a) Write Maxwell's *macroscopic* equations in their most complete form, including contributions from free-charge and free-current densities, as well as those from polarization and magnetization source terms. Explain the meaning of each symbol that appears in these equations.

b) Derive the charge-current continuity equation directly from Maxwell's equations, and explain the meaning of this equation. Be brief but precise.

c) Define the bound-electric-charge and bound-electric-current densities. Use these entities to eliminate the D and H fields from Maxwell's equations. (In other words, rewrite Maxwell's equations with the help of bound-charge and bound-current densities in such a way that only the E and B fields would appear in the equations.)

d) Show that the bound-charge and bound-current densities of part (c) satisfy their own charge-current continuity equation.

A. A flat mirror is in front of you at a distance of 600 mm. The mirror is circular, has a diameter of 150 mm, and the normal line to the mirror center is horizontal and passes through your eye. Assuming you are looking at the mirror using only one eye, determine the maximum field of view that you can see reflected off the mirror. Assume the eye pupil diameter is negligible. Give the full field of view in degrees.

B. A spherical mirror is placed at the center of the flat mirror and has a diameter of 50 mm. Determine the radius of curvature of the spherical mirror that will provide twice the field of view as that of the flat mirror. Is the spherical mirror concave or convex? The images on the smaller spherical mirror must be erect.

C. A truck is behind you at a distance of 10 meters. In comparison to the image produced by the flat mirror, approximately how far away the truck would appear on the smaller spherical mirror? Explain the reasoning for your estimate.

A sinusoidal amplitude transmission grating is illuminated by a unit-amplitude, uniform plane wave at normal incidence. Assume that the amplitude transmittance of the grating is:

$$t(x) = \frac{A}{2}\left[1 + \cos\left(\frac{2\pi}{d}x\right)\right],$$

where d is the grating pitch. Note that since there is no y-dependence in this problem, you do not need to keep track of any terms involving the y-direction in your answers below.

(a) Write and simplify the mathematical expression for the electric field given by the **Fraunhofer** far field diffraction pattern at a distance z_0.

(b) Write and simplify the mathematical expression for the electric field diffraction pattern at any distance based on the **angular spectrum** method.

(c) What happens when $z_0 = \dfrac{\lambda}{1 - \sqrt{1 - \left(\frac{\lambda}{d}\right)^2}} \approx \dfrac{2\,d^2}{\lambda}$? What is this effect called?

(d) In the limit of large z_0, does the answer to part (b) reduce to the answer in part (a)? Explain why or why not. A full mathematical derivation is not necessary, but a written explanation of the physical reasons behind the agreement or disagreement of the two models is necessary.

Reference information:

A selection of diffraction integrals:

$$U(x_0, y_0, z_0) = -j\frac{e^{jkz_0}\, e^{\frac{jk}{2z_0}(x_0^2 + y_0^2)}}{\lambda z_0} \iint U_S^+(x_s, y_s)\exp\left[\frac{jk}{2z_0}(x_s^2 + y_s^2)\right]\exp\left[-j\frac{2\pi}{\lambda z_0}(x_s x_0 + y_s y_0)\right]dx_s\, dy_s$$

$$U(x_0, y_0, z_0) = \iint \left[\iint U_S^+(x_s, y_s)\, e^{-j2\pi(x_s\xi + y_s\eta)}dx_s\, dy_s\right]\exp(jk\gamma z_0)\, e^{j2\pi(x_0\xi + y_0\eta)}\, d\xi\, d\eta,$$
$$\text{where } \gamma = \sqrt{1 - (\lambda\xi)^2 - (\lambda\eta)^2} \text{ is the } z\text{-direction cosine.}$$

$$U(x_0, y_0, z_0) = -j\frac{e^{jkz_0}\, e^{\frac{jk}{2z_0}(x_0^2 + y_0^2)}}{\lambda z_0} \iint U_S^+(x_s, y_s)\,\exp\left[-j\frac{2\pi}{\lambda z_0}(x_s x_0 + y_s y_0)\right]dx_s\, dy_s$$

Fourier pairs:

$f(x)$	$g(\xi) \equiv \displaystyle\int_{-\infty}^{\infty} f(x)e^{-j2\pi\xi x}\, dx$
1	$\delta(\xi)$
$f(x)\cos(a\,x)$ ← cosine-modulated function	$\dfrac{1}{2}\left[g\left(\xi - \dfrac{a}{2\pi}\right) + g\left(\xi + \dfrac{a}{2\,\pi}\right)\right]$

Fall 2018 Qualifying Exam
OPTI 511, Day 1

In this problem, you will work with operators and wavefunctions associated with a one-dimensional coordinate space. The position variable is x. All operators and wavefunctions below are expressed in terms of x (ie, we're using position space, or the position representation, to express these quantities), and you should do the same with your answers. Each part of this question builds on the preceding parts. [10 pts total.]

[a]. Give a function of x that is an eigenfunction of the operator $-i\hbar\frac{\partial}{\partial x}$.

[b]. What is the eigenvalue associated with your answer to part [a], and what are the dimensional units of this eigenvalue?

[c]. An eigenfunction of $-i\hbar\frac{\partial}{\partial x}$ is not a physically realistic wavefunction for a particle of mass m. Why is that the case?

[d]. Despite it not being physically realistic, we'll consider a particle of mass m that has a wavefunction $\psi(x)$ (at time $t = 0$) that *is* an eigenfunction of $-i\hbar\frac{\partial}{\partial x}$. Let $\psi(x)$ be equal to your answer to part [a]. The particle evolves according to a Hamiltonian $\hat{H}$ that has only a kinetic energy term, and no potential energy. Is $\psi(x)$ also an eigenfunction of $\hat{H}$? If so, what is the associated eigenvalue; if not, why not?

[e]. Now write down a new function that is a superposition of two eigenfunctions of $-i\hbar\frac{\partial}{\partial x}$ that are associated with eigenvalues that are **equal in magnitude** but **opposite in sign**. **Let the superposition coefficients be identical.** Denote this new function as $\varphi(x)$, and assume that it specifies the wavefunction for a particle of mass m at some time $t = 0$. Here and in the rest of this problem, you may incorporate or ignore an overall scaling coefficient for $\varphi(x)$, but we won't worry about overall scaling or dimensional units for $\varphi(x)$ in the questions below.

[f]. Is $\varphi(x)$ an eigenfunction of $-i\hbar\frac{\partial}{\partial x}$? Is it an eigenfunction of $\hat{H}$?

[g]. If the *momentum* of the particle with wavefunction $\varphi(x)$ is to be measured, what are the possible results, and what is the probability of obtaining each result?

[h]. Calculate and sketch the probability density distribution for the particle with wavefunction $\varphi(x)$ over some range of positions that clearly illustrates significant physical properties.

[i]. If the *position* of a particle with wavefunction $\varphi(x)$ is to be measured, what are the most likely positions or regions in which the particle would be found?

[j]. If the *position* of the particle with wavefunction $\varphi(x)$ is measured and found to be exactly x_0, what is the wavefunction for the particle immediately after the measurement?

A monochromatic plane-wave of frequency ω traveling in free space is reflected from a perfectly-conducting, flat mirror surface at an oblique incidence angle θ, as shown in the figure. In the half-space $z \leq 0$, the electric and magnetic fields of the incident and reflected waves are

$$\begin{cases} \boldsymbol{E}^{(\text{inc})}(\boldsymbol{r},t) = E_o(\cos\theta\hat{\boldsymbol{x}} - \sin\theta\hat{\boldsymbol{z}})\exp\{i(\omega/c)[(\sin\theta)x + (\cos\theta)z - ct]\}, \\ \boldsymbol{H}^{(\text{inc})}(\boldsymbol{r},t) = (E_o/Z_o)\hat{\boldsymbol{y}}\exp\{i(\omega/c)[(\sin\theta)x + (\cos\theta)z - ct]\}. \end{cases}$$

$$\begin{cases} \boldsymbol{E}^{(\text{ref})}(\boldsymbol{r},t) = -E_o(\cos\theta\hat{\boldsymbol{x}} + \sin\theta\hat{\boldsymbol{z}})\exp\{i(\omega/c)[(\sin\theta)x - (\cos\theta)z - ct]\}, \\ \boldsymbol{H}^{(\text{ref})}(\boldsymbol{r},t) = (E_o/Z_o)\hat{\boldsymbol{y}}\exp\{i(\omega/c)[(\sin\theta)x - (\cos\theta)z - ct]\}. \end{cases}$$

a) Find the tangential component of the E-field at the mirror surface, and verify that it satisfies the relevant boundary condition.

b) Find the tangential component of the H-field at the mirror surface, and determine the current density $\boldsymbol{J}_s(x, y, z = 0, t)$ that must exist on the surface in order to satisfy the relevant boundary condition.

c) Find the perpendicular component of the E-field at the mirror surface, and determine the charge density $\sigma_s(x, y, z = 0, t)$ that must exist on the surface in order to satisfy the relevant boundary condition.

d) Show that the surface charge and current densities obtained in parts (b) and (c) satisfy the continuity equation $\boldsymbol{\nabla} \cdot \boldsymbol{J}_s + \partial\sigma_s/\partial t = 0$.
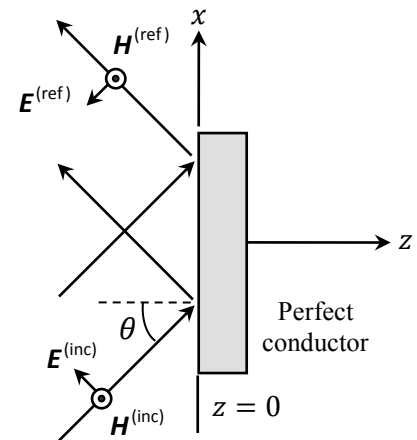
A field flattener is a negative lens placed near the image plane of an objective lens. The objective lens has a focal length $f_1 = 100mm$. The field flattening lens is located $95mm$ to the right of the objective. The image plane for both lenses in combination is located $105mm$ to the right of the objective lens. The aperture stop is at the objective lens and has a diameter of $25mm$. Assume both elements are thin lenses. Answer the following:

(a) What is the focal length of the field flattening lens needed to ensure a distant object is imaged onto the image plane?

(b) What is the total power of the combined system?

(c) Where are the Cardinal Points of the lens system located?

(d) Where are the entrance and exit pupils located?

(e) What are the diameters of the entrance and exit pupils?

(f) What is the $f/\#$ of the system?

The Large Binocular Telescope (LBT) sits on top of Mt. Graham in southern Arizona. Some of
its features are:

- Number of Primary Mirrors: 2
- Primary Spacing: 14.4 meter center-to-center
- Primary Physical Diameter: 8.4 meter

Some useful equations and graphs are provided at the end of this
problem.
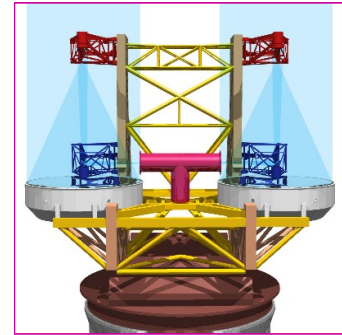
a.) (3 pts.) Ignore effects of the atmosphere. Assume that the LBT is looking at Vega,
which is the 5th brightest star in the night sky. Vega has an astronomical diameter
(Dsrc/zsrc) of 1.67e-8 rad. If the observation wavelength is 1μm, is the light
emitted from Vega spatially coherent when it reaches the telescope over the large
dimension of the telescope mirrors? Justify your answer.

b.) (4 pts.) For the sake of calculation, assume that both mirrors act as if they are part of
one larger mirror and that they focus to a common point a distance of 100 m
behind the mirrors. Also, you may assume that the mirrors are square, with a side
length of 8.4m and a center-to-center separation of 14.4m. If the wavelength of
observation is 1 μm, make a rough sketch of the normalized MTF of the system
along the (ξ',0) and (0,η') profiles. Include scales of spatial frequency on your
axes.

c.) (3 pts.) Ideally, what are the approximate dimensions of the smallest object that the telescope can resolve, relative to the two directions that are 1) across the wide dimension and 2) across the small dimension of the telescope system? State any assumptions that you make. Express your answer as the angular subtense in object space, which is the spatial resolution at the image plane divided by the focal length.

$$F_\xi\left[\text{rect}\left(\frac{x}{L}\right)\right]=L\,\text{sinc}\left(L\xi\right)$$

$$B_{\rho_\xi}\left[\text{circ}\left(\frac{\rho}{D}\right)\right]=\frac{\pi}{4}D^2\,\text{somb}\left(D\rho_\xi\right)$$
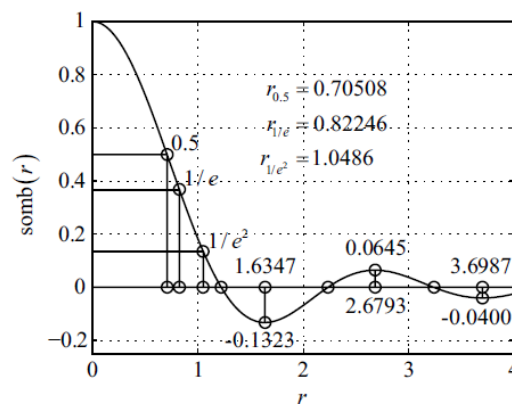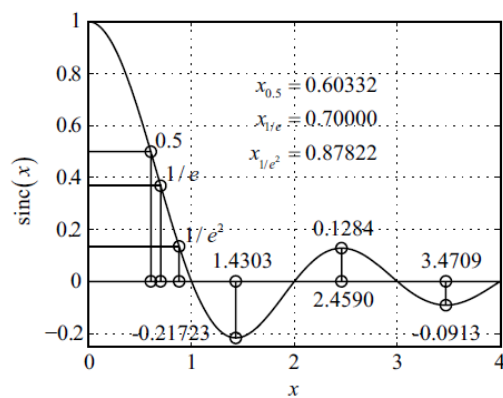
The wavefuction for the ground state of the hydrogen atom is given by $\psi_{100} = \frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o}$, where a_o is the Bohr radius, the quantum numbers associated with this state are given in the usual form Ψ_{nlm}, and $r = |\vec{r}|$. In solving some parts of this question, recall that in spherical coordinates $x = r\sin(\theta)\cos(\phi), y = r\sin(\theta)\sin(\phi), z = r\cos(\theta)$, and a differential volume element is $dxdydz = r^2 \sin(\theta)drd\theta d\phi$. The following integral relationship will also be helpful: $\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}$.

(a). Find $\langle r \rangle$ for an electron in the ground state of hydrogen ($not < \vec{r} >$). Express your answer in terms of the Bohr radius a_o.

(b). By how much does the internal energy of the atom change if it undergoes a transition from the ground state to the state ψ_{610}? If you do not remember the ionization energy of the hydrogen atom, express your answer in terms of the ground-state energy E_1 for partial credit.

(c). In going from the quantum state ψ_{100} to ψ_{610}, by how much does the magnitude of orbital angular momentum of the electron change?

(d). What polarization of light is needed to induce a transition from ψ_{100} to ψ_{610}?

(e). Now assume the atom is in a state described by the normalized wavefunction $\Psi = \frac{1}{\sqrt{8\pi a_o^3}} e^{-r/2a_o}$. Note that this is not the ground state. Determine the probability that a measurement of the atom's internal energy will produce a result corresponding to the ground state of the hydrogen atom.