broadened to a full width of 4 GHz (at the 1/e points). Assume the laser operates on a single TEM_{00} transverse mode.

a. **(2 pt)** If the gain medium is pumped sufficiently above threshold, will this laser operate on a single longitudinal mode in the steady state, or is lasing action on multiple longitudinal modes possible? Please clearly explain in a few sentences.

b. **(2 pt)** If the unsaturated gain coefficient at the center of the lasing transition is 3 times greater than the round trip loss coefficient, what is the maximum number of longitudinal modes that can lase?

c. **(2 pt)** Consider a single longitudinal mode slightly detuned from the center of the gain peak, where the unsaturated gain coefficient is 1.5 times the round trip loss coefficient. If the saturation intensity of the atomic transition is $I_{sat} = 2W/cm^2$, what would be the on axis intensity of that particular mode?

d. **(2 pt)** If you wanted to shift the lasing frequencies of the laser by 1 MHz, what change in cavity length would be required?

e. **(2 pt)** If you were to mode-lock this laser, what would be the time between pulses in the pulse train, and what would be the shortest possible pulse you could obtain (approximately)?

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Question 4

A unit-amplitude on-axis plane wave illuminates an aperture in air with the transmission function $t_{ap}(x_s, y_s)$.

a) (3pts) Find the Fraunhofer diffraction pattern $U_0(x_0, y_0)$ along the profile $(x_0, 0)$ a distance L away from the aperture if

$$t_{ap}(x_s, y_s) = \text{gaus}\left(\frac{x_s - \Delta x_s}{a}\right)\text{rect}\left(\frac{y_s}{b}\right) \text{, where } b >> a.$$

?, * Note add!

b) (2pts) Find the Fraunhofer *irradiance* $I_0(x_0, 0)$ if

$$t_{ap}(x_s, y_s) = \left[\text{gaus}\left(\frac{x_s - d/2}{a}\right) + \text{gaus}\left(\frac{x_s + d/2}{a}\right)\right]\text{rect}\left(\frac{y_s}{b}\right) \text{, where } b >> a.$$

c) (3pts) Assume that $d >> a$ and provide a plot of $I_0(x_0, 0)$ indicating key features including width w of the irradiance profile at the $1/e^2$ maximum irradiance and the spacing between adjacent zeros of the irradiance profile.

d) (2pts) If identical slit apertures of width a are used in place of the Gaussian apertures in part (b), explain how the overall shape and structure of your plot from part (c) changes.

You may find the following information useful:

$\text{gaus}(x) = e^{-\pi x^2}$

$\mathbf{F}_\xi[\text{gaus}(x)] = \text{gaus}(\xi)$

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Question 4

This problem involves a plane-wave $E_0 e^{ikz}$ propagating along the z-axis that impinges upon a screen at $z = 0$ with field transmission $t(x')$, resulting in a field just after the screen $E(x', z = 0) = E_0 t(x')$. Here we assume the screen and field are homogeneous along the y-axis and restrict the analysis to one transverse dimension. Then within the Fresnel approximation the field in the Fraunhofer region a distance L beyond the screen is given by

$$E(x, L) = \int_{-\infty}^{\infty} dx' E(x', 0) e^{-\frac{ikxx'}{L}},$$

where for simplicity a prefactor multiplying the integral has been omitted.

(a - 2pts) First consider the case of a single Gaussian aperture with transmission $t(x') = \exp(-[x' - x_0]^2/a^2)$, x_0 being the position of the aperture center and 'a' is a measure of the aperture width. Using the information above derive an expression for the diffracted field $E(x, L)$ in the Fraunhofer region. As a check you should find that the diffracted field includes a Gaussian envelope factor $\exp(-x^2/w^2)$, where $w = 2L/ka$.

(b - 2pts) Next consider the case with two identical Gaussian apertures separated by a distance d with transmission $t(x') = \exp(-[x' - d/2]^2/a^2) + \exp(-[x' + d/2]^2/a^2)$. Use your result from part (a) to obtain an expression for the resulting transverse intensity profile $|E(x, L)|^2$.

(c - 1pt) Using your result from part (b) obtain an expression for the spacing Δ between adjacent zeros in the transverse intensity profile.

(d - 3pts) Assuming $d >> a$ provide a sketch of $|E(x, L)|^2$ versus x indicating key features including the width of the transverse intensity profile and the positions of zeros in the intensity profile.

(e - 2pts) If identical slit apertures of width $a << d$ were used in place of the Gaussian apertures *explain* how the overall shape and structure of your plot from part (d) would change.

The following tabulated integral may be of use

$$\int_{-\infty}^{\infty} ds\ e^{-isq-bs^2} = \sqrt{\frac{\pi}{b}} e^{-q^2/4b}.$$

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Question 5

Answer the following questions, 2 points each.

a. Is the binding energy of exciton in a 2D semiconductor larger or smaller than that in 3D and by how much for the n=1 exciton?

b. Plot the 2D density of states and compare it with 3D.

c. Describe field-assisted transition or field-induced tunneling in a semiconductor. Plot the schematic of the absorption spectrum to describe your answer.

d. What is quantum confined Stark effect? Describe it by plotting the schematic absorption spectrum.

e. Describe the use of quantum confined Stark effect in an optical modulator.

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Question 5

Answer the following questions related to Maxwell's equations, the time-dependent wave equation, and sources of light. All parts equally weighted.

(a) Write Maxwell's 4 equations in differential form, identifying all scalar and vector quantities.

(b) Derive the time-dependent wave equation for **E** in vacuum. Do not assume a source free region of space. Show all steps.
(Hint: you may need the vector identity $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$)

(c) What equation does the Green's function associated with the wave equation in (b) satisfy? Explain (write equation) how it is used to solve defined-source problems.

(d) What are the criteria for fields to constitute propagating electromagnetic radiation (light)? Which source term(s) in (b) can give rise to light?

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Question 5

Consider the simplest model for lattice vibrations, namely the monatomic chain with atoms of mass M and springs with spring constant f. Write down the classical equations of motion for the displacements u_j and derive the dispersion relation.

(10 points)

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Question 6

Given an optical system with primary spherical aberration, you want to add defocus to improve the final image. The wavefront $W(\rho,\theta)$ from this system is given by

$$W(\rho,\theta) = W_{020}\rho^2 + W_{040}\rho^4 = W_{020}(x^2 + y^2) + W_{040}(x^2 + y^2)^2$$

where (ρ,θ) are the normalized pupil coordinates, θ is measured in the clockwise direction from the y axis, and $x = \rho\sin\theta$ and $y = \rho\cos\theta$. Show all work.

A. (3 points) Derive the expressions for the transverse ray error ε_x and ε_y in the x and y directions, respectively.

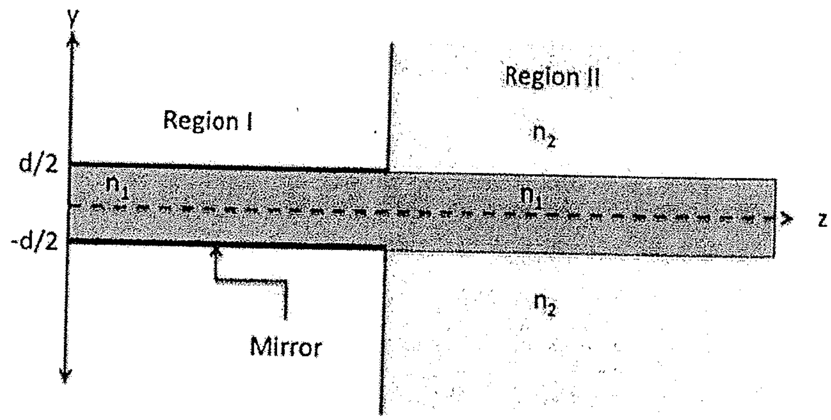
B. (3 points) What value of defocus W_{020} causes the marginal rays to come to focus?

C. (1 points) Determine the transverse ray error in the radial direction ε_ρ:

$$\varepsilon_\rho = \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$$

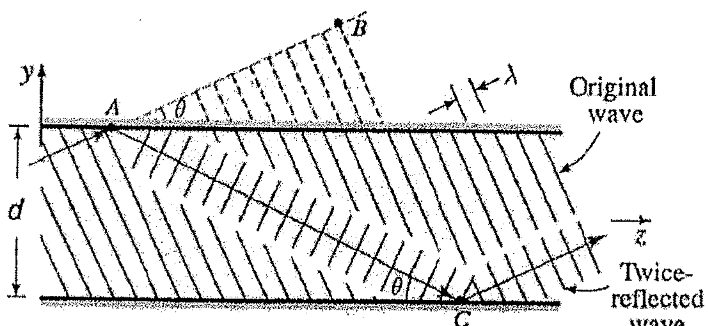
D. (3 points) For spherical aberration, derive the value of defocus W_{020} that minimizes the rms spot size:

$$\text{rms spot size} = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1 \varepsilon_\rho^2 \rho\, d\rho\, d\theta\right]^{1/2}$$



Mirror/dielectric slab waveguide transition problem - **R. A. Norwood**

Problem 1: Consider the slab waveguide structure above. There is a slab mirror waveguide in Region I and a dielectric slab waveguide in Region II. The cladding material in Region II can be considered to extend to infinity along the +/- **y** axis and along the positive **z** axis. The waveguides in both Region I and Region II can be taken to extend to infinity along the **x** axis. The mirror in Region I is taken to be perfectly reflecting and lossless. Light of free space wavelength λ_0 propagates in the positive z direction starting in the mirror waveguide (Region I), which is single-mode and TE polarized. In the dielectric waveguide region (Region II), the refractive index of the waveguide core is given by n_1 and that of the cladding is given by n_2 ($n_2 < n_1$) as indicated.



a) What is the self-consistency condition for the single TE mode of the mirror waveguide Region I in terms of the <u>material</u> wavelength λ ($=\lambda_0/n_1$), thickness d, mode angle θ^I, and phase shifts at the mirror interfaces? (3 points).

b) What is the self-consistency condition for the single TE mode of the dielectric waveguide in Region II in terms of the <u>material</u> wavelength λ, thickness d, mode angle θ^{II}, and ϕ, the phase shift on reflection at the dielectric interface? (3 points)

c) It is determined that the phase shift, ϕ, at the dielectric interfaces is π/2. The TIR condition at the interface provides the following relationship between ϕ and θ^{II}

$$\tan\frac{\phi}{2} = \sqrt{\frac{\sin^2 \bar{\theta}_c}{\sin^2 \theta''} - 1}$$

where $\bar{\theta}_c$ is the complement of the critical angle at the dielectric interface. Use this expression and the self-consistency condition from Part b) to derive an expression for n_2 in terms of n_1, λ, and d. (2 points)

d) The solution for the transverse electric field profile in Region I is $\psi_I(y)$ and the solution for the transverse electric field profile in Region II is $\psi_{II}(y)$. Write the expression for the fraction of the power propagating in Region I that continues to propagate in Region II in terms of $\psi_I(y)$ and $\psi_{II}(y)$. (2 points)

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Question 6

Given an optical system with primary spherical aberration, you want to add defocus to improve the final image. The wavefront $W(\rho,\theta)$ from this system is given by

$$W(\rho,\theta) = W_{020}\rho^2 + W_{040}\rho^4 = W_{020}(x^2 + y^2) + W_{040}(x^2 + y^2)^2$$

where (ρ,θ) are the normalized pupil coordinates, θ is measured in the clockwise direction from the y axis, and $x = \rho\sin\theta$ and $y = \rho\cos\theta$. Show all work.

A. (3 points) Derive the expressions for the transverse ray error ε_x and ε_y in the x and y directions, respectively.

B. (3 points) What value of defocus W_{020} causes the marginal rays to come to focus?

C. (1 points) Determine the transverse ray error in the radial direction ε_ρ:

$$\varepsilon_\rho = \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$$

D. (3 points) For spherical aberration, derive the value of defocus W_{020} that minimizes the rms spot size:

$$\text{rms spot size} = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1 \varepsilon_\rho^2 \rho \, d\rho \, d\theta\right]^{1/2}$$

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Question 6

An iPhone 3G camera has a lens with a focal length f=3.85 mm and f/# = 2.8. The image sensor (CMOS detector) has 1600 (vertical) x 1200 (horizontal) pixels with a pixel size of 2.2μm x 2.2μm.

a. (2 points) If an object 1 m from the front principal point of the lens is in focus on the sensor, where is the image located relative to the rear principal point of the lens?
b. (1 point) Assuming the lens is a thin lens with the stop at the lens, what is the numerical aperture (NA) of the beam in image space?
c. (1 point) Explain why this fixed focus camera with no focus adjustments can work well over a range of object distances from 0.5 m to infinity.
d. (1 points) What is the field of view of the camera for the object at 1 m distance?
e. (1 points) Assume this camera is a diffraction-limited imaging system. Based on the Rayleigh criterion, what is the minimum separation of two object points that can be resolved in the object plane at 1 m distance (For this problem you can assume a wavelength of 550 nm).
f. (2 point) Show a plot of the transfer function for this imaging system and calculate the value for the maximum spatial frequency that can be transferred to the image at the wavelength of 550 nm.
g. (2 points) Explain the concept of the Nyquist sampling theorem and determine whether or not the condition is met by the iPhone.

WRITTEN PRELIM EXAM – SECOND DAY

Fall 2010

September 29, 2010
8:30 a.m. to 12:00 p.m.

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$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$

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$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

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$\oint_C \mathbf{F} \cdot d\boldsymbol{\ell} = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

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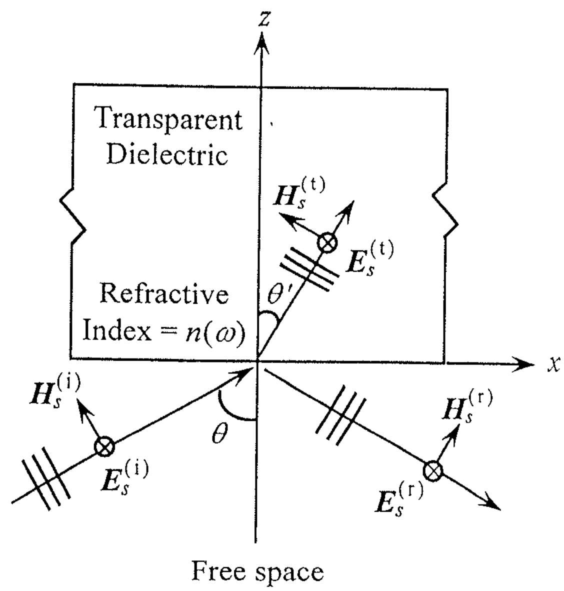
Question 7

System of units: MKSA

A monochromatic plane-wave of frequency ω arrives from free space at an angle of incidence θ at the flat surface of a transparent, semi-infinite, dielectric medium of refractive-index $n(\omega)=\sqrt{\varepsilon(\omega)}$. The plane-wave is linearly polarized in the s-direction, as shown in the figure, and the magnetic permeability of the dielectric is assumed to be unity, i.e., $\mu(\omega)=1$.



(4 pt) a) Write expressions for the incident, reflected, and transmitted plane-waves, identifying their respective k-vectors, E-fields, and H-fields. (The incident E-field is assumed to be known, but all the other E- and H-fields must eventually be related to the known parameters.)

(4 pts) b) Write the relevant boundary conditions and determine the Fresnel reflection and transmission coefficients, ρ_s and τ_s, for the s-polarized incident plane-wave.

(2 pts) c) Verify conservation of energy by calculating the rate-of-flow of energy per unit cross-sectional area per unit time for the incident, reflected, and transmitted beams.

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Question 8

Design a 4-f image relay system out of two 100 mm focal length thin lenses in air. This relay works at unit negative magnification and its working f-number is f/4. The object is a slide measuring 24 mm x 36 mm. The system is doubly telecentric.

Each part is worth 1 point.

1) Make a clear sketch of the system including the lenses, the object, the image, and the aperture stop, and the optical axis.
2) Determine the image distance as measured from the object plane.
3) Determine the distance between the lenses and the location of the pupils.
4) Name and determine the location of the six cardinal points.
5) Determine the size of the stop and of the lenses to have no vignetting.
6) Determine the size of the image.
7) Trace in your drawing the chief ray and the marginal ray and clearly mark them as chief and marginal rays.
8) What is the transverse magnification? What is the longitudinal magnification?
9) If the lens system (both lenses and stop) move 10 mm along the optical axis, how much does the image move relative to the object plane? (The object plane remains stationary).
10) If the lens system (both lenses and stop) is tilted 10 degrees (the lens optical axis is now intersecting the object plane at 10 degrees), how much is the image plane tilted in relation to the object plane?

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Question 9

A 2-level quantum system is realized by placing an electron at rest in a uniform and constant magnetic field oriented along the $\hat{z}$ axis (B_z). The Hamiltonian describing the energy of the electron is:

$$\hat{H} = -\gamma B_z \hat{S}_z \tag{1}$$

where γ is the gyromagnetic ratio. Suppose at time $t = 0$ the electron is in the superposition spin state:

$$\chi = a_1 \begin{pmatrix} 1 \\ 2-i \end{pmatrix}$$

This state vector is written in the basis of eigenstates of the $\hat{S}_z$ operator.

a. **(1 pt)** What is the spin quantum number of the electron?

b. **(1 pt)** Write the 2x2 matrix associated with the $\hat{S}_z$ operator.

c. **(1 pt)** What is the energy difference between the two energy eigenstates?

d. **(1 pt)** Solve for the normalization coefficient a_1.

e. **(1 pt)** What is the probability of measuring $+\hbar/2$ if a measurement of the angular momentum along the z direction is made?

f. **(1 pt)** Does your answer to part (e) above change if the measurement is instead performed at a later time, t? Explain your answer in a sentence or two.

g. **(2 pt)** Write an expression for the *time dependent* state vector, $\chi(t)$.

h. **(2 pt)** Evaluate $\langle S_z \rangle$ for the state $\chi(t)$

Consider an atomic medium consisting of four-level atoms, with the atomic energy levels labeled 1 through 4 in order of increasing energy. In this problem, we will consider the conditions for population inversion on the 4-2 transition only, in order to evaluate whether it might be possible to achieve lasing on this transition.

Denote the population density of atoms in levels j as N_j. In this medium, the atoms are "cascade pumped" with two separate pumping transitions: pumping of population density from level 1 to level 3 occurs at a rate P_A, and pumping from level 3 to level 4 occurs at a rate P_B. Suppose that there are also significant relaxation rates γ_{ji} between all levels j and i (these decay rates may be due to spontaneous radiative decay, collisions, etc.).

(a) [3 pts] Write the population density rate equations using only the pumping and decay processes defined above. Assume here and in the following parts that the optical gain on the lasing transition is below threshold.

(b) [5 pts] In terms of the relaxation rates, for what values of P_A and P_B can a steady-state population inversion occur between N_4 and N_2? That is, for what conditions is $N_4/N_2 > 1$? You do not need to derive the population density difference N_4-N_2 in terms of the total density of atoms.

(c) [2 pts] What is the relationship between γ_{21} and γ_{42} that must hold in order to have $N_4 > N_2$ for *any* positive values of the pumping rates and the other decay rates?

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Question 10

An extended, quasimonochromatic incoherent source is used to illuminate a grating with period d. The source is in the shape of a thin ring with radius R. Mathematically, the source radiance distribution is described as $m_R(r) = \delta(r - R)$. The grating is spaced a distance z_{src} from the source.

a.) (2.5pts) Calculate the spatial coherence visibility factor $\mu_{12}\left(\frac{\theta_{ph}}{\lambda}\right)$ at the grating. You may want to use the relationships shown below for the calculation.

b.) (2.5pts) Does rotation of the grating have any effect on visibility of the diffracted orders? Justify your answer.

c.) (2.5pts) With the parameters below, will diffracted orders from the grating be observed with $V > 0.2$, based on interference between adjacent openings in the grating? Justify your answer. You may want to use the graph of J_0 shown below.

$R = 0.5\text{mm}$

$\lambda = 532\text{nm}$

$d = 0.140\text{mm}$

$z_{src} = 100\text{mm}$

d.) (2.5pts) With the parameters below, will diffracted orders be observed from the grating with $V > 0.2$, based on interference between adjacent openings in the grating? Justify your answer. You may want to use the graph of J_0 shown below.

$R = 0.5\text{mm}$

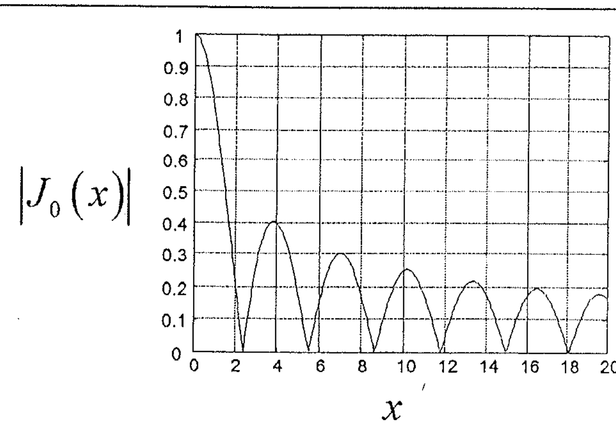
$\lambda = 532\text{nm}$

$d = 0.070\text{mm}$

$z_{src} = 100\text{mm}$

$$\mathrm{B}_\rho\left[g_R(r)\right] = 2\pi\int_0^\infty g_R(r) J_0(2\pi r\rho) r\,dr$$

$$\mathrm{B}_\rho\left[\delta(r-a)\right] = 2\pi a J_0(2\pi a\rho)$$



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Question 10

This problem involves propagation of light along the z-axis through a telescopic (afocal) optical system composed of a thin lens of focal length $f_1 > 0$ followed by a section of free-space of length $(f_1 + f_2)$, and finally a second thin lens of focal length $f_2 > 0$. The ray transfer matrices for free-space and a thin lens are

$$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

(a - 1pt) Write down the ray transform that relates the incident ray vector $\binom{x}{x'}_i$ to the final ray vector $\binom{x}{x'}_f$ across a general first-order optical system.

(b - 2pts) Calculate the ray transfer matrix for the telescopic optical system.

(c - 1pt) Using your result from part (b) demonstrate that the optical system is indeed telescopic and provide an expression for the magnification m of the optical system.

(d - 2pts) Consider the case $f_2 > f_1$ and incident rays propagating parallel to the z-axis. Sketch the telescopic optical system and include some ray paths through the optical system that elucidate its magnification properties, in particular the relation between the incident ray heights and slopes and the final ray heights and slopes.

(e - 2pt) The ABCD law for transformation of the complex beam parameter $Q = \frac{1}{R} + \frac{2i}{kw^2}$ across a first-order optical system is $Q_f = \frac{C+DQ_i}{A+BQ_i}$. Consider an incident collimated Gaussian beam with flat phase front and spot size w_i, and associated Rayleigh range $z_i = kw_i^2/2$. Assuming that $|m| >> (f_1 + f_2)/z_i$ derive expressions for the final inverse radius of curvature $(1/R_f)$ and the ratio (w_f/w_i) of the final and incident spot sizes.

(f - 2pts) Discuss the relation between your results from parts (d) and (e).

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Question 11

Consider a 3-dimenstional system of electrons in thermal equilibrium at temperature T. Assume there is only one two-fold degenerate energy band,

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

which is parabolic and isotropic ($k = |\vec{k}|$). The distribution function is given as

$$f(k) = \frac{1}{e^{\beta(\varepsilon_k - \mu)} + 1}$$

with $\beta = 1/k_B T$.

(a) What is the name of this distribution function? Without any proof or calculation, sketch the function vs. energy at T=0. Properly label the axes and all special points.

(3 points)

(b) Assume you know the temperature and electron density n. Write down a general equation that would allow you to compute the chemical potential.

(2 points)

(c) In the high temperature limit, $\beta\mu$ is negative and very large. Write down the corresponding distribution function. What is the name of this distribution function? Find an analytical expression of the density as function of temperature and chemical potential through direct integration of the relevant k-integral. Use the following definite integral:

$$\int_0^\infty dx\, x^2\, e^{-a^2 x^2} = \frac{\sqrt{\pi}}{4a^3}$$

(5 points)

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(5 points)

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Question 11

The goal of this problem is to show pictorially that you understand how a silicon photodiode works.

Label all sketches and graphs clearly, indicating any significant features.

The bandgap of silicon is 1.1 eV

All parts are equally weighted.

(a) Sketch the absorption coefficient of silicon vs. wavelength. What is meant by a cutoff wavelength and what is its numerical value?

(b) Sketch the dispersion relation (energy vs. one component of the wavevector), indicating the bandgap. Label the valence and conduction bands in this sketch.

(c) On the same scale as in part (b), indicate the dispersion relation for light and show how the optical transitions responsible for the absorption coefficient occur. Explain the distinction between direct and indirect bandgaps.

(d) Sketch the physical structure of a PN junction, indicating where donors and acceptors are present and where electrodes might be attached. Indicate the depletion region on this sketch and briefly explain what this term means.

(e) With the help of the sketch in part (d), explain what happens when light is absorbed in the depletion region. How is this absorption observed in an electrical circuit connected to the electrodes? Is there an observable effect from light absorbed outside the depletion region?

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Question 12

Step index fiber

(a) The lowest order propagating modes of a weakly guiding step index fiber are the LP_{01}, LP_{11}, and LP_{02} modes. Sketch the transverse intensity distribution of the three modes. (3 points)

(b) If the fiber has a core radius of 5 microns, a core index of 1.45 and refractive index change $\Delta = 0.003$, what is the shortest wavelength for which the fiber is a single mode waveguide? What is the numerical aperture of the fiber? (3 points)

(c) Explain why it is desirable to transmit signal using LP_{01} mode (2 points).

(d) The dispersion of the fiber at 1550nm is $D_\lambda = +17$ ps/km-nm. A Gaussian pulse has a time-bandwidth product given by $\Delta f_p \tau_p \approx 0.44$, where Δf_p is the pulse bandwidth in Hz and $\tau_p = 1$ps is the pulse width in time. The pulse propagates over a distance L and the pulse width increases to 5ps. What is L? (2 points)

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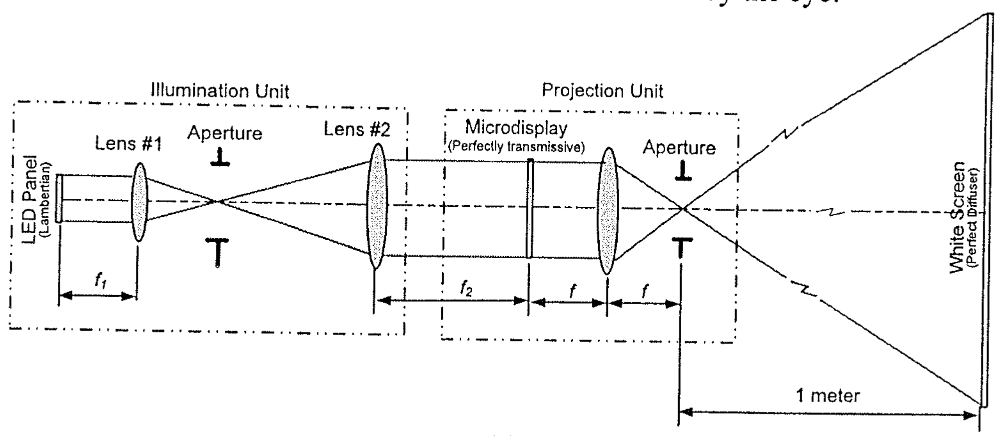
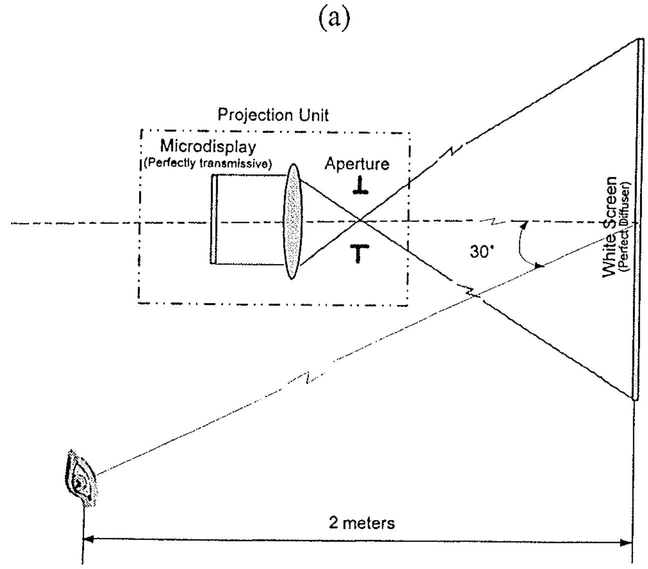
Question 12

Figure (a) illustrates a design of a pico projector, which consists of two major optical units: an illumination unit and a projection unit. In the illumination unit, a small LED panel (0.25 cm^2 area) in combination with a double-telecentric illumination optics is used to illuminate a microdisplay panel (1 cm^2 area). The LED panel is assumed to be a perfect Lambertian surface having a radiance of 1×10^5 $W/(m^2 sr)$. The microdisplay panel (1 cm^2 area) is assumed to be perfectly transmissive (i.e. with a uniform transmittance of 1.0). A projection screen, which is assumed to be a Lambertian surface, is placed at a 1-meter distance to receive the image. The projection optics is telecentric in the microdisplay space and has a working F/# of 2.0 and the projected image area is 0.25 m^2 at a distance of 1 meter. For the sake of simplicity, circular apertures are assumed for all the components of the system.

(1) (2 points) Determine the throughput of the projection optics;
(2) (3 points) Determine the adequate numerical aperture of the two lens elements in the double-telecentric illumination unit in order to match the throughput of the projection optics;
(3) (2 points) Calculate the total radiant flux of the pico projector and the irradiance of the projected image.
(4) (3 points) A person views the projected image at a distance of 2 meters and from an angle of 30 degrees relative to the projection direction, as illustrated in Fig. 1(b). Let us assume a 3-mm pupil dilation. Estimate the total radiant flux received by the eye.



(a)



(b)

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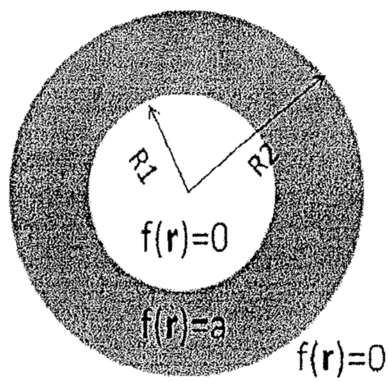
Question 12

Answer the following questions related to the 2D Radon Transform, the fundamental concept behind imaging techniques such as X-ray computed tomography, SPECT, and PET. All parts weighted equally.

(a) Write the equation for the 2D Radon transform, illustrate all variables on a diagram, and describe in words what it accomplishes.

(b) Compute the 2D Radon transform for an arbitrary projection angle ϕ for the annular object as shown in the figure below. Assume that the value of the property being imaged is constant with value a inside the annular region and zero everywhere outside, including in the center.



(c) Sketch the result as accurately as you can. Label axes.

(d) Using diagrams and words, explain the *central slice theorem* and its value in tomographic reconstruction.

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Question 12

Equal amounts of two illuminants with the following chromaticity coordinates are combined.

$x_1 = 0.1$ $y_1 = 0.3$

$x_2 = 0.4$ $y_2 = 0.5$

$L_{V1} = L_{V2}$

What are the chromaticity coordinates of the resulting color?