WRITTEN PRELIM EXAM – FIRST DAY

Spring 2012

February 21, 2012
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

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$\sin 2A = 2\sin A \cos A$

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$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

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$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

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$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

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$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

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$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Spring 2012 Written Comprehensive Exam

Opti 501

System of units: MKSA

3 pts a) Write Maxwell's *macroscopic* equations in their most complete form, including contributions from free-charge and free-current densities, as well as those from polarization and magnetization. Explain the meaning of each symbol that appears in these equations.

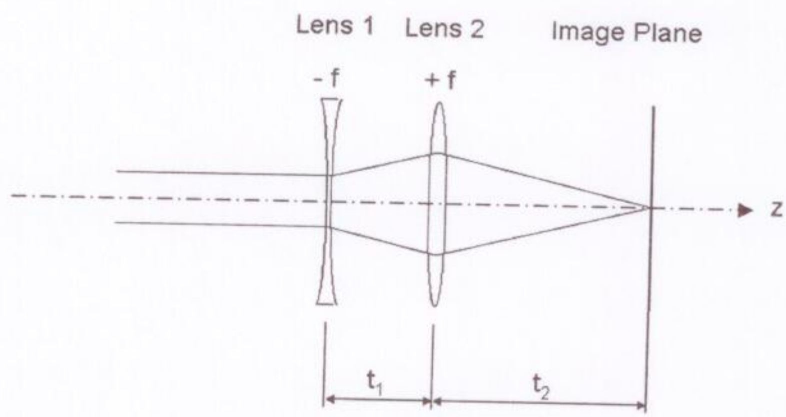
2 pts b) Derive the charge-current continuity equation directly from Maxwell's equations, and explain the meaning of this equation (be brief yet precise).

3 pts c) Define the bound-electric-charge and bound-electric-current densities. Use these entities to eliminate the $\boldsymbol{D}$ and $\boldsymbol{H}$ fields from Maxwell's equations. (In other words, rewrite Maxwell's equations with the help of bound-charge and bound-current densities in such a way that only the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields would appear in the equations.)

2 pts d) Show that the bound-charge and bound-current densities of part (c) satisfy their own charge-current continuity equation.

Spring 2012 Written Comprehensive Exam
Opti 502

A zoom lens varies its focal length by moving lens elements while the focal position is unchanged. Consider the following zoom optical system. The two lens elements have equal, but opposite, focal lengths. Lens 1 is a negative thin lens having a focal length of -f. Lens 2 is a positive thin lens having a focal length of +f. The object is located at infinity.



(a) (3 points) When $t_2 = 2\,f$
Determine the distance between Lens 1 and Lens 2: t_1,
Determine the system focal length: f_{syst}
Locate the rear principal plane with respect to the image plane.

(b) (4 points) Sketch the trajectories of Lens 1 and Lens 2 when t_2 varies from 1.5 f to 3 f. (Show the lens positions relative to the image plane as a function of t_2.)

(c) (3 points) Prove that the distance between Lens 1 and the image plane, t_1+t_2, is minimized when $t_2 = 2\,f$.

Spring 2012 Written Comprehensive Exam
Opti 511R

Consider a stable 2-mirror laser cavity of length $L = 20$ cm, with radii of curvature $R_1 = \infty$ and $R_2 = 50$ cm. The laser gain medium has a length $l = 10$ cm. The purely homogeneously broadened unsaturated gain coefficient is given by $\gamma_o(\nu) = ae^{-(\nu-\nu_o)^2/\Delta\nu^2}$, where a is a real constant, $\Delta\nu \sim 2 \times 10^{12}$ Hz, and $\nu_o \sim 300 \times 10^{12}$ Hz.

a. (2.5 pts) Write an expression for the functional form of the electric field at the flat mirror of the cavity for the $TEM_{1,1}$ transverse mode of the cavity. Express the field in form of $E(x,y) = A \cdot f(x,y)$, where A is a constant. Include a numerical value for the beam radius w_o.

b. (2.5 pts) If both mirrors have a power reflectivity of 99% and the laser gain medium has a single-pass linear scattering loss of 1%, calculate the threshold gain coefficient (γ_T).

c. (2 pts) Assuming the laser is operating above threshold on a single cavity mode, graph the steady-state gain coefficient, $\gamma(\nu)$, and $\gamma_o(\nu)$ on a single plot, highlighting key differences. Label γ_T, ν_o, and the laser frequency ν_l. (Note: the exact location of the lasing transition is not critical but should be indicated on the plot)

d. (1 pt) If this laser operates under mode-locked conditions, what would be the pulse repetition rate and approximate pulse duration?

e. (2 pts) Assume the laser gain medium is a solid cylindrical rod of diameter 5 mm with an atomic gain number density of $N = 5 \times 10^{18}/\text{cm}^3$. If the laser is Q-switched and emits a single pulse of the form $P(t) = P_o e^{-t^2/\tau^2}$, where $\tau = 100$ ns, what is the maximum peak power, P_o, that could possibly be generated? For this calculation, assume all the atoms are initially in the excited state and the cavity mode fills nearly the entire volume of the gain medium. The pulse therefore extracts the maximum possible energy from the laser rod.

Hint: $\int_{-\infty}^{\infty} e^{-(t/\tau)^2} dt = \tau\sqrt{\pi}$

Spring 2012 Written Comprehensive Examination
OPTI 544

A two-level atom coupled to a single mode of an optical cavity has the following Hamiltonian in the Schrodinger picture:

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar g(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$$

where ω is the bare cavity mode frequency, ω_0 is the atomic resonance frequency, and g is the interaction strength (assume real). The operators $\hat{a}$ and $\hat{a}^\dagger$ are the usual photon annihilation and creation operators. The operators $\hat{\sigma}_z$, $\hat{\sigma}_+$, and $\hat{\sigma}_-$ are the usual Pauli z, raising, and lowering operators (respectively) that act on the bare states of the two-level atom.

[a - 1 pt]. State the common name for the Hamiltonian $\hat{H}$.

[b - 1 pt]. Aside from the two-level atom and the single-mode approximations that are used to define this model, there are two approximations that are made to reach this form of $\hat{H}$. What are these two approximations? (Hint: both are also commonly made in the semi-classical model of atom-light interaction).

[c - 3 pt]. Write a general expression for the eigenvalues of $\hat{H}$. You may either state the eigenvalue expression (if you remember it) or make a good guess if you are short on time, or you can calculate the eigenvalues. You might even find it useful to solve the next part first if you remember what the spectrum looks like. Define all terms and variables used that are not already defined above.

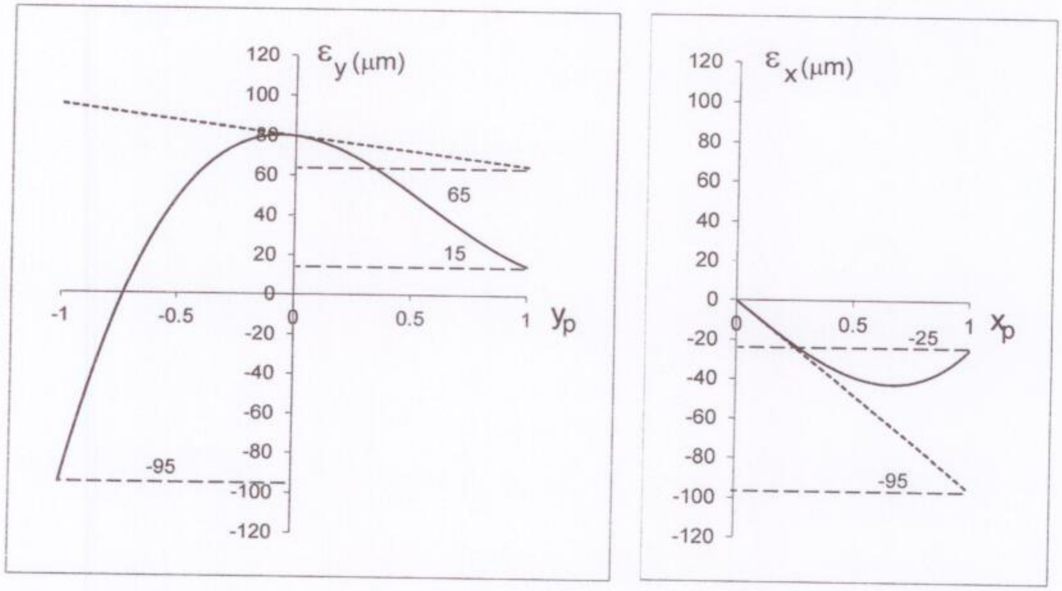
[d - 3 pt]. Sketch an energy level diagram for the lowest few (5 or 6) energy levels of the coupled system as a function of ω_0, from ω_0 slightly less than ω to slightly above ω. Your sketch should show the main characteristic features of the energy levels of the coupled system.

[e - 2 pt]. Give the value of the vacuum Rabi splitting in terms of the quantities used in $\hat{H}$, and indicate this splitting on your energy level diagram.

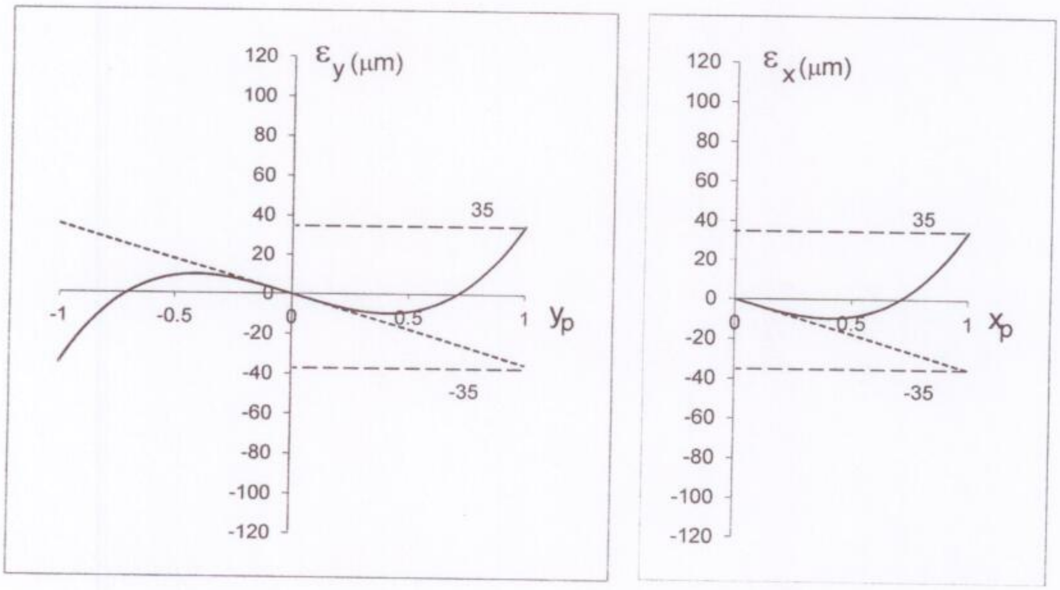
Spring 2012 Written Comprehensive Exam
Opti 503

You are given the following set of ray fans. The ray fans are plotted at H = 0 and H = 1.0 for an f/5 system. The dashed lines indicate the slope of the rayfans through the origin. Only first and third order aberrations are present. Calculate the aberration coefficients W_{XYZ} for each aberration. The units on the plots are microns.

H = 1



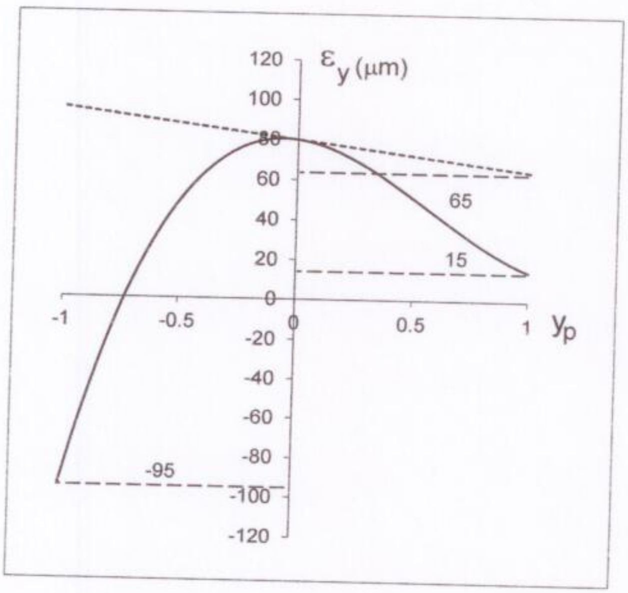
H = 0

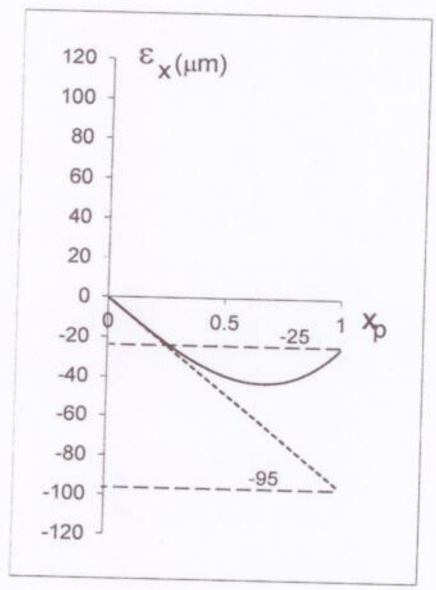


Spring 2012 Written Comprehensive Exam
Opti 509

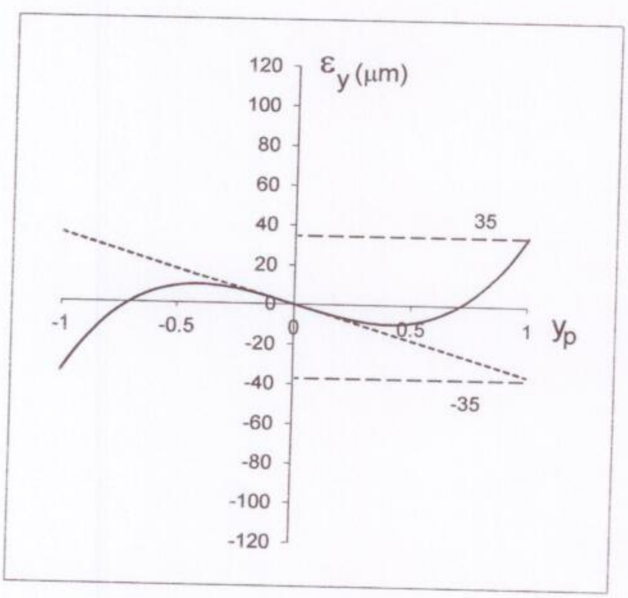
You are given the following set of ray fans. The ray fans are plotted at H = 0 and H = 1.0 for an f/5 system. The dashed lines indicate the slope of the rayfans through the origin. Only first and third order aberrations are present. Calculate the aberration coefficients W_{XYZ} for each aberration. The units on the plots are microns.

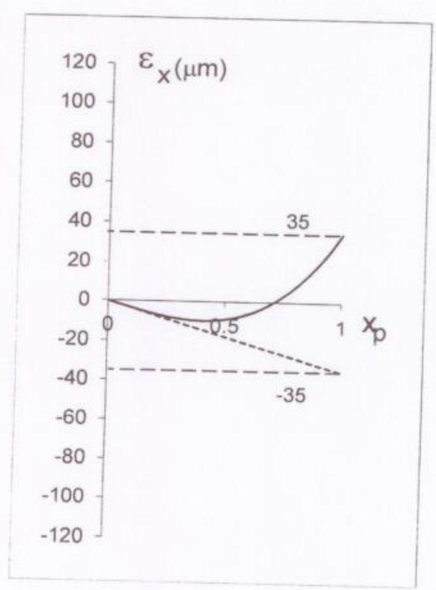
H = 1





H = 0





Spring 2012 Written Comprehensive Exam
OPTI 510

This is a problem on the topics of the Drude model, dispersive media and pulse propagation

(a) What is dispersion? Name three types of dispersion and explain what they are. (2 points)

(b) In the Drude model, which can be used to describe the optical properties of metal, the effective permittivity can be written as

$\epsilon_e = \epsilon_0(1 - \frac{\omega_p^2}{\omega^2})$,

where ω_p is called the plasma frequency. For a plane wave with wavenumber k, describe how the wave propagates when $\omega < \omega_p$. (Note: the wavenumber is related to the propagation constant and the attenuation coefficient by $k = \beta - \frac{j\alpha}{2} = \omega\sqrt{\epsilon_e \mu_0}$.) (2 points)

(c) For a plane wave with wavenumber k, describe the amplitude and phase of the wave as it propagates in the medium when $\omega > \omega_p$. What is the refractive index? (Hint: $\beta = \omega n(\omega)/c_0$) (2 points)

(d) Write an expression for the phase velocity and the group velocity for $\omega > \omega_p$. (2 points)

(e) An optical pulse with $\omega > \omega_p$ and spectral width σ_ν is propagating inside this material. What is the temporal spread σ_τ after propagation over a distance z? (2 points)

Spring 2012 Written Comprehensive Exam
OPTI 536

For parts (a) - (d), write a mathematical expression for each of the specified waves:

(a) [15%] A complex scalar representation of a monochromatic plane wave of frequency ν_0 propagating in free space in the direction specified by the unit vector $\hat{\mathbf{s}} \equiv (s_x, s_y, s_z)$. Any auxiliary variables you use should be clearly defined in terms of the unit vector and the frequency.

(b) [10%] The same wave as in (a) but evaluated on the plane $z = 0$. This time express the result in terms of the components of a 2D spatial-frequency vector $\boldsymbol{\rho}$.

(c) [15%] A complex scalar representation of a monochromatic spherical wave of frequency ν_0 propagating in free space and emanating from a point source located at $\mathbf{r}_0 \equiv (x_0, y_0, z_0)$. Clearly define all variables you use and state any restrictions on them.

(d) [10%] The same wave as in (c) but evaluated on the plane $z = 0$ and with the Fresnel approximation.

For parts (e) and (f), the desired result is an integral expression.

(e) [20%] How can plane waves as in part (b) be used to express an arbitrary scalar field $u_0(\mathbf{r})$ in the plane $z = 0$? Define any new symbols you introduce.

(f) [30%] Given the expansion in part (e), how can you find the field $u_{z_0}(\mathbf{r})$ in another plane $z = z_0$? Discuss the result.

Spring 2012 Written Comprehensive Exam
Opti 505.

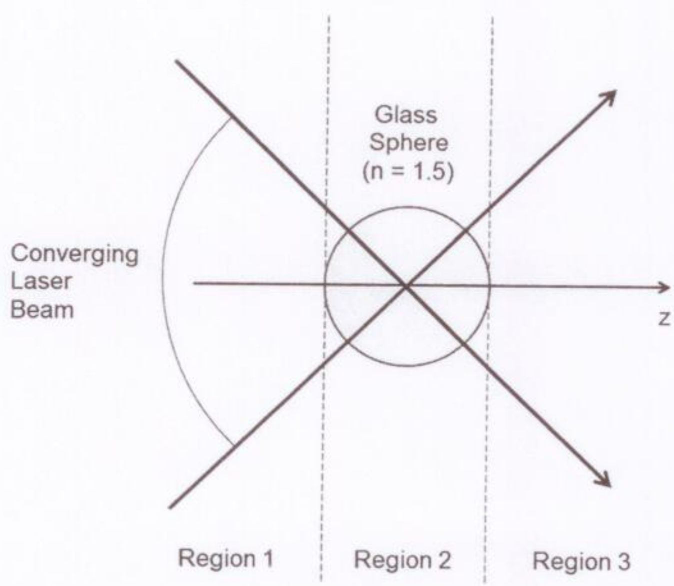
A converging laser beam in air is focused at the center of a perfect glass sphere. You can assume that the focusing laser in region 1 is a perfectly collapsing spherical wave, which is bounded on the outside by the arrows shown in the figure. The converging spherical wave and the glass sphere are concentric. The glass has $n = 1.5$ index of refraction. The laser wavelength is 650 nm. You can ignore wave components having less than 1% of the incident beam power in each region.

A) (2.5pts) Draw on your paper to show any fringes that might occur in regions 1, 2 and 3 due to the laser beam and its reflections, if any, that are due to the incident wave, its transmissions and its reflections above the 1% threshold. Ignore diffraction. Draw lines representing fringes centers. You do not need to show all fringes. However, show enough to indicate fringe shape in each region, if any.

B) (2.5pts) What is the fringe shape in each region (plane, sphere, hyperboloid, ellipsoid, paraboloid)?

C) (2.5pts) What are distances between adjacent fringe intercepts with the z axis in each region, if any?

D) (2.5pts) What is the fringe visibility in each region?



Spring 2012 Written Comprehensive Exam
Opti 546

This problem explores the ray and Gaussian beam optics of an optical resonator. The ABCD law for the complex beam parameter $\frac{1}{q} = \frac{1}{R} + \frac{2i}{kw^2}$ for a single round trip of the resonator is given by

$$\frac{1}{q_{n+1}} = \frac{C + D/q_n}{A + B/q_n}, \qquad M_L = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \qquad M_R = \begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix},$$

M_L and M_R being the ray transfer matrices for free-space and a mirror.

(a - 2pts) We consider a hemi-confocal optical resonator composed of one flat mirror and one concave curved mirror of radius of curvature R separated by a distance $L = R/2$. Calculate the ray transfer matrix M_1 for a single round trip of this resonator taking the flat mirror as the reference plane.

(b - 2pts) For the ray optics approach consider an initial ray vector $\binom{x_0}{x_0'}$. By examining the ray vector $\binom{x_1}{x_1'}$ after one round trip argue that a single round trip of the hemi-confocal resonator corresponds to an optical system that produces a Fourier transform.

(c - 2pts) Building on part (b), show that for an initial on-axis ray ($x_0 = 0$) the ray also crosses the axis after two and four round trips, $x_2 = x_4 = 0$. (Hint: You need to calculate M_2 and M_4 for two and four round trips.)

(d - 2pts) Turning now to the Gaussian beam optics of the hemi-confocal resonator, consider an initial Gaussian beam at the flat mirror of infinite radius of curvature and spot size w_0. Show that the Gaussian beam spot sizes w_2 and w_4 after two and four round trips of the resonator, respectively, are both equal to the initial spot size $w_2 = w_4 = w_0$. Thus, the initial field is reproduced in spot size after every two round trips of the resonator, irrespective of the value of w_0, analogous to the ray crossing in part (c).

(e - 2pts) Obtain an expression for the spot size w_1 after one round trip of the resonator given the initial spot size w_0. By demanding that $w_1 = w_0$, so that the spot size repeats after every round trip, obtain an expression for the stable Gaussian mode size w_0 of the hemi-confocal optical resonator in terms of k and L.

#6

Spring 2012 Written Comprehensive Examination
OPTI-537

Answer the following questions related to radiative transport. The radiative transport equation (RTE), or Boltzmann equation, describes the time evolution of a phase-space distribution function w in terms of four processes: absorption, emission, propagation, and scatter

$$\frac{dw}{dt} = \left[\frac{\partial w}{\partial t}\right]_{abs} + \left[\frac{\partial w}{\partial t}\right]_{emiss} + \left[\frac{\partial w}{\partial t}\right]_{prop} + \left[\frac{\partial w}{\partial t}\right]_{scat} \qquad (1)$$

(a) What is w a function of and what are its units? What does it describe?

(b) In class, we derived forms for each of the terms in the RTE and wrote an overall spatio-temporal-integro-differential transport equation of the form:

$$\frac{dw}{dt} = -c_m \mu_{total} w + \Xi_{p,E} - c_m \hat{\mathbf{s}} \cdot \nabla w + \boldsymbol{K} w \qquad (2)$$

where $\boldsymbol{K}$ is an integral operator. Please associate each term in equation (1) with the terms in equation (2). Describe and write the units of each of the variables in equation (2).

(c) Write an expression for the $\boldsymbol{K}$ operator for both inelastic collisions and elastic collisions.

(d) What is meant by the "steady-state solution" of the RTE? What changes in equation (2) when we assume steady state?

(e) Now assume that the intensity of the source is modulated with a known time dependence: $e^{j\omega t}$ where ω is the modulation frequency. Also assume that the time dependence of the solution also has only an $e^{j\omega t}$ dependence. Using just this information, derive a steady-state like radiative transport equation for modulated sources that is surprisingly similar to your result in part (d). (NOTE) For this problem, please allow the distribution function and the source to go negative. This is not physical but a simple shift can fix this issue.)

Spring 2012 Comprehensive Exam
OPTI 507

Consider light scattering involving phonons in III-V semiconductors (e.g. GaAs).

(a) Write down the energy and momentum conservation relations for the case of phonon emission.

(2 points)

(b) Make an approximate sketch of the dispersion curves of the phonons involved in Raman and Brillouin scattering, respectively.

(2 points)

(c) Give an approximate value for the phonon frequency Ω in III-V semiconductors such as GaAs or GaP.

(1 point)

(d) Using the thermal distribution function of phonons, estimate the number of phonons at room temperature ($k_B T \simeq 25\,\text{meV}$).

(2 points)

(e) For Raman scattering to be effective, do you need to choose a light frequency close to the phonon frequency? ("yes" or "no" answer is sufficient)

(1 point)

(f) Sketch the Raman spectrum as function of the difference between scattered and incident light frequency. Indicate the Stokes and Anti-Stokes line.

(2 points)

WRITTEN PRELIM EXAM – SECOND DAY

Spring 2012

February 22, 2012
8:30 a.m. to 12:00 p.m.

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$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

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$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Spring 2012 Written Comprehensive Exam

Opti 501

System of units: MKSA

Consider a homogeneous, linear, isotropic medium of permeability $\mu(\omega)=1.0$, real-valued and positive permittivity $\varepsilon(\omega)$, and refractive index $n(\omega)=\sqrt{\varepsilon(\omega)}$. Within this medium, two plane-waves having equal E-field amplitudes but differing frequencies propagate along the z-axis. Both plane-waves are linearly-polarized along the x-axis, their E-field amplitudes being E_o, and their respective frequencies being ω_1 and ω_2. The center frequency is $\omega_o=\frac{1}{2}(\omega_1+\omega_2)$, and the frequency difference $\Delta\omega=(\omega_2-\omega_1)$ is much smaller than ω_o.

3 pts a) Write expressions for the *real-valued* E- and H-field distributions of both plane-waves as functions of the space-time coordinates (x, y, z, t).

4 pts b) Write a complete expression for the rate of flow of electromagnetic energy (per unit area per unit time) associated with the superposition of the two plane-waves. Simplify the expression so that the energy flux associated with the beat-signal of frequency $\Delta\omega$ can be clearly identified. **Hint**: $\cos a \cos b = \frac{1}{2}[\cos(a+b)+\cos(a-b)]$.

3 pts c) Ignoring the rapidly-oscillating terms in the expression obtained in part (b), show that the energy flow-rate associated with the beat signal moves along the z-axis at the group velocity V_g, which is derived from $n(\omega)$ in the vicinity of the beat signal's center frequency ω_o.

Spring 2012 Written Comprehensive Exam
Opti 502

A two thin-lens collimator in air has lens focal lengths of $f_1 = 60$ mm and $f_2 = -120$ mm. The first lens is located at a distance $t_0 = 90$ mm from the object. The second lens is located $t_1 = 60$ mm from lens 1. The object is circular with an object semi-diameter (chief ray height) $\bar{y}[0] = 1.5$ mm. The collimator is telecentric in object space. The paraxial marginal ray angle in object space is $u[0] = 0.2$.

a. (2 points) Where is the aperture stop of this thin-lens collimator located and what is its diameter?

The output of the collimator now needs to couple into a rather long two-lens imager. The imager's first lens has a focal length $f_3 = 120$ mm. The aperture stop of the imager is located 160 mm after lens 3 and has a diameter of 8 mm. The imager's second and final lens is located 280 mm after lens 3 and has a focal length of $f_4 = 120$ mm.

Both the collimator and the imager have exactly the same Lagrange invariant. Thus, unless the collimator and imager are precisely aligned, vignetting will occur.

b. (2 points) Describe the conditions for coupling together two optical systems with the same Lagrange invariant to ensure that vignetting is avoided.

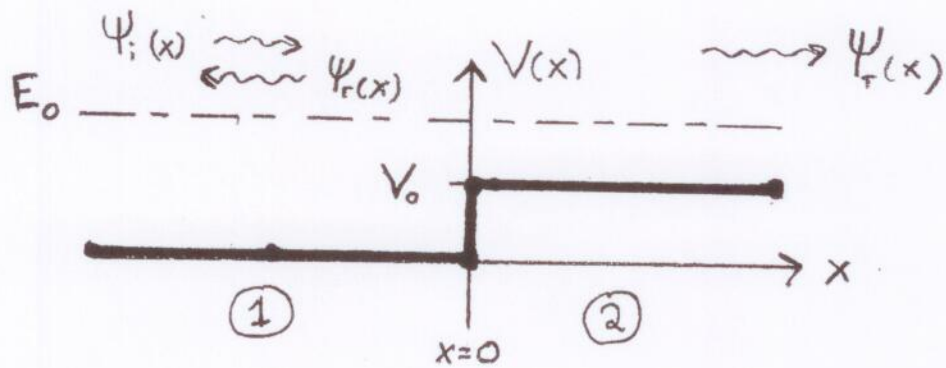
c. (2 points) Considering the answer to part b), what is the proper spacing t_2 between lenses 2 and 3?

d. (2 points) How far past lens 4 does the final image form?

e. (2 points) What is the magnification of the combined system?

Spring 2012 Written Comprehensive Exam
Opti 511R

Consider a 1-dimensional model for a free electron in a thin wire, represented by the potential energy function defined below. The electron may be emitted from the end of the wire since its energy E_o is greater than the potential energy barrier V_o. The incident electron wavefunction, $\psi_i(x)$, can be described by a plane wave with momentum $p_o = \hbar k_o$.



a. (1 pt) Write down the 1-dimensional time-independent Schrödinger equation for region 2 (include in your expression the electron's mass m_e).

b. (1 pt) Express k in terms of E_o for region 2.

c. (1 pt) Solve the time-independent Schrödinger equation and write down the general expression for $\psi(x)$ in both regions.

d. (1 pt) Write down the specific boundary conditions for this problem.

e. (1 pt) Make a plot of $Re\{\psi(x)\}$ across the potential energy barrier that illustrates these boundary conditions. Indicate quantitative differences in the deBroglie wavelength of the particle in regions 1 and 2.

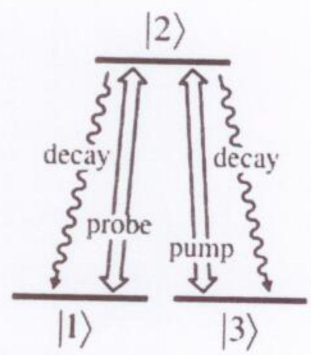
f. (3 pts) Determine the probability the electron will be reflected from the barrier. To do this, you will need to use the boundary conditions to solve for the ratio of the coefficients for $\psi_i(x)$ and $\psi_r(x)$ from part (c). Express your final answer in terms of E_o and V_o.

g. (1 pt) If $E_o < V_o$, what is the probability the particle will be reflected?

h. (1 pt) If $E_o < V_o$, what is the functional form of $\psi(x)$ in region 2?

Spring 2011 Written Comprehensive Exam
OPTI 544

Consider a medium of atoms with the level structure shown at right. The excited state lifetime is A^{-1}, and decay into each ground state is equally probable. The $|1\rangle \to |2\rangle$ and $|3\rangle \to |2\rangle$ transitions are both at wavelength $\lambda = 1.0\mu m$, and have equal photon scattering cross sections which are a factor of two smaller than for a two-level atom with the same A. There is no collision or Doppler broadening. The medium forms a slab of length $l = 2cm$ and thickness $d << l$, and the atom number density is $N = 10^{15} m^{-3}$.



We first send an optical probe beam lengthwise through the slab. It is resonant with the $|1\rangle \to |2\rangle$ transition, and its photon flux Φ is well below saturation, $A >> \sigma\Phi$. At $t = 0$ all atoms are in state $|1\rangle$, $\rho_{11}(t=0) = 1$. There is no field present to drive the $|3\rangle \to |2\rangle$ transition.

(a) (2 pts) Calculate the transmission T at $t = 0$.

(b) (2 pts) What is the steady state transmission T_{SS}? Explain.

(c) (3 pts) Write down a set of rate equations for the populations ρ_{11}, ρ_{22} and ρ_{33}. Simplify as much as possible by dropping negligible terms. Then derive equations for ρ_{11} and ρ_{33} alone by adiabatic elimination of the equation for ρ_{22}, valid in the limit $A >> \sigma\Phi$. Solve these and derive an expression for the time dependent transmission $T(t)$.

We next add a pump field that passes through the slab in the thin direction, such that the entire medium is uniformly illuminated. The pump field is resonant with the $|3\rangle \to |2\rangle$ transition, and its photon flux Φ' is well below saturation, $A >> \sigma\Phi'$.

(d) (3 pts) Modify your rate equations from (c) to include the presence of the pump. Once again, simplify as much as possible by dropping negligible terms. Then find the steady state value of ρ_{11} and the steady state transmission of the probe in the presence of the pump field.

The xy-chromaticity coordinates and luminance values of three narrow-band red, green, and blue LEDs are:

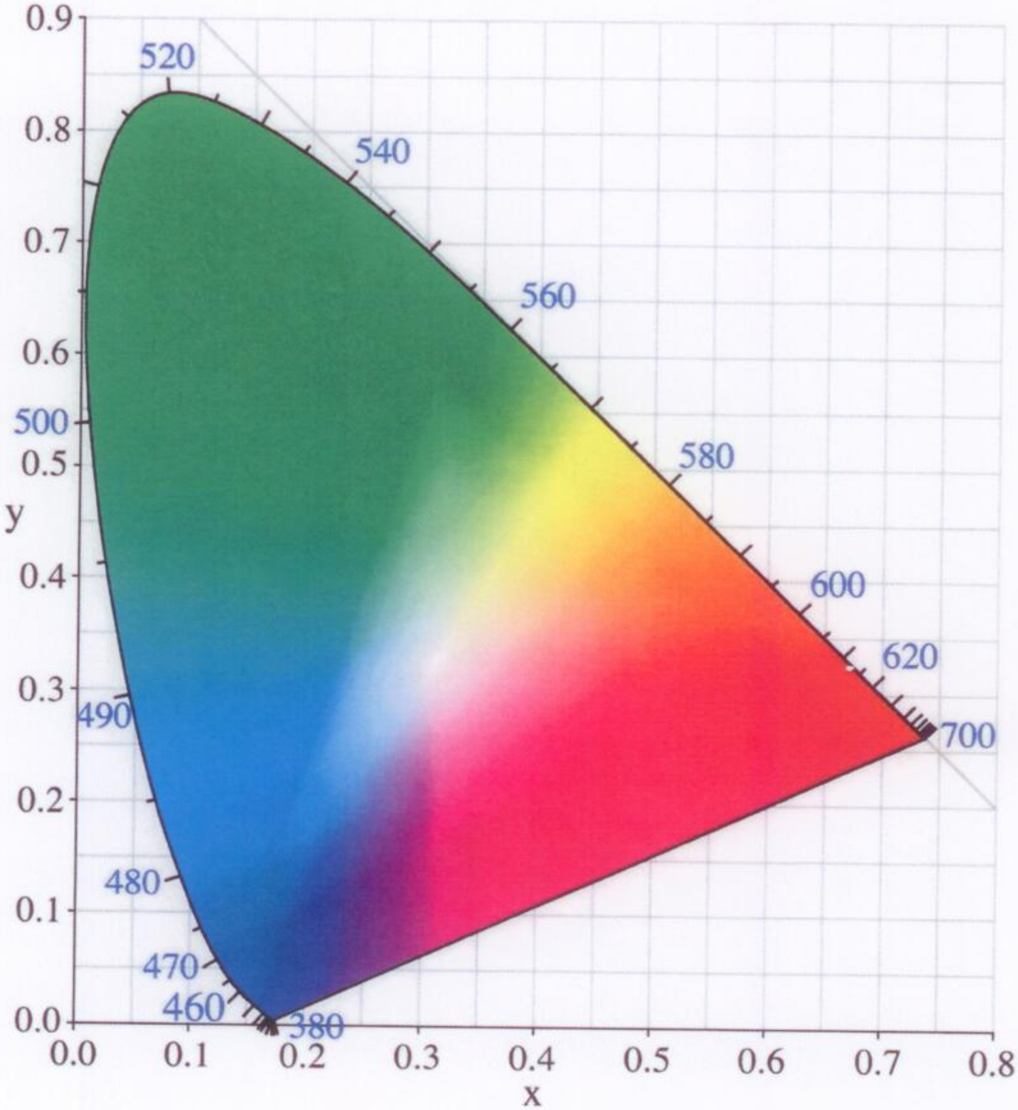
Red:	x_r=0.7	y_r=0.28	L_r=100 lm/m^{2}sr
Green:	x_g=0.2	y_g=0.65	L_g=300 lm/m^{2}sr
Blue:	x_b=0.15	y_b=0.1	L_b=10 lm/m^{2}sr

(3 points) Compute the XYZ-tristimulus values of the three diodes individually.

(4 points) When these LEDs are combined, what are the chromaticity coordinates and luminance of the resulting color? Show the locations of the original three LEDs and the combined color on the blank chromaticity diagram provided below.

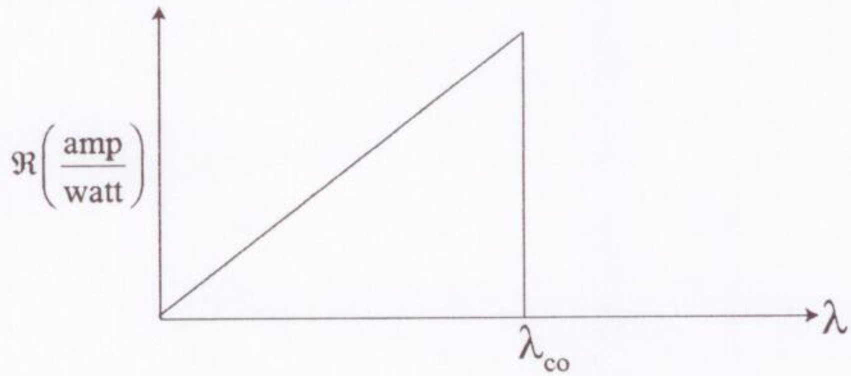
(3 points) Determine the approximate dominant wavelength of the resulting color when the LEDs are combined.

For Solution:



Spring 2012 Written Comprehensive Exam
Opti 509

The ideal photodetector (quantum efficiency equals one, $\eta = 1$) spectral responsivity versus wavelength curve is shown below:



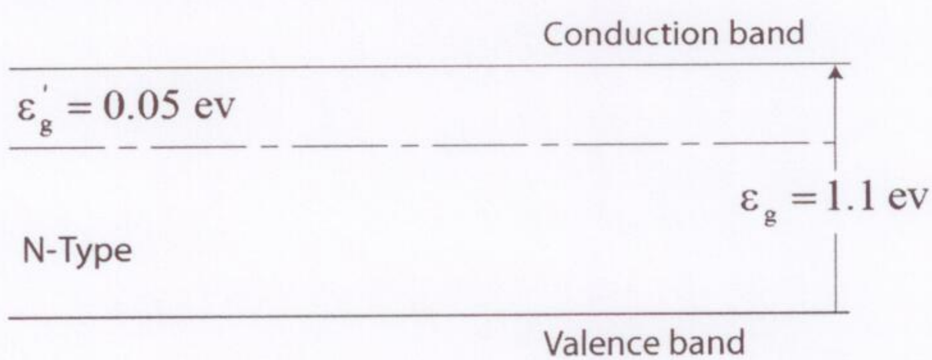
a. (3 points) Sketch the quantum efficiency vs. wavelength for a real photodetector with non-unit quantum efficiency and discuss why it deviates from the ideal photodetector at short wavelengths and at long wavelengths of its response.

b. (3 points) What determines the cutoff wavelength (λ_{co}) for:

(1) Photodiode
(2) Photoconductor

c. (1 point) Why is the responsivity vs. wavelength degraded over the spectral response? (i.e., is a photodetector less sensitive at wavelengths well below λ_{co} compared to those near λ_{co} ?).

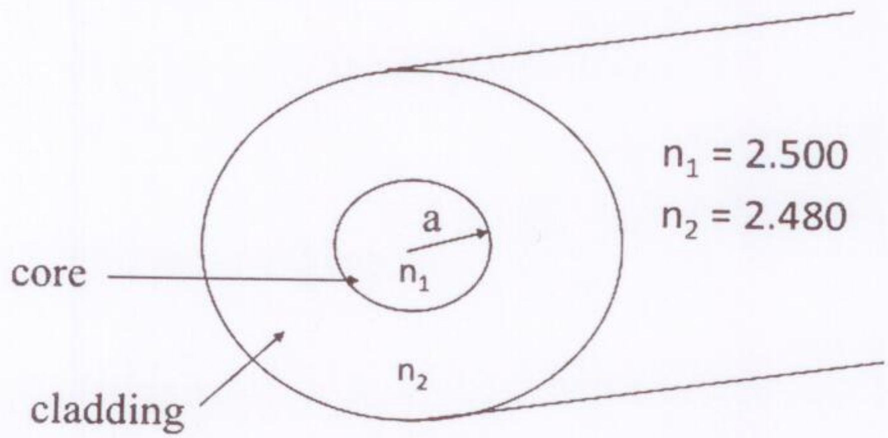
d. (1 point) For a photoconductor material of an extrinsic semiconductor (N-type) with energy diagram shown below, what is the cutoff wavelength?



e. (2 points) Which of the three material groups, metals, semiconductors or insulators, are most easily used for photodetectors? Why are they more difficult than the other two types of material?

Spring 2012 Written Comprehensive Exam
Opti 510R

This problem concerns the step index optical fiber shown below:



The fiber is made of a very high refractive index glass, with the core and cladding being of slightly different compositions. The refractive indices are given at 1.550 microns (1550nm), the wavelength at which light is propagated.

a) Determine the radius a below which the fiber is single mode. (4 points)

b) Estimate the magnitude of the reflection loss when an ordinary doped fused silica fiber with refractive index ~ 1.450 is butt coupled (flat end faces) to the high index fiber. Express the result in dB. You can ignore mode field diameter mismatch (3 points).

c) A pulse is launched into the high index optical fiber with an initial pulse profile given by

$$I(t) = \frac{I_0}{\cosh^2(t/\tau)}$$

Write down the time dependent phase, $\phi(t)$, of the pulse at fiber length L in the presence of a nonlinear refractive index, n_2 where the intensity dependent index is given by

$$n(I) = n_0 + n_2 I,$$

the carrier frequency is ω_o and the vacuum wavelength is λ_o. Calculate the instantaneous frequency shift associated with the time dependent phase as a function of time at fiber length L (3 points).

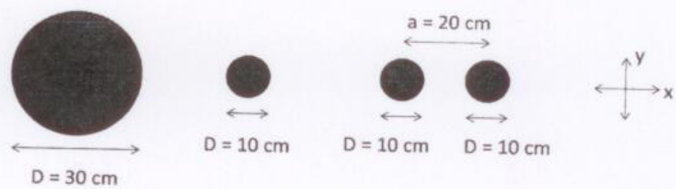
Spring 2012 Written Comprehensive Examination
OPTI 536

Binoculars are basically two Keplerian telescopes side by side. (1 point for each part)

a) Show the general optical layout for one of these telescopes with the marginal and chief rays shown for an object at infinity. You can assume thin lenses but be sure to show focal lengths and the separation between elements. You can ignore any prisms that might be used in actual binoculars for this diagram.
b) Why would there be prisms used in the binoculars?
c) What is the definition of principal planes in an optical system and where are the principal planes located for this telescope? You should be able to determine this based on the ray diagram in part a.
d) Where are the stop, entrance pupil, and exit pupil located? You can assume the standard configuration for this type of telescope.
e) For binoculars with markings 10X 30 on the side, what do these numbers mean?
f) The angular resolution of the eye is 1.5 arc min = 4.36 x 10^{-4} radians. What is the angular resolution in the object one can observe using the binoculars described in part e.
g) Why are we discussing the operation of this imaging system in terms of angular resolution rather than spatial resolution in the object as we might do for a microscope.
h) When one views a scene with binoculars, there is typically a finite circular field of view observed. Show how this is achieved (i.e. show the type of "component" and where it is located in the optical system to provide a distinct field of view).
i) Assuming the exit pupil diameter of the binocular matches or exceeds the entrance pupil diameter of the eye, does the brightness of the image viewed with binoculars (irradiance on the retina) appear brighter, darker, or equal to that without the binoculars? Explain your answer.
j) Assume one is viewing the moon. How would one classify this imaging system in terms of the kind of radiation involved, the property being imaged, the imaging mechanism, whether this is direct or indirect imaging, and whether it is passive or active imaging.

Spring 2012 Written Comprehensive Exam
OPTI 505

Consider three imaging systems being used to image quasi-monochromatic point objects at infinity at a wavelength $\lambda = 500$ nm. All three systems are operating with a focal length of $f = 90$ cm. The first has a circular exit pupil of diameter 30 cm. The second has a circular exit pupil of diameter 10 cm. The third is a system with an exit pupil made up of two 10-cm-diameter circles with center-to-center separation of 20 cm (note, this is a single imaging system with a single detector, not a set of binoculars with independent detectors). Assume that all systems are operating at the diffraction limit.



A. (6 pts) Sketch the MTF and PSF of each of these systems along the $(x,0)$ and $(0,y)$ axes in image space, being sure to label all physical quantities with appropriate units.
B. (4 pts) What angular separation is required between two point objects in the horizontal (x) and vertical (y) directions in order to resolve them for each of the systems? Be sure to justify the metric you use to determine resolvability.

Spring 2012 Written Comprehensive Exam
Opti 546

This problem deals with scattering of a plane-wave of wavelength $\lambda = 1\ \mu$m from an acousto-optic cell of length $L = 0.5$ cm with a spatially periodic refractive-index modulation $\Delta n(x) = n_1 \sin(Kx)$, where n_1 is the magnitude of the index modulation, and $K << k$ the magnitude of the wavevector of the acoustic wave, k being the magnitude of the optical wavevector.

(a - 2pts) Sketch the characteristic arrangement of the incident optical field wavevector, acoustic wavevector, and the scattered field components for Raman-Nath diffraction for the situation described above.

(b - 3pts) For the case of Raman-Nath diffraction of a plane-wave incident along the z-axis the field exiting the acousto-optic cell is

$$\mathcal{E}(x, L) = \mathcal{E}_i e^{i2\pi\Delta n(x)L/\lambda}, \tag{1}$$

where $\mathcal{E}_i$ is the amplitude of the incident plane-wave. Using the Bessel function identity

$$e^{i\delta \sin\phi} = \sum_{m=-\infty}^{\infty} J_m(\delta) e^{im\phi}, \tag{2}$$

with $J_m(\delta)$ the Bessel function of the first kind of order m, show that the field exiting the acousto-optic cell is composed of scattered waves with components of their wavevectors along the x-axis given by $k_x^{(m)} = mK, m = 0, \pm 1, \pm 2, \ldots$.

(c - 2pts) Based on your solution from part (b) argue that in the far field region beyond the acousto-optic cell the intensity pattern will composed of scattered waves traveling at angles $\theta_m = mK/k$ with respect to the z-axis, and with intensities $I_m(L) = I_i J_m^2 (2\pi n_1 L/\lambda)$, with I_i the incident intensity.

(d - 1pt) Calculate the minimum value of n_1 required to produce zero intensity in the direction of the incident plane-wave. (Hint: You will need the result $J_0(2.4) = 0$).

(e - 2pts) Based on your answer from part (d) suggest how Raman-Nath diffraction from an acousto-optic cell may be used to create an optical modulator that can vary the transmitted intensity between zero and I_i.

Spring 2012 Comprehensive Exam
OPTI 507

Consider a simple model for an optical absorption process in a thin semiconductor quantum well (with z being the direction normal to the quantum well). Assume you have only one conduction and one heavy-hole valence subband, and the bands to be parabolic and isotropic. Also, assume the wave vector of light to be negligible and neglect Coulomb effects.

(a) Assume you are absorbing light from a monochromatic light source with frequency $\omega = 2.7347\times10^{15}\ \mathrm{s}^{-1}$. Let the quantum well gap be $E_G^{qw} = 1.6\,\mathrm{eV}$, the electron mass $m_e = 0.1\,m_0$, and the hole mass $m_h = 0.2\,m_0$ where m_0 = electron mass in vacuum. Sketch the energy bands as function of k_x (with $k_y = 0$) and as function of k_y (with $k_x = 0$). For $k_y = 0$, calculate the value of k_x at which the transition occurs and indicate that transition in your sketch. How would your result change if you were to calculate the value of k_y for $k_x = 0$?

(8 points)

(b) In the $k_x - k_y$ plane, indicate the location of the wave vectors at which the optical transition takes place. Label your k_x and k_y axes in units of inverse Angstrom.

(2 points)

(Note: Use $\hbar = 0.6582\,\mathrm{meV\,ps}$ and $\hbar^2/m_0 = 7.62\times10^{-16}\ \mathrm{eV\,cm^2}$.)

Spring 2012 Written Comprehensive Examination
OPTI-537

Answer the following questions related to semiconductor detectors. All parts weighted as indicated.

(a) (10%) What is the definition of a Bravais lattice? What is a primitive lattice vector? Write an expression for a translation vector that spans the equivalent locations in a real-space Bravais lattice. How many distinct Bravais lattices are there?

(b) (15%) Write the 3D time-independent Shrödinger equation for a single electron in a Bravais lattice. Show where periodicity is expressed in the Hamiltonian and explain what assumptions are necessary to invoke the time-independent form of the Shrödinger equation to describe the electronic structure and related properties of crystals.

(c) (15%) Consider a translation operator T_R such that $T_{\mathbf{R}}\left[\psi(\mathbf{r})\right]=\psi(\mathbf{r}+\mathbf{R})$. Show that if $\mathbf{R}$ is a translation vector in the Bravais lattice that describes the periodic crystal, then T_R commutes with the one-electron Hamiltonian of question (b) above.

(d) (15%) Sketch a one dimensional band diagram for the nearly-free electron model and compare it with the free electron curve (hint: x-axis units are momentum, y-axis units are energy) in the reduced-zone scheme. Point out the band gaps and explain why they arise.

(e) (15%) What is the definition of the Fermi level (energy)? Write the expression for the Fermi-Dirac distribution and state what it means.

(f) (15%) Explain what kind of dopant is added to Si to create an N-type material. What is the majority carrier in an N-type material? What is the minority carrier? Sketch the location (in an *energy versus position* band diagram) of the dopant states for an N-type material relative to the bottom of the conduction band and the top of the valence band. Indicate where the Fermi level is at room temperature.

(g) (15%) Sketch a graph that plots the concentration of majority and minority carriers in N-type Si as a function of temperature, starting at absolute zero and going well above room temperature. Label which curve is which, and show some temperature points on the horizontal scale – say every 100K.