WRITTEN PRELIM EXAM – FIRST DAY

Spring 2011

February 15, 2011
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}$ J · s $= 4.134 \times 10^{-15}$ eV · s

$e = 1.6 \times 10^{-19}$ C

$c = 3.0 \times 10^{8}$ m/s

$k_B = 1.38 \times 10^{-23}$ J/K

$\sigma = 5.67 \times 10^{-8}$ W/K^4 · m^2

$\epsilon_0 = 8.85 \times 10^{-12}$ F/m

$\mu_0 = 1.26 \times 10^{-6}$ H/m

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

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$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

Spring 2011 Comprehensive Exam
Opti 501
Question 1

System of units: MKSA

In this problem you are asked to describe and explain Maxwell's macroscopic equations in their most general differential form, and also to describe the related boundary conditions. Explain (in words) the meaning of the various parameters, operators, fields, and sources that appear in these equations. **Be brief but precise.**

(3 pts) a) Explain the meaning of the various terms in Maxwell's 1st equation, $\nabla \cdot \boldsymbol{D}(\boldsymbol{r},t) = \rho_{\text{free}}(\boldsymbol{r},t)$. How is the displacement field related to the permittivity of free space, ε_o, to the E-field, and to the polarization $\boldsymbol{P}(\boldsymbol{r},t)$? What boundary condition is derived from Maxwell's 1st equation? What does this boundary condition describe?

(4 pts) b) Explain the various terms in Maxwell's 2nd equation, $\nabla \times \boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{J}_{\text{free}}(\boldsymbol{r},t) + \partial \boldsymbol{D}(\boldsymbol{r},t)/\partial t$. What boundary condition is derived from this equation, and what does the boundary condition imply for the fields and sources at and around the boundary?

(2 pts) c) Explain the various terms in Maxwell's 3rd equation, $\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\partial \boldsymbol{B}(\boldsymbol{r},t)/\partial t$. How is the B-field related to the permeability of free space, μ_o, to the H-field, and to the magnetization $\boldsymbol{M}(\boldsymbol{r},t)$? What boundary condition is derived from Maxwell's 3rd equation, and what does the boundary condition imply for the fields in the immediate vicinity of the boundary?

(1 pt) d) Explain the various terms in Maxwell's 4th equation, $\nabla \cdot \boldsymbol{B}(\boldsymbol{r},t) = 0$. What boundary condition is derived from this equation, and what does this boundary condition imply for the fields in the immediate vicinity of the boundary?

Spring 2011 Comprehensive Exam
Opti 502
Question 2

Problem in paraxial optics
(Each part worth 1 point, except parts g and h which are worth 2 points each)

A three thin-lens optical system in air is telecentric in object space and should also be telecentric in image space. The lens focal lengths are $f_1 = 60$ mm, $f_2 = -120$ mm, and $f_3 = 480$ mm. The object is circular with an object semidiameter (chief ray height) $\bar{y}_0 = 1.5$ mm. The paraxial marginal ray angle in object space is $u_0 = 0.2$. The first lens is located at a distance $t_0 = 90$ mm from the object. The second lens is located $t_1 = 60$ mm from lens 1. The paraxial image is formed following lens 3.

a. What is the minimum lens diameter d_1 for lens 1 which avoids vignetting?
b. What is the minimum lens diameter d_2 for lens 2 which avoids vignetting?
c. Determine t_2, the separation between lenses 2 and 3, such that the system is telecentric in image space?
d. What are the paraxial image height and the transverse magnification?
e. What is the marginal ray angle u_3 in image space?
f. What is the overall length from object to image?

After the system is built and mounted, the system requirements change. The new specifications require that the magnification be decreased to one half of the initial magnification. However, since the system has already been built, the position of the image plane is no longer easily changed. To make minimal optical system changes, the optical designer (you) decides to only move lens three and change its focal length, keeping all other lenses and their positions the same. As a result, the image space cannot remain telecentric. The marginal and chief rays remain the same from the object through lenses 1 and 2, and in the space after lens 2. The paraxial image location is unchanged but the new transverse magnification is one half its original value.

g. (2 points) What is the new marginal ray slope u_{new3} in image space?
h. (2 points) Find a new focal length f_{new3} and position t_{new2} for lens 3 (spacing between lens 2 and 3) which halves the transverse magnification while keeping the overall length fixed.

Spring 2011 Comprehensive Exam
Opti 511
Question 3

Suppose a hydrogen atom is initially in its ground state Ψ_{100}. At time $t = 0$ a monochromatic laser field of the form $E = \frac{1}{2}\hat{z}E_o e^{-i\omega t} + c.c.$ is turned on. The laser frequency is near resonance with the $n = 1$ to $n = 2$ transition $\omega_o \equiv (E_2 - E_1)/\hbar$, with detuning $\Delta = \omega - \omega_o$, where $\Delta \ll \omega_o$. In this problem, assume the following: (1) spontaneous emission can be neglected; (2) the 2-level atom, electric dipole and rotating wave approximations are all valid.

a. (1 pt) To what specific excited energy eigenstate(s), Ψ_{nlm}, can a transition occur?

b. (1 pt) If the atom is found in this excited state after the field is shut off, what would we measure for (1) the z-component of the electron's orbital angular momentum, and (2) the *magnitude* of the electron's orbital angular momentum? Give numerical answers in units of $\hbar$ (the fundamental unit of angular momentum).

c. (2 pt) Write out expressions that define the on-resonance Rabi frequency Ω_o, and the generalized Rabi frequency Ω, in terms of the dipole matrix element $\wp$ and any other terms already defined above.

d. (2 pt) Write the expression for the probability to find the atom in the energy eigenstate $n = 2$ as a function of time in terms of the Rabi frequencies.

e. (3 pt) If $\Delta = 0$, calculate the earliest time in which we can be *certain* to find the atom in the excited state $n = 2$ if the field strength $E_o = (\frac{3\pi\hbar}{ea_o}) \times 10^8 \approx 11.6 \times 10^3\ [\frac{J}{C\cdot m}]$. Provide a numeric answer. See the bottom of this page for information you may find useful.

f. (1 pt) This semi-classical picture of light-matter interaction does not predict spontaneous emission to occur from the excited state after the laser field is turned off. In a brief sentence or two, describe what is missing in the semi-classical theory that is needed to account for the phenomena of spontaneous emission.

Below are useful expressions for calculating the dipole matrix element $\wp = -e\langle\psi_b|\vec{r}|\psi_a\rangle \cdot \hat{\epsilon}$, where $\hat{\epsilon}$ is the laser polarization and e is the electron charge, and a_o used below is the Bohr radius ($0.5\times10^{-9}[m]$).

$\langle l = 0,\ m = 0\ |\hat{\mathbf{r}}|\ l = 1,\ m = 0\rangle = (0,\ 0,\ \sqrt{\tfrac{1}{3}})$

$\langle l = 0,\ m = 0\ |\hat{\mathbf{r}}|\ l = 1,\ m = 1\rangle = (-\sqrt{\tfrac{1}{6}},\ -i\sqrt{\tfrac{1}{6}},\ 0)$

$\langle l = 0,\ m = 0\ |\hat{\mathbf{r}}|\ l = 1,\ m = -1\rangle = (\sqrt{\tfrac{1}{6}},\ -i\sqrt{\tfrac{1}{6}},\ 0)$

$\langle n = 1,\ l = 0\ |r|\ n = 2,\ l = 1\rangle = 1.29a_0$

$\langle n = 1,\ l = 0\ |r|\ n = 3,\ l = 1\rangle = 0.517a_0$

$\langle n = 2,\ l = 0\ |r|\ n = 3,\ l = 1\rangle = 3.07a_0$

$\langle n = 2,\ l = 1\ |r|\ n = 3,\ l = 0\rangle = 0.95a_0$

Spring 2011 Comprehensive Exam
Opti 544
Question 3

In this problem we consider classical and semiclassical, 2-level models for the interaction between a gas of atoms with resonance frequency ω_0 and a monochromatic plane wave with frequency ω.

For sufficiently low values of the atom density N, the electron oscillator (Lorentz atom) model predicts the following dependence of the real and imaginary indices of refration:

$$n_R(\Delta) = 1 + \tfrac{1}{2}\eta\, g_R(\Delta), \qquad n_I(\Delta) = \eta\, g_I(\Delta),$$

where the detuning $\Delta = \omega_0 - \omega$, and the functions $g_R(\Delta)$ and $g_I(\Delta)$ are normalized to have maximum values of one.

(a) Write down expressions for $g_R(\Delta)$ and $g_I(\Delta)$. (2pts)

(b) Write down the relationship between the complex k and ω for the medium. Use this to find an expression for the absorption coefficient $a(\Delta)$ (the rate of decay of the intensity of the plave wave). (2pts)

Now consider a quantum mechanical, 2-level description of the atoms.

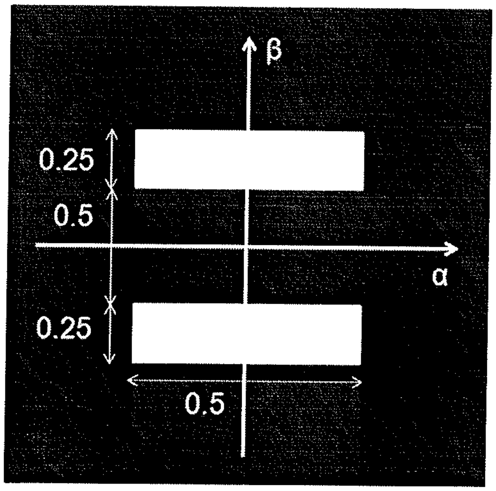
(c) Under what conditions do we expect the electron oscillator and quantum 2-level descriptions to give identical results for $n_R(\Delta)$ and $n_I(\Delta)$? (2pts)

(d) In the limit identified in (d), write down an expression for the frequency dependent photon scattering cross section $\sigma(\Delta)$, and for the corresponding absorption coefficient $a(\Delta)$. Based on this, find an expression for the constant η that depends only on the transition wavelength and the density N. (2pts)

(e) Compute the maximum value of $n_R(\Delta)$ for a gas of ultracold atoms with $\lambda = 589nm$, $N = 10^{13}cm^{-3}$ (typical for a Na Bose-Einstein condensate). Use the low density approximation and assume there is no Dobbler broadening. (2pts)

Spring 2011 Comprehensive Exam
Opti 505
Question 4

The aperture below is used in the entrance pupil of an optical system that is used to image a self-luminous object. The white rectangular areas are open parts of the aperture. α and β are direction cosines in the entrance pupil.



a) (5pts) Sketch OTF(ξ,0) and OTF(0,η), where $\xi = \alpha/\lambda$ and $\eta = \beta/\lambda$. Let $\lambda = 1\,\mu m$.

b) (5pts) Polarizers are placed over the apertures such that transmission through the upper rectangle is polarized along β and transmission through the lower rectangle is polarized along α. Sketch OTF(ξ,0) and OTF(0,η).

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Opti 546
Question 4

(a - 2pts) In the treatment of the Hermite-Gaussian modes of free-space the terminology TEM_{mn} is employed to designate the modes. In this notation what do the letters TEM stand for, and what can be inferred about spatial structure of the mode labeled with the subscripts (m, n).

(b - 2pts) Sketch the form of the scaled Gaussian beam spot size $(w(z)/w_0)$ and scaled inverse radius of curvature $(z_0/R(z))$ versus scaled propagation distance in the range $-2 < (z/z_0) < 2$ around the beam focus at $z = 0$, making sure to indicate key features. Here w_0 is the focused Gaussian spot size and z_0 the corresponding Rayleigh range.

(c - 1pt) List two distinct optical systems, that are not trivially related, whose output transverse intensity profile is proportional to the modulus squared of the Fourier transform of the input beam.

(d - 2pts) Sketch the layout of a Mach-Zehnder interferometer and state whether it is based on division of amplitude or wavefont.

(e - 1pt) What roles do the transverse and longitudinal electric fields play in the description of a system of atoms interacting with the electromagnetic field?

(f - 2pts) Provide sketches of the characteristic arrangement of the incident optical field wavevector, acoustic wavevector, and the scattered field components for both Bragg diffraction and Raman-Nath diffraction.

Spring 2011 Comprehensive Exam
Opti 507
Question 5

Consider a quantum well with infinite potential barriers. To be specific, consider only the lowest state with wave function

$$\xi(z) = A\cos(k_z z)$$

and the quantum well being extending from $-L_z/2$ to $+L_z/2$.

(a) Write down the boundary conditions for the wave function.

(2 points)

(b) Using

$$\varepsilon = \frac{\hbar^2 k_z^2}{2m}$$

with $m = 0.1 m_0$ (m_0 = electron mass in vacuum) determine the quantum well thickness L_z required for a quantum confinement shift of 200 meV. (Note: use $\hbar^2 / m_0 = 7.62\times10^{-16}$ eV cm^2.)

(8 points)

Spring 2011 Comprehensive Exam
Opti 537
Question 5

Answer the following questions related to semiconductor detectors. All parts weighted equally.

(a) What is the definition of a Bravais lattice? What is a primitive lattice vector? Write an expression for a translation vector that spans the equivalent locations in a real-space Bravais lattice.

(b) Write the 3D time-independent Shrödinger equation for a single electron in a Bravais lattice. Show where periodicity is expressed in the Hamiltonian and explain what assumptions are necessary to invoke the time-independent form of the Shrödinger equation to describe the electronic structure and related properties of crystals.

(c) What is the Born-von Karman boundary condition for a simple orthorhombic crystal ($|\mathbf{a}| \neq |\mathbf{b}| \neq |\mathbf{c}|, \alpha = \beta = \gamma = 90^{\circ}$,) if there are $L \times M \times N$ copies of the unit cell in the **a**, **b**, and **c** directions respectively? Why is it invoked?

(d) What is Bloch's theorem? Show that the application of Born-von Karman boundary condition (for example for the case above) in combination with Bloch's theorem leads to a discrete set of possible **k** (momentum) values for the electrons to occupy.

(e) Sketch a one dimensional band diagram for the nearly-free electron model and compare it with the free electron curve (hint: x-axis units are momentum, y-axis units are energy). Point out the bandgaps and explain why they arise.

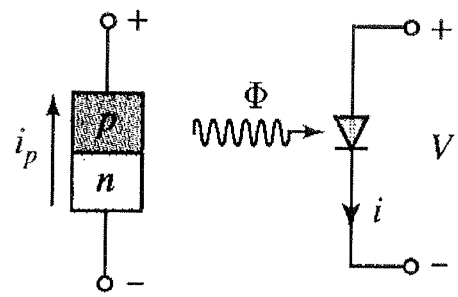
(f) What is the definition of the Fermi level (energy)? Write the expression for the Fermi-Dirac distribution and state what it means. Use a sketch to show why pure Si and other intrinsic semiconductors are insulators at absolute zero, but have modest conductivity at room temperature.

(g) Explain why donor and acceptor impurities are added to Si. Sketch the location (in an *energy versus position* band diagram) of donor and acceptor dopant states relative to the conduction and valence bands and indicate where the Fermi levels are for N- and P-type Silicon.

(h) Draw the band structure of the PN junction, under conditions of a) no bias, b) forward bias and c) reverse bias. Which are useful for photodetection?

Spring 2011 Comprehensive Exam
Opti 510
Question 6

Problem 1: Semiconductor photodiode and fiber optics link



(a) A photodetector is made of a semiconductor p-n junction. Sketch the ideal voltage and current characteristic (i versus V) of the p-n junction under no light illumination and some light illumination. (2 points)
(b) Describe the differences between a homojunction and a heterojunction. (1 points)
(c) The detector is actually a p-i-n junction. What are the advantages of a p-i-n junction over a p-n junction? (2 points)
(d) The photodetector responsivity is given by 0.5A/W. If the wavelength is 1.55 micron, and the minimum detectable sensitivity is 10^{-9} A, what is the photodiode sensitivity in dBm? (2 points)
(e) The detector is used in a 10 km optical fiber link with attenuation coefficient of 0.2dB/km. We can assume the total connectors and coupling loss to be 6 dB and a safety margin of 6 dB, what is the minimum power (W) of the source laser, so that we can use this detector to detect the transmitted signals. You can assume an attenuation-limited system. (3 points)

Spring 2011 Comprehensive Exam
Opti 503
Question 6

For the wavefront error $W = W_{040}\rho^4$:

A. (1 points) What is the peak-to-valley wavefront error?

B. (4 points) Show that the wavefront variance $\sigma_w^2 = \frac{4}{45} W_{040}^2$.

C. (2 points) Based on this result, what is the approximate Strehl ratio?

D. (1 points) The Maréchal criterion says that a system with $\sigma_w < \frac{\lambda}{14}$ can be considered to be diffraction limited. What value of W_{040} does this correspond to?

E. (2 points) How does the value of W_{040} calculated in part D compare to Rayleigh's criterion for a diffraction limited system?

Spring 2011 Comprehensive Exam
Opti 509
Question 6

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E. (2 points) How does the value of W_{040} calculated in part D compare to Rayleigh's criterion for a diffraction limited system?

Spring 2011 Comprehensive Exam
Opti 536
Question 6

Pick one of the following tomographic imaging systems:

PET

SPECT

MRI

CT

OCT

Answer the following questions for the chosen system:

a. (1 point) What do the initials stand for?

b. (1 point) What physical quantity is represented in the image?

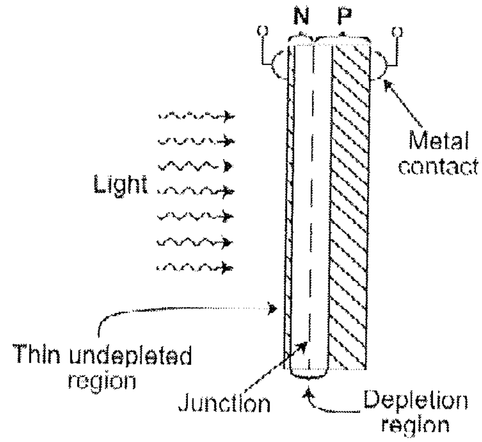
c. (1 points) What kind of detector is used? Briefly, how does the detector work?

d. (2 points) How is the detector signal related to the physical quantity being imaged? Use equations or sketches as needed in this part.

e. (2 points) What are the important noise sources? What measures can be used to improve the signal-to-noise ratio in the raw data?

f. (3 points) How are the data processed in order to go from the detector signals to a reconstruction of the physical quantity of interest? Give a specific example of a useful reconstruction algorithm, stating the mathematical equation on which it is based. Define all symbols used.

Spring 2011 Comprehensive Exam
Opti 506
Question 6



The figure above shows a silicon photodiode. The thin undepleted region is made of silicon also. The refractive index of silicon can be taken as n = 3.4 and the bandgap is E_g = 1.1 eV. All parts are equally weighted.

a. Explain what happens when light of wavelength 500 nm is incident on the diode. In what regions can the light be absorbed?

b. Explain what happens when light of wavelength 5 μm is incident on the diode. In what regions can the light the absorbed in this case?

c. How does an optical photon absorbed in the depletion region result in a current in the diode

d. Sketch an appropriate circuit using an operational amplifier to sense the photocurrent; explain briefly how it works.

e. Sketch the responsivity of an ideal silicon diode (in A/W) as a function of wavelength. Compute the peak responsivity and indicate it on your sketch.

WRITTEN PRELIM EXAM – SECOND DAY

Spring 2011

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$\cos 2A = 2 \cos^2 A - 1$

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$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

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$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Spring 2011 Comprehensive Exam
Opti 501
Question 7

System of units: MKSA

An electromagnetic plane-wave of frequency ω resides in free space. There are no free charges and free currents, nor are there any polarization or magnetization in this space, i.e., $\rho_{\text{free}}(\boldsymbol{r},t)=0$, $\boldsymbol{J}_{\text{free}}(\boldsymbol{r},t)=0$, $\boldsymbol{P}(\boldsymbol{r},t)=0$, and $\boldsymbol{M}(\boldsymbol{r},t)=0$. In general the plane-wave is elliptically polarized.

(2 pts) a) Write an expression for the E-field distribution in space and time. Identify the propagation direction (in terms of the k-vector), and the field amplitude. Specify the units of k, ω, and the E-field.

(1 pt) b) What property of the E-field distribution in part (a) distinguishes a linearly-polarized plane-wave from one that is circularly or elliptically polarized?

(1 pt) c) Write an expression for the H-field distribution in space and time. Specify the units of the H-field.

(1 pt) d) What constraint does Maxwell's 1st equation, $\nabla\cdot\boldsymbol{D}(\boldsymbol{r},t)=0$, impose on the various parameters of this plane-wave?

(1 pt) e) What constraint does Maxwell's 2nd equation, $\nabla\times\boldsymbol{H}(\boldsymbol{r},t)=\partial\boldsymbol{D}(\boldsymbol{r},t)/\partial t$, impose on the various parameters of this plane-wave?

(1 pt) f) What constraint does Maxwell's 3rd equation, $\nabla\times\boldsymbol{E}(\boldsymbol{r},t)=-\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$, impose on the various parameters of this plane-wave?

(1 pt) g) What constraint does Maxwell's 4th equation, $\nabla\cdot\boldsymbol{B}(\boldsymbol{r},t)=0$, impose on the various parameters of this plane-wave?

(2 pts) h) Combining the various constraints obtained in parts (d)-(g) above, find the *dispersion relation* in vacuum, which is the relationship among $\boldsymbol{k}$, ω, and the vacuum speed of light, c.

Spring 2011 Comprehensive Exam
Opti 502
Question 8

An optical system consists of two thin lenses in air. The second thin lens has a focal length of +20 mm. The first thin lens is placed at the front focal point of the second thin lens. Light travels from left to right and the object is at infinity. There are nine parts into this problem each worth different points as indicated.

a) Make a neat drawing of the optical axis and both thin lenses. (0.5 points)
b) What are the units of optical power? (0.5 points)
c) What should the optical power of the first lens be to produce a longitudinal focus shift of +1 mm with respect to the rear focal plane of the second lens (the focus will be farther away from the second lens)? (3 points)

From now on assume that the focal length of the first lens is -20 mm. The Lagrange invariant for the complete thin lens combination is 1.0. The object remains at infinity.

d) What is the optical power of the combination of the two thin lenses? (2 points)
e) Locate a stop aperture as to make lens combination telecentric in image space. Add the stop to the drawing in a). (0.5 points)
f) Assume the aperture stop to have a diameter of 2 mm. What is the f/# of the lens combination? (1 point)
g) What is the formula that relates the radius of curvature of the two surfaces of a thin lens in air to the lens focal length? (0.5 points)
h) What is the image height? (1 point)
i) Neatly draw the system layout showing the stop aperture, the marginal ray, the chief ray and the location of the image plane. (1 point)

Spring 2011 Comprehensive Exam
Opti 544
Question 9

Consider two non-interacting and distinguishable photons, each of which can be in an arbitrary coherent superposition of x and y polarized states. We define a polarization observable S_j for photon j as $S_j = |x_j\rangle\langle x_j| - |y_j\rangle\langle y_j|$, where $j \in \{1, 2\}$. $|x_j\rangle$, $|y_j\rangle$ are the x and y polarized states, respectively, with $\langle x_j|y_j\rangle = 0$.

A basis for the two-photon state space is defined by the following state vectors:
$|x_1\ x_2\rangle$ (both photons are x-polarized)
$|x_1\ y_2\rangle$ (photon 1 is x-polarized, photon 2 is y-polarized)
$|y_1\ x_2\rangle$ (photon 1 is y-polarized, photon 2 is x-polarized)
$|y_1\ y_2\rangle$ (both photons are y-polarized).

We now define the state $|\varphi\rangle$ as

$$|\varphi\rangle = \frac{1}{2}\,|x_1\ x_2\rangle + \frac{1}{2}|x_1\ y_2\rangle + \frac{1}{\sqrt{2}}|y_1\ y_2\rangle\,.$$

(a) 2 pts. If the two-photon system is in state $|\varphi\rangle$, what is the probability of finding photon 2 to be x-polarized? What is the probability of finding photon 2 to be y-polarized?

(b) 2 pts. If the system is initially in state $|\varphi\rangle$ and then a measurement of the polarization of photon 2 returns the result that this photon is y-polarized, what is the normalized state vector of the two-photon system immediately after this measurement?

(c) 5 pts. Suppose that at time $t = 0$ the system is in state $|\psi(0)\rangle = |\varphi\rangle$. The system then evolves under the Hamiltonian

$$H = \epsilon_1 S_1 \ + \ \epsilon_2 S_2.$$

where ϵ_1 and ϵ_2 are constants. Write an expression for $|\psi(t)\rangle$, the state of the system at time t. Then calculate the probability $P_\beta(t)$ that at time t the system would be found in the state $|\beta\rangle$ defined by

$$|\beta\rangle = \frac{1}{\sqrt{2}}\,|x_1\ x_2\rangle + \frac{1}{\sqrt{2}}|y_1\ y_2\rangle\,.$$

Simplify your final answer as much as possible.

(d) 1 pt. Plot the function $P_\beta(t)$ that you calculated in part (c).

Spring 2011 Comprehensive Exam
Opti 511
Question 9

Consider a particle of mass m in a 1-dimensional potential of the form $V(x) = \frac{1}{2}m\omega^2 x^2$, where ω is the characteristic frequency of the harmonic potential. At time $t = 0$ it is known that the wavefunction for the particle is given by: $\Psi(x,0) = \frac{1}{\sqrt{3}}\psi_0 + \frac{1}{\sqrt{2}}e^{i\pi}\psi_2 + \alpha\psi_5$, where ψ_n are the energy eigenfunctions for this potential and α is a real and positive constant.

a. (1 pt) Calculate the numerical value for α.

b. (2 pt) Write out the wavefunction for the particle at some later time t in terms of the energy eigenstates and ω.

c. (2 pt) If a measurement of the particle's energy is made at some time $t > 0$, what are the possible results of the measurement and the probability for each? Express your answer in terms of ω.

Assume instead that the particle is now in the ground state of the potential well. It's wavefunction is therefore given by: $\psi_o = (\frac{m\omega}{\pi\hbar})^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$. Now suppose at some time t_1 the strength of the potential well *instantaneously* increases ($V(x) \to V'(x)$)such that the new characteristic frequency of the well is now $\omega' = 2\omega$. The wavefunction for the particle in this new potential is *initially* unchanged, such that at time t_1 the wavefunction can still be written: $\Psi(x,t_1) = (\frac{m\omega}{\pi\hbar})^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$.

d. (1 pt) Is the particle in a stationary state of the new potential well $V'(x)$?

e. (2 pt) What is the probability that a measurement of the particle's energy would now return the value of $\hbar\omega/2$?

f. (2 pt) Calculate the probability that a measurement of the particle's energy would now return the value $\hbar\omega$.

Hint for part (f): $\sqrt{\pi} = \int_{-\infty}^{+\infty} e^{-u^2} du$

Spring 2011 Comprehensive Exam
Opti 546
Question 10

This question is related to electro-optics in a uniaxial crystal with point group $\bar{4}2m$ with a DC electric field E_{DC} applied along the Z-axis, for which the index ellipsoid may be written as

$$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} + 2r_{63}E_{DC}XY = 1, \tag{1}$$

(X, Y, Z) being the principal axes system defined with respect to the orthonormal basis vectors $\vec{X}, \vec{Y}$, and $\vec{Z}$.

(a - 2pts) Go through the explicit calculation to transform the index ellipsoid to the new axes system (x, y, z) defined with respect to the orthonormal basis vectors $\vec{x}, \vec{y}$, and $\vec{z}$, where $Z = z, X = (x-y)/\sqrt{2}, Y = (x+y)/\sqrt{2}$.

(b - 3pts) Show that with respect to the new axes system (x, y, z) the crystal is effectively biaxial in the presence of the DC electric field, and evaluate the principal refractive-indices n_x, n_y, and n_z (you may assume $|n_0^2 r_{63} E_{DC}| << 1$).

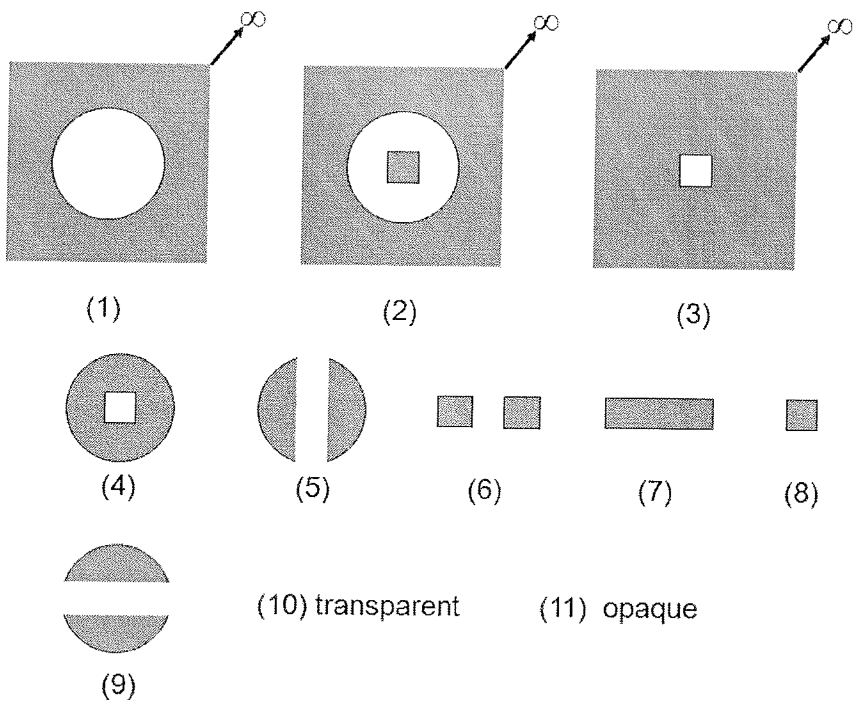
(c - 1pt) Explain whether the electro-optic effect under consideration is the Pockels effect or the Kerr effect.

(d - 3pts) For propagation along the z-axis in a crystal of length L, electro-optic coefficient $r_{63} = 26\times10^{-12}$ m/V, $n_o = 1.5$, and a free-space wavelength of $\lambda = 800$ nm, calculate the minimum voltage $V = |E_{DC}|L$ that needs to be applied along the z-axis of the crystal to realize a half-wave plate.

(e - 1pt) Is it possible to make an electro-optic half-wave plate as in part (d) using an isotropic crystal?

Spring 2011 Comprehensive Exam
Opti 505
Question 10

The diffracted amplitude and phase from each aperture shown below is measured at a distance z from the aperture. The field measurements are denoted $f_i(x,y)$, where $i = 1, 2, \ldots 11$. Find four combinations of apertures such that $f_i(x,y) = f_j(x,y) + f_k(x,y)$, where $i \neq j \neq k$. Assume identical measurement conditions for each aperture. (Shading indicates an opaque region.)



$f_i(x,y) = f_j(x,y) + f_k(x,y)$

(2.5 pts for each correct answer)

i	j	k

Spring 2011 Comprehensive Exam
Opti 537
Question 11

Answer the following questions related to radiative transport. All parts weighted equally.

The radiative transport equation (RTE), or Bolzmann equation, describes the time evolution of a *phase-space distribution function* w in terms of four processes: absorption, emission, propagation, and scatter

$$\frac{dw}{dt} = \left[\frac{\partial w}{\partial t}\right]_{abs} + \left[\frac{\partial w}{\partial t}\right]_{emiss} + \left[\frac{\partial w}{\partial t}\right]_{prop} + \left[\frac{\partial w}{\partial t}\right]_{scat}$$

(a) What are the variables that w is a function of, and what are its units? What does it describe?

(b) What is the relationship between the distribution function w and spectral photon radiance $L_{p,E}$?

(c) In class we derived forms for each of the terms in the transport equation and wrote an overall spatio-temporal-integro-differential transport equation of the form

$$\frac{dw}{dt} = -c_m\left(\mu_{abs} + \mu_{scat}\right)w + \Xi_{p,E} - c_m\hat{\mathbf{s}}\cdot\nabla w + \boldsymbol{K}w$$

where $\boldsymbol{K}$ is an integral operator representing the *scatter-in* process. Show, by considering a steady state solution in a homogeneous, source-free medium with no absorption or scatter, that we can recover the optics result that *radiance is constant along a ray*.

(d) Now consider a steady state situation with propagation and absorption in a homogeneous medium, but no source or scatter (in or out). If we know the distribution function at $z = 0$, and we choose propagation direction $\hat{\mathbf{s}}$ to point along the positive z-axis, what is the value of the distribution function at some arbitrary location $z = z'$ along the positive z-axis.

(e) What name is commonly given to the relationship found in d ?

Spring 2011 Comprehensive Exam
Opti 507
Question 11

The figure below shows the phonon dispersion of a 3-dimensional cubic crystal. The horizontal axis is the wave vector in units of $k_R = 2\pi / a$, where a is the lattice constant. The vertical axis is the frequency ν in units of Hz (note that the angular frequency is $2\pi\nu$).

(a) Consider the left-most segment of the figure, labeled $(\xi, 0, 0)$. Indicate the longitudinal acoustic (LA) and transverse acoustic (TA) branch(es). Give a brief justification for your answer. Instead of indicating it directly in the figure, you can just draw that segment on your own paper and indicate the LA and TA branches there.

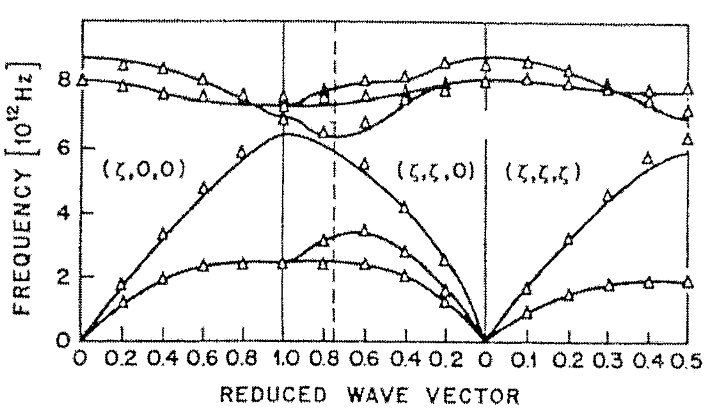
(3 points)

(b) Give an approximate estimate of the phonon energy $\hbar\omega$ (in units of meV) at which infrared absorption can be expected to be strong. Indicate clearly in the figure (or your own drawing of the figure) the point that you are using for the estimate. Note: you cannot deduce an exact number from the figure, but you can, and should, be correct to within approximately 10%. ($\hbar = 0.658$ meV ps)

(3 points)

(c) Estimate the sound velocity (in units of m/s) in the crystal for longitudinal sound waves, assuming $a = 5$ Å.

(4 points)



Spring 2011 Comprehensive Exam
Opti 506
Question 12

A. [4 points] Radiometric quantities are defined fundamentally in terms of derivatives of the flux. For each of the radiometric quantities listed below, give the customary symbols, the differential definitions and the SI units. Be clear about the meaning of all symbols you use.

a. Radiance

b. Irradiance

c. Radiant intensity

B. [2 points] It is often said that 'radiance is constant along a ray'. Explain briefly what this statement means and list the assumptions under which it is true. Provide a sketch if helpful to your explanation.

C. [4 points] Consider a uniform Lambertian source 5 cm in diameter that is centered on the optical axis and imaged at 1:1 magnification by an ideal (unaberrated) thin lens of focal length 2.5 cm and diameter 2.5 mm. Denote the radiance at the source as L_0.

a. Provide a sketch showing how the *radiance* at a point on the optical axis in the image plane varies with position and flux direction.

b. Provide a similar sketch for the *radiance* at a point 2 cm off the optical axis in the image plane.

c. Compute the *irradiance* at a point on the optical axis in the image plane.

d. Sketch the *irradiance* in the image plane as a function of distance from the optical axis. Explain any salient features of the sketch and label any pertinent dimensions.

Spring 2011 Comprehensive Exam
Opti 536
Question 12

Many optical instruments are designed to aid human vision in seeing objects or resolving details of objects that would otherwise not be visible

a. (2 points) The human visual system is clearly limited in its ability to resolve the fine detail in many objects. It is stated that the human visual system has an angular resolution of 1.5 arc minutes. Explain what angular resolution means, and why the human eye has such a limit. (i.e. what properties of the eye determine why the angular resolution is not 1.5 arc sec).
b. (1 point) Given the angular resolution of 1.5 arc min for the eye, what is the minimum separation of two self-luminous point objects that can just be resolved. (For this part you can assume that in viewing these two points you have the ability to move them as close to or as far away from you as you like while keeping their separation constant).
c. (1 point) Let's assume the physical separation of the two self-luminous point objects is 1.09 microns and you can't resolve them as two points with the naked eye. What magnifying power (MP) would you need for your instrument to allow you to just resolve these as two points.
d. (2 points) You work at Optical Sciences and have many optical components and instruments available to you. Describe what you would use to achieve the required MP in part c. (Be specific in describing what you are using. For example, if it is a simple lens, what is its focal length, conjugates, ?).
e. (2 points) Now assume that these two point objects are stars with a separation of 0.015 arc min. Describe the instrument you would use in this case in terms of what will give the necessary MP in order for you as the observer to resolve these two points. (Again, be specific in describing the instrument properties – configuration, focal lengths, etc).
f. (1 point) Does the size of the aperture of the instrument you described in part (e) come into play here in terms of what you can resolve using the instrument as an aid to your normal vision? Explain.
g. (1 point) Does the size of the instrument aperture come into play in terms of how bright an object you can see? Explain.

Spring 2011 Comprehensive Exam
Opti 503
Question 12

Two narrow-band red and green laser diodes are combined to create a new color. The red laser diode has XYZ-tristimulus values of (X_R=1.2, Y_R=0.5, Z_R=0) (unit: $watts \cdot m^{-2} \cdot sr^{-1}$) and the green laser diode has XYZ-tristimulus values of (X_G=0.3, Y_G=1, Z_G=0) (unit: $watts \cdot m^{-2} \cdot sr^{-1}$).

- (2 points) Compute the luminance values of the Red and Green laser diodes individually.
- (2 points) Compute the total luminance of the combined color.
- (3 points) Compute the xy-chromaticity coordinates of the Red and Green laser diodes, individually.
- (3 points) Compute the xy-chromaticity coordinates of the combined color.

Spring 2011 Comprehensive Exam
Opti 509
Question 12

A. [4 points] Radiometric quantities are defined fundamentally in terms of derivatives of the flux. For each of the radiometric quantities listed below, give the customary symbols, the differential definitions and the SI units. Be clear about the meaning of all symbols you use.

a. Radiance

b. Irradiance

c. Radiant intensity

B. [2 points] It is often said that 'radiance is constant along a ray'. Explain briefly what this statement means and list the assumptions under which it is true. Provide a sketch if helpful to your explanation.

C. [4 points] Consider a uniform Lambertian source 5 cm in diameter that is centered on the optical axis and imaged at 1:1 magnification by an ideal (unaberrated) thin lens of focal length 2.5 cm and diameter 2.5 mm. Denote the radiance at the source as L_0.

a. Provide a sketch showing how the *radiance* at a point on the optical axis in the image plane varies with position and flux direction.

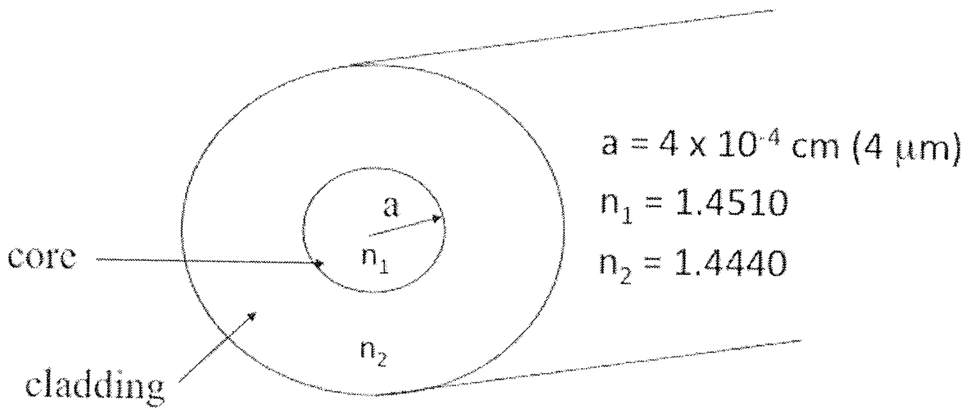
b. Provide a similar sketch for the *radiance* at a point 2 cm off the optical axis in the image plane.

c. Compute the *irradiance* at a point on the optical axis in the image plane.

d. Sketch the *irradiance* in the image plane as a function of distance from the optical axis. Explain any salient features of the sketch and label any pertinent dimensions.

Spring 2011 Written Comprehensive Exam
Opti 510
Question 12

This problem concerns the step index optical fiber shown below:



The fiber is made of fused silica, with the cladding pure fused silica and the core doped with germanium ions in order to achieve the elevated refractive index required. The refractive indices are given at 1.550 microns (1550nm), the wavelength at which light is propagated.

a) Calculate the V parameter and numerical aperture for this fiber. (2 points)

b) Determine whether the fiber is single-mode or not at 1550nm. (2 points)

The fiber described above is then heated at a temperature such that the germanium ions in the core diffuse out to a greater distance; we idealize the diffusion such that the new expanded core fiber obtained is still step index, but with a core radius of 8 x 10^{-4}cm (8 μm).

c) Obtain the V parameter and numerical aperture for this new expanded core fiber. You will need to estimate the refractive index in the new core. (3 points)

d) For hermetic packaging, we need to coat optical fiber with a layer of gold. Would it be better to choose the original fiber or the expanded core fiber for hermetic packaging? Why? (2 points)

e) When coupling to other optical waveguides or fibers, is the original fiber or the expanded core fiber more sensitive to angular misalignment? (1 point)