WRITTEN PRELIM EXAM – FIRST DAY

Spring 2013

February 12, 2013
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

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$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Spring 2013 Written Comprehensive Exam
Opti 501

System of units: MKSA

A linearly-polarized light pulse of duration T, frequency ω_0, E-field amplitude E_0, and cross-sectional area A, propagates in free space. (The pulse is long enough and broad enough that one can ignore its spectral content and treat it simply as a section from a plane wave.) The pulse arrives at normal incidence at the flat surface of a linear, isotropic, homogeneous, semi-infinite material of refractive index n, where n is real-valued and greater than unity. You may assume that the transparent dielectric material is non-magnetic [i.e., $\mu(\omega)=1$] and non-dispersive (i.e., n does *not* vary with frequency ω within the bandwidth of the light pulse).

2 pts a) What is the total energy content of the light pulse in free space?

3 pts b) Describe the properties of the reflected light pulse (e.g., frequency, wavelength, duration, polarization state, total optical power).

3 pts c) Find the E-field and H-field amplitudes of the light pulse that enters the glass medium. Describe the properties of the light pulse that propagates within the glass medium.

2 pts d) Show that the total energy of the pulse is conserved upon reflection/transmission at the glass surface.

Spring 2013 Written Comprehensive Examination

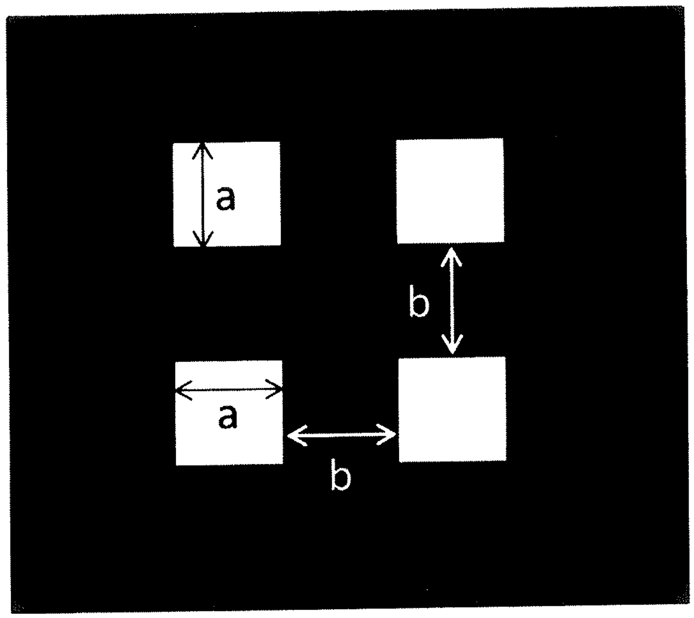
OPTI 502

To build an afocal system with magnification m=-2, the focal length of the first lens L1 is 50 mm and the diameter of L1 is 20mm

1. What is the required focal length of the second lens L2? Draw a diagram showing the distance between the lenses L1 and L2. (1 point)
2. Assume the diameter of the second lens L2 is 80 mm and there is no other aperture stop except the lenses themselves. Which lens acts as an aperture stop for this afocal system? What are the locations and diameters of the entrance and exit pupil? (3 points)
3. If an aperture stop is placed at the rear focal point of the first lens L1, what are the locations and diameters of the entrance and exit pupil of the system? (1 point)
4. The afocal system with the aperture stop at the back focal point of the first lens L1 is now to be used as an image relay. Assume lenses L1 and L2 are both plano-convex lenses. The object size is 5 mm. Provide an appropriate imaging configuration showing the location of the object and the image. Also note the correct orientation of the lenses. (3 points)
5. In the configuration of Part 4, the object is shifted 1mm toward the first lens L1. What are the location and size of the image? (2 points)

Spring 2013 Written Comprehensive Examination
OPTI 505

The 2×2 array aperture shown below is illuminated by a normally incident plane wave with wavelength λ.



a. (1.5 pts) How far from the aperture must we be in order to observe the *Fraunhofer* diffraction pattern without the aid of additional optics?

b. (2.0 pts) What is the observed irradiance at a distance z from the aperture in the Fraunhofer region for this aperture if only one of the four openings is uncovered? How does this irradiance change as we change which of the four is uncovered?

c. (4.0 pts) Assume now that $a = b$. Sketch the observed intensity in the Fraunhofer region when all four openings are uncovered. What happens to the diffraction pattern as b is increased while a is held constant?

d. (2.5 pts) Now sketch the observed irradiance in the Fraunhofer region when only the upper left and lower right openings are uncovered.

Spring 2013 Written Comprehensive Exam
Opti 546

This problem reveals the common physical origin of the electro-optic Pockels effect and harmonic generation in a crystal based on the following anharmonic oscillator model for the optical polarization $P(t)$

$$\frac{d^2P}{dt^2} + \omega_0^2 P + \kappa P^2 = \omega_0^2 \epsilon_0 \chi^{(1)} E. \tag{1}$$

Here $E(t)$ is the electric field, ω_0 the medium resonant frequency, κ controls the magnitude of the anharmonic effects, and $\chi^{(1)}$ is the linear susceptibility, the linear dielectric constant being given by $\epsilon^{(1)} = \epsilon_0(1 + \chi^{(1)})$.

(a - 1pt) Consider first the case of linear optics ($\kappa = 0$) with a monochromatic field $E(t) = \frac{1}{2}[E(\omega)e^{-i\omega t} + c.c.]$ that is off-resonant $\omega << \omega_0$. For this case show that one may safely neglect the time derivative term in Eq. (1), resulting in the linear polarization $P(t) = P^{(1)}(t) = \epsilon_0 \chi^{(1)} E(t)$.

(b - 1pt) Within the same off-resonant approximation as in part (a), and for $\kappa \neq 0$, find a solution for $P(t)$ in terms of $E(t)$. (Hint: You need the solution for which $P(t) = 0$ for $E(t) = 0$.)

(c - 2pts) In practice anharmonic effects are small meaning that terms at most linear in κ need to be retained from your solution from part (b). Then writing the polarization as $P(t) = P^{(1)}(t) + P^{(2)}(t)$, with $P^{(1)}(t)$ as in part (a), prove that the second-order polarization $P^{(2)}(t) = \epsilon_0 \chi^{(2)} E^2(t)$ and give an expression for the parameter $\chi^{(2)}$. (Hint: $\sqrt{1+x} = 1 + x/2 - x^2/8 + \ldots$).

(d - 3pts) Consider the case $E(t) = E_s + E_o(t)$, with E_s a static field and $E_o(t)$ the optical field, where for electro-optics $|E_s| >> |E_o|$. Based on this show that the polarization for the optical field may be written as $P_o(t) = \epsilon \chi E_o(t)$with $\chi = \chi^{(1)} + 2\chi^{(2)} E_s$, and show that this produces a change in dielectric constant $\Delta\epsilon_o = 2\epsilon_0 \chi^{(2)} E_s$ for the optical wave.

(e - 3pts) In nonlinear optics the anharmonic terms are due solely to the optical field. For the case of an incident monochromatic electric field demonstrate that the second-order polarization from part (c) yields polarization source terms at frequencies $\Omega = 0, 2\omega$.

Spring 2013 Written Comprehensive Exam
Opti 511R

Consider a laser composed of a linear cavity with 2 identical mirrors with radii of curvature of 20cm, each having power reflectivity coefficients $R = 0.99$ across the entire visible spectrum. A homogeneously broadened 10 cm long gain medium inside the cavity is modeled by the 4-level system diagramed below, with population densities N_0, N_1, N_2, N_3. $N_T = 1 \times 10^{19} cm^{-3}$ is the total population density of this closed 4-level system. The energy difference between levels 2 and 1 is $\Delta E_{21} = 3 \times 10^{-19}$ J and the energy difference between levels 1 and 0 is $\Delta E_{10} = 3.7 \times 10^{-19}$ J. The spontaneous decay rates between dipole allowed transitions (Γ_{ij}) are also shown. In this problem we will assume the decay from level 3 to 2 is instantaneous, such that $N_3 \approx 0$. An external pumping rate P is required for population inversion. Recall that the small-signal gain coefficient can be written as: $\gamma_o = \Delta N \cdot \sigma_0$, where ΔN is the population difference between any 2 energy levels. The maximum resonant absorption cross section for this gain medium is $\sigma_0 = 3 \times 10^{-20} cm^2$. You may assume the optical path length through the gain medium is negligible,but there is a power scattering loss of 1% per pass in the gain medium.

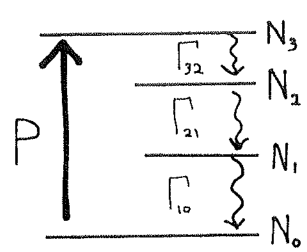
a. What is the longest cavity length for which a stable optical cavity exists?

b. What is the threshold gain coefficient for lasing, γ_t?

c. Assuming a shutter is placed inside the cavity to prevent lasing, set up the rate equations for this system and write down the steady-state solutions for N_2/N_1 and N_1/N_0 in terms of the remaining parameters.

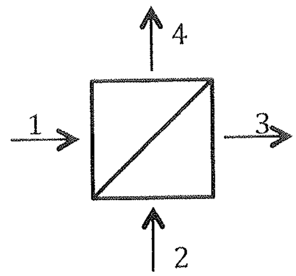
d. If $\Gamma_{10} = 1800$ s^{-1} and $\Gamma_{21} = 180$ s^{-1}, at what wavelength would the laser operate assuming the pumping rate is sufficiently high and the shutter is removed? Calculate the maximum small-signal gain coefficient for this system assuming a pumping rate of $P = 1800$ s^{-1} and determine if it is possible for lasing to occur.

e. On the same graph, plot and compare the gain coefficient (γ) versus pumping rate for an ideal 3-level laser versus an ideal 4-level laser. (Note: γ is the actual gain coefficient and not the same as the small-signal gain coefficient γ_o). Your plot should not contain quantitative numbers, but should start at $P = 0$ and increase beyond the lasing threshold. Be sure to label $\gamma = 0$ and $\gamma = \gamma_t$ on the plot.



Spring 2013 Written Comprehensive Exam
OPTI 544

A 50/50 optical beamsplitter has input ports 1 and 2, and output ports 3 and 4, as illustrated in the following figure.



Recall that if the beamsplitter input state (ports 1 and 2 together) is described in terms of photon creation operators $a_1^\dagger$ and $a_2^\dagger$ acting on the vacuum, the 2-mode output (in the Schrodinger picture) can be found by utilizing the following relationships between the photon creation operators of the four modes:

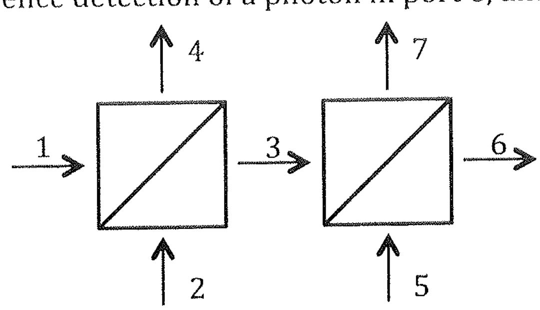
$$a_1^\dagger = \frac{1}{\sqrt{2}} a_3^\dagger + \frac{i}{\sqrt{2}} a_4^\dagger \qquad \text{and} \qquad a_2^\dagger = \frac{i}{\sqrt{2}} a_3^\dagger + \frac{1}{\sqrt{2}} a_4^\dagger.$$

(a - 2 pts) Consider an input state $|\psi_{in}\rangle = |1\rangle_1 |0\rangle_2$, which is interpreted as a one-photon number state in port 1 and vacuum input in port 2. Find the output $|\psi_{out}\rangle$ in terms of photon number states of modes 3 and 4.

(b - 2 pts) For input state $|\psi_{in}\rangle = |1\rangle_1 |1\rangle_2$, find the output state $|\psi_{out}\rangle$. What does your answer imply about the probability of coincidence detection of a photon in port 3 and one in port 4, given this input state?

(c - 4 pts) Consider the two-beamsplitter arrangement illustrated at the bottom of the page, with 3 input ports (1, 2, and 5), and 3 output ports (4, 6, and 7). Given an input state $|\psi_{in}\rangle = |1\rangle_1 |1\rangle_2 |0\rangle_5$, find the output state in terms of number states of modes 4, 6, and 7.

(d - 2 pts) For input state $|\psi_{in}\rangle = |1\rangle_1 |1\rangle_2 |0\rangle_5$, as in part (c), give the probability of detecting 0, 1, or 2 photons in ports 4. Do the same for ports 6, and 7. What is the probability of coincidence detection of a photon in port 6, and one in port 7?



Spring 2013 Written Comprehensive Examination
OPTI 503

Assume that the wavelength of light is 0.0005 mm, thin lenses in air, and an object at infinity.

1. For a rotationally symmetric optical system that has 10 waves of third-order astigmatism aberration at full field and no other aberrations, plot the wave fans in the principal sections: meridional and sagittal.
2. Describe how in a rotationally symmetric system this astigmatism aberration varies with the field of view and the aperture.
3. The system is an achromatic doublet that has a focal length of 100 mm and an aperture of 10 mm. The glass specification for the crown glass is 500640 and for the flint glass is 600360. For the half field of field position determine the longitudinal distance between the astigmatic focal line segments. The stop aperture is at the lens.
4. Determine the focal lengths of the crown and flint elements in this 100 mm focal length achromatic doublet.
5. The Lagrange invariant of the system is 1.0 mm. What is the full field of view in degrees of this achromatic doublet?

Spring 2013 Written Comprehensive Exam
OPTI 506

For each of the following radiometric quantities, give the usual symbols, the differential definition and the SI units. Define any auxiliary quantities that you introduce.

a. Radiance

b. Irradiance

c. The spectral radiant intensity (per unit wavelength)

d. Spectral photon radiant intensity (per unit photon energy)

Answer each of the following questions by giving an appropriate integral. Carefully state the variables of integration, the limits of integration and the variable, if any, remaining in the result of the integration. You should provide a sketch for each part, but you do not have to perform any of the integrals. Be sure that all symbols are clearly defined, and state any approximations you make

e. How can you calculate the irradiance at an arbitrary point on a surface if the radiance is known?

f. How can you compute the irradiance in the image plane of an ideal thin lens viewing a Lambertian object? You may assume that the lens has a small numerical aperture, but do not assume that the object radiance is spatially constant.

g. How do you compute the flux on a circular detector of area A_{det} on the optical axis in the image plane of a lens satisfying the Abbé sine condition and viewing a Lambertian object as in part e? In this case, do not assume that the lens has a small numerical aperture, and again do not assume that the object radiance is spatially constant or even that it is slowly varying across the detector. You may, however, assume that the detector diameter is small compared to the distance from the lens to the detector.

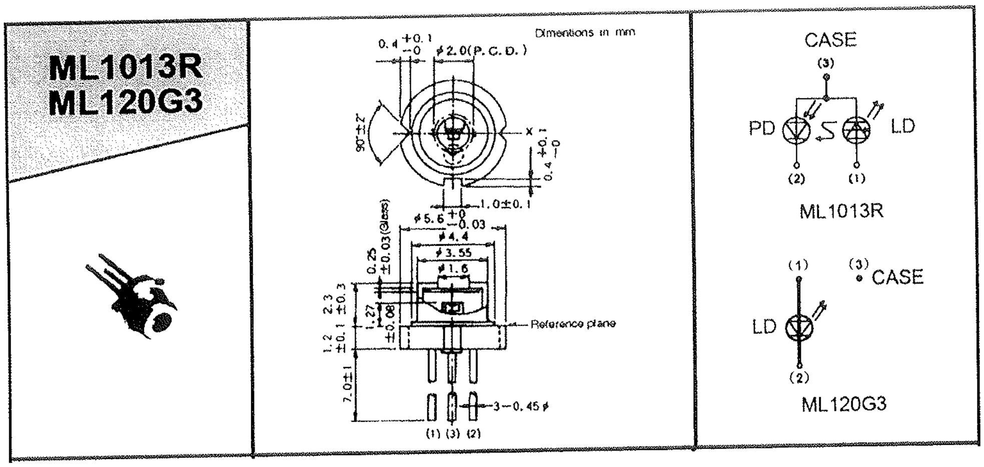
Parts a-d are worth one point each, parts e-g are worth two points each.

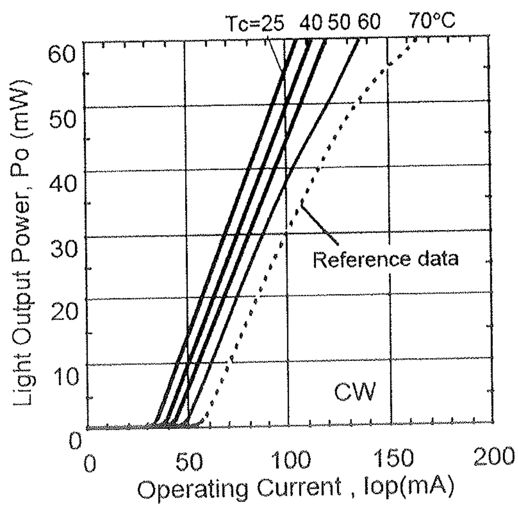
Spring 2013 Written Comprehensive Examination
OPTI 509

Assume that the wavelength of light is 0.0005 mm, thin lenses in air, and an object at infinity.

1. For a rotationally symmetric optical system that has 10 waves of third-order astigmatism aberration at full field and no other aberrations, plot the wave fans in the principal sections: meridional and sagittal.
2. Describe how in a rotationally symmetric system this astigmatism aberration varies with the field of view and the aperture.
3. The system is an achromatic doublet that has a focal length of 100 mm and an aperture of 10 mm. The glass specification for the crown glass is 500640 and for the flint glass is 600360. For the half field of field position determine the longitudinal distance between the astigmatic focal line segments. The stop aperture is at the lens.
4. Determine the focal lengths of the crown and flint elements in this 100 mm focal length achromatic doublet.
5. The Lagrange invariant of the system is 1.0 mm. What is the full field of view in degrees of this achromatic doublet?

Spring 2013 Written Comprehensive Exam
OPTI 510
Problem 1: Semiconductor laser





Above is the light current curve of a window-mirror-facet AlGaInP semiconductor laser with output at 685nm (Mitsubishi ML1013R). The operating voltage of the semiconductor laser is 2.5V at 50mW output power.

(a) Explain why there are three pins on the laser diode. What voltages do you apply to pin 1 to 3 for normal operation? (2 points)
(b) Name two factors that determine the response time of the laser? (1 point)
(c) What are the threshold current I_t (A) and the differential responsivity, R_d (W/A), of the laser at 25C? (2 points)
(d) In general, how do the threshold current and differential responsivity change with increasing operating temperature and why? (2 points)
(e) Can you estimate the number of electrons that are injected into the junction to extract a single photon at T=25C above threshold, assuming 2.5V and 50mW output power? (3 points)

Spring 2013 Written Comprehensive Examination
OPTI 536

Consider a brightfield (transillumination) microscope with Koehler illumination. The microscope is infinity corrected with a tube lens focal length of 200 mm. It has a 4X objective with an NA (object space NA) of 0.1. The stop is at the rear focal plane of the objective. The microscope has a 10X FN 22 ocular (designed for the 22 mm diameter field at the intermediate image plane of this microscope). The condenser lens in the illuminator has a focal length of 12.5 mm.

a. (1 point) For the objective lens, what does 4X mean?
b. (1 point) For the ocular, what does 10X mean?
c. (1 point) What is the magnifying power of the microscope?
d. (1 point) What is the lateral magnification of this optical imaging system (including the eye) from object plane to retina? You can assume the eye is a simple lens with a focal length of 17.1 mm (22.8 mm rear focal length).
e. (1 point) What is the field of view at the object plane?
f. (1 point) What is the diameter for the condenser aperture in order to achieve proper Koehler illumination?
g. (2 points) What is the spatial resolution achievable in the object assuming this is an incoherent diffraction-limited imaging system? (State your result in terms of the Rayleigh criterion for the distance between two just-resolvable point objects.) You can assume this is a situation where the microscope, not the eye, is limiting the achievable resolution performance and the wavelength of light is 500 nm.
h. (2 points) If the field aperture of the Koehler illuminator is adjusted to just fill the object plane field of view (your answer to part c) and you measure a power of 1mW (10^{-3} Watts) getting through the condenser aperture, what is the radiance distribution at the object plane? You can assume the condenser aperture was adjusted to just fill the NA of the objective and there is no object present to attenuate or redistribute the illumination light distribution at the object plane.

Spring 2013 Written Comprehensive Exam
OPTI 507

Consider the dispersion relation $\omega(k_x)$ of a surface plasmon polariton (sometimes simply called a surface plasmon), which can be written as

$$k_x^2 = \frac{\omega^2}{c^2}\frac{\varepsilon_1\varepsilon_2(\omega)}{\varepsilon_1+\varepsilon_2(\omega)}$$

Assume the dielectric to have a dielectric constant of $\varepsilon_1 = 1.5$ and the metal to have the dielectric function

$$\varepsilon_2(\omega) = 1 - \frac{\omega_{pl}^2}{\omega^2}$$

with $\omega_{pl} = 10^{12}\ \mathrm{s}^{-1}$. Determine (i) the dispersion relation at small frequencies and (ii) the asymptotic value of the frequency (in units of s^{-1}) in the short-wavelength limit. Use that information to plot a qualitatively correct dispersion relation $\omega(k_x)$ for the surface plasmon. Properly label the axes, all special points and the slope (given in terms of the speed of light, c) in the long-wavelength limit.

(10 points)

Spring 2013 Written Comprehensive Examination
OPTI 537

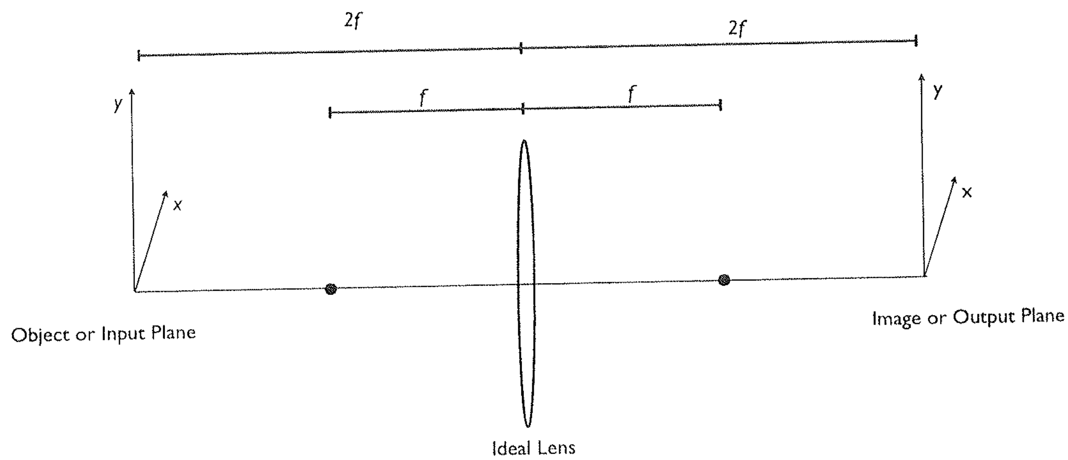
Consider imaging systems that are imaging coherent, monochromatic objects with wavelength λ. Fresnel diffraction for a system described by an ABCD matrix is given by:

$$u_{out}(\vec{r}) = \frac{-i}{B\lambda} \int_{-\infty}^{\infty} d^2 r_0 \, u_{in}(\vec{r_0}) \exp\left(\frac{i\pi}{\lambda B}[A|\vec{r_0}|^2 + D|\vec{r}|^2 - 2\,\vec{r}\cdot\vec{r_0}]\right) \qquad Eq\ (1)$$

where both $\vec{r}$ and $\vec{r_0}$ are 2 vectors in the x-y plane. (A selection of ABCD matrices is given in the table on the next page.)

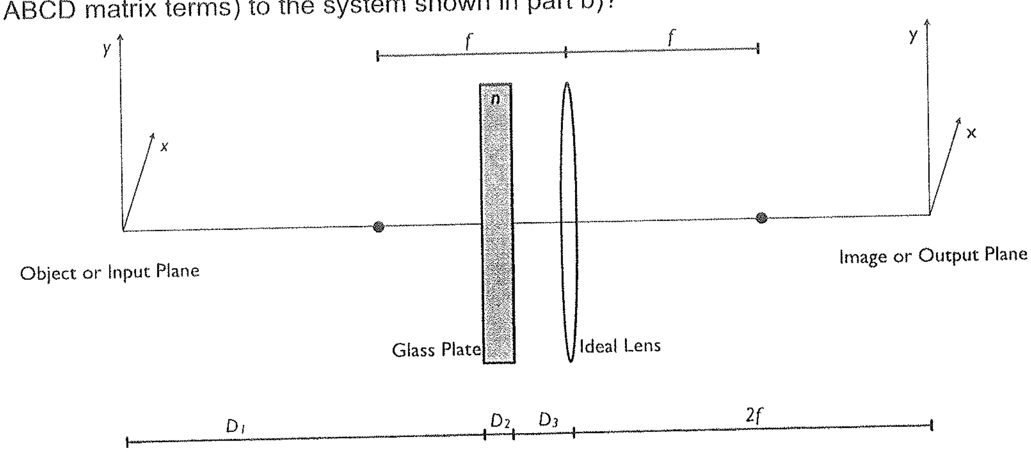
(a) Show that Equation 1 above simplifies to standard Fresnel transform when considering free-space propagation.

(b) Now consider the simple imaging system below,



Write down the ABCD matrix for this system and describe how to use Equation 1 to describe the propagation of a field from the input plane to the output plane.

(c) Finally, consider the following system where we have inserted a piece of glass with index of refraction n before the lens. The distance from the lens to the image plane remains $2f$. However the distances D_1, D_2, and D_3 no longer add up to $2f$. Compute the ABCD matrix for this system. What condition must be true for this imaging system to be equivalent (in ABCD matrix terms) to the system shown in part b)?



ABCD Matrices:

Propagation in a medium of constant refractive index	$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$	d = distance along optical axis
Refraction at a flat interface	$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$	n1 = initial refractive index n2 = final refractive index
Refraction by a thin lens	$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$	F = focal length (f>0 means convex lens)

WRITTEN PRELIM EXAM – SECOND DAY

Spring 2013

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8:30 a.m. to 12:00 p.m.

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$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$

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$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

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$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

Spring 2013 Written Comprehensive Exam
Opti 501

System of units: MKSA

The classical theory of electrodynamics describes various relationships among fields and their sources. (Some of the fields are composite constructs in the sense that they are defined as combinations of fields and sources.) Maxwell's macroscopic equations govern the evolution of the fields in free space and in the presence of the sources.

1 pt a) Use a sentence or two to describe each source of the electromagnetic field in a qualitative way.

1 pt b) Give a brief description of each field that appears in Maxwell's equations.

2 pts c) Using Maxwell's equations, prove that free charge and free current densities satisfy the charge-current continuity equation.

3 pts d) Explain the notions of bound-charge and bound-current in the classical theory. Use the most general form of Maxwell's equations to derive expressions for the bound-charge and bound-current densities associated with polarization $\boldsymbol{P}(\boldsymbol{r},t)$ and magnetization $\boldsymbol{M}(\boldsymbol{r},t)$.

3 pts e) Show that bound-charge and bound-current densities satisfy the charge-current continuity equation.

Spring 2013 Written Comprehensive Examination

OPTI 502

Consider an optical system depicted in Fig. 1. The object O is located at the center of a sphere having a radius of $|R| = 100$ mm. The index of refraction of the sphere is $n = 1.5$. A concave mirror concentric with the sphere has a radius of curvature $|R_{mirror}| = 1.5|R| = 150$ mm. An aperture stop is placed on the concave mirror. The object height is 10 mm.

1) (3 points) Sketch the marginal ray and the chief ray.
2) (3 points) Determine the image position and the magnification between the object and image.
3) (4 points) Determine the height of the chief ray at the surface of the sphere (assume paraxial rays).

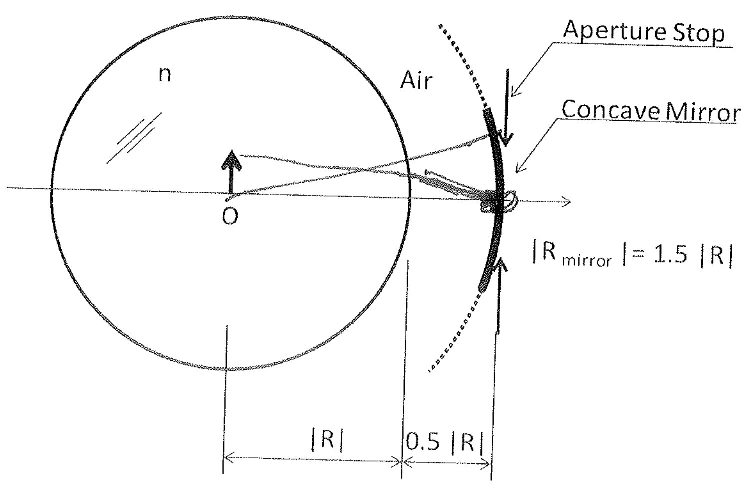


Fig. 1

Spring 2013 Written Comprehensive Exam
OPTI 505

A 200 mm focal length is used to image an object. The stop is located at the back focal length of the lens and is 10 mm in diameter. The object is 1 m from the lens. The lens suffers no aberration.

a.) (2.5 pts) If the target is illuminated with a 550 nm wavelength on-axis laser beam, what is the highest image spatial frequency (expressed in mm^-1) recorded?

b.) (2.5 pts) The object is now illuminated with a 550 nm wavelength spatially incoherent light source. What is the cutoff frequency of the modulation transfer function (expressed in mm^-1) in image space?

c.) (2.5 pts) Two points in the object are illuminated coherently with a 550 nm wavelength on-axis laser beam. What is the minimum separation of the two points in object space before they can no longer be resolved? Provide justification for your answer.

d) (2.5 pts) Two points in the object are illuminated incoherently with a 550 nm source. What is the minimum separation of the two points in object space before they can no longer be resolved? Provide justification for your answer.

Spring 2013 Written Comprehensive Exam
Opti 546

This problem explores ray and Gaussian beam propagation across an optical system. The ABCD law for the evolution of the complex beam parameter $\frac{1}{q} = \frac{1}{R} + \frac{2i}{kw^2}$ across the optical system is

$$\frac{1}{q_1} = \frac{C + D/q_0}{A + B/q_0}, \qquad M_L = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \qquad M_f = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix},$$

M_L and M_f being the ray transfer matrices for free-space and a thin lens.

(a - 3pts) A non-imaging optical system which is prescribed by a ray transfer matrix with elements A, B, C, D may be realized using a lens of focal length f_1 followed by a section of free-space of length B and finally a thin lens of focal length f_2. Find expressions for the focal lengths f_1 and f_2 in terms of the ray matrix elements A, B, C, D so that the above statement is true.

(b - 2pts) For the remainder of this question we consider a telescopic or afocal optical system with $A = 2, B = 20$ cm. Using your results from part (a) sketch an optical system that would realize this optical system making sure to quote the required values for f_1 and f_2 in cm.

(c - 2pts) Consider that an image represented by collimated rays is incident on the optical system of part (b). Using arguments based on ray transformations describe the form of the image at the output.

(d - 2pts) Next consider that a collimated Gaussian beam of incident spot size w_0 is incident at the input to the optical system from part (b). Assuming that $|A| >> |B|/z_R$, with $z_R = kw_0^2/2$ the Rayleigh range of the incident Gaussian beam, derive expressions for the output inverse radius of curvature $(1/R_1)$ and the ratio (w_1/w_0) of the output and incident spot sizes.

(e - 1pts) Discuss the relation between your results from parts (c) and (d).

Spring 2013 Written Comprehensive Exam
Opti 511R

Consider a particle of mass m trapped in a 2-dimensional simple harmonic oscillator potential well. The potential well is characterized in the usual way, with oscillation frequencies ω_x and ω_y in the x and y directions, respectively. In this problem, $\omega_x = \omega_y$. The solutions to the time-independent Schrödinger equation may be written as the product state $\psi_{m,n}(x,y) = \psi_m(x) \cdot \psi_n(y)$. For example, the state $\psi_{10}(x,y) = a_{10} \cdot x \cdot e^{-(x^2+y^2)/(2\sigma^2)}$, where a_{10} is the normalization coefficient.

a. (1.5 pts) Write out the Hamiltonian for the 2-dimensional simple harmonic oscillator.

b. (1.5 pts) Write an expression for the eigenvalues of the Hamiltonian.

The operator for the z-component of angular momentum is given by $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$.

c. (2 pts) Show that the states ψ_{10} and ψ_{01} are not eigenstates of the $\hat{L}_z$ operator.

Now assume the particle is in the following superposition state:

$$\phi(x,y) = \frac{1}{\sqrt{2}}(\psi_{01} + i\psi_{10}) \tag{1}$$

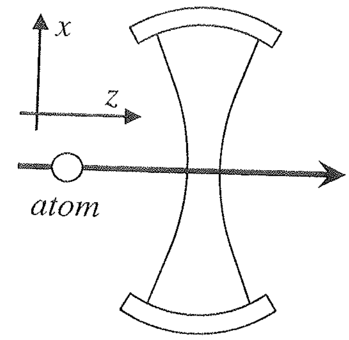
d. (2 pts) Show that $\phi(x,y)$ is an eigenstate of $\hat{L}_z$ and determine the eigenvalue.

e. (1.5 pts) Is $\phi(x,y)$ an eigenstate of the energy operator? Please show your work to justify your answer, or explain your reasoning.

f. (1.5 pts) Is it possible to simultaneously determine both the exact energy and angular momentum of the particle in state $\phi(x,y)$? Justify you answer.

Spring 2013 Written Comprehensive Exam
OPTI 544

In celebration of the 2012 Nobel Prize in Physics, we here consider atom-light interaction in optical cavities. Our configuration consists of a two-level atom traveling along the z-axis and crossing the cavity mode parallel to the wave fronts as shown in the accompanying figure. For simplicity we assume in the following that the cavity field is resonant with the atomic transition, that the transit time is short enough that we can ignore decay of both the atomic excited state and the intracavity light field, and that any light induced forces are small enough that the atomic velocity v remains constant.



We begin with a classical description of the intracavity field, in which case the atom-light coupling is characterized by a Rabi frequency $\chi(z) = \chi_0 \exp(-z^2/2\sigma^2)$.

(a) Write down an expression for the angle θ between the Bloch vectors before and after the atom has crossed the cavity, as a function of χ_0, σ and v. For an atom initially in the ground state, sketch the corresponding trajectory on the Bloch sphere.

(b) Based on your answer in (a), find the velocities v_g at which the atom exits the cavity in the ground state. Similarly, find the velocities v_e at which the atom exits in the excited state. Make a plot that shows the probability P_e for the atom to exit in the excited state as function of v, without worrying too much about the detailed behavior as $v \to 0$.

Next, we switch to a quantum description of the intracavity field, in which case the atom-light coupling is characterized by a vacuum Rabi frequency $g(z) = g_0 \exp(-z^2/2\sigma^2)$.

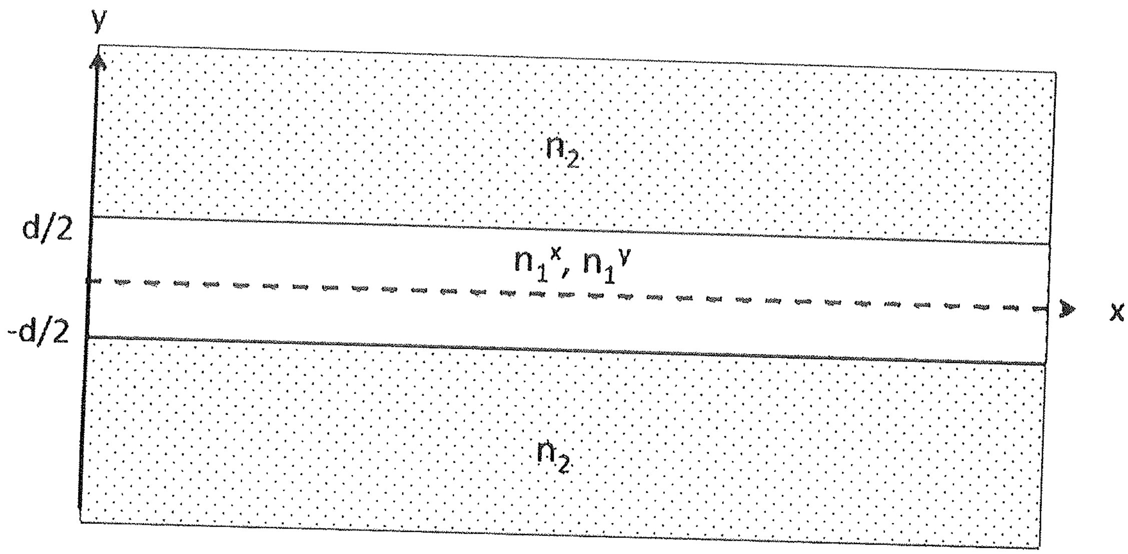
(c) For an atom initially in the ground state and the cavity field initially in a number state $|n\rangle$, find the velocities v_g and v_e as function of n.

(d) Assume the cavity field is known to be in either the ground or first excited state. Based upon the results from (c), describe how you might determine which of these states is actually present with 100% accuracy, by shooting a single atom though the cavity and measure whether it exits in the ground or excited state.

Helpful Math: $\int_{-\infty}^{\infty} Exp(-a^2b^2)\,da = \sqrt{\pi/b^2}$

Birefringent slab waveguide

This problem concerns the waveguide structure shown below



A slab waveguide exists as shown. The core material is birefringent with refractive indices n_1^x and n_1^y for the **x** and **y** directions as shown. Propagation at wavelength λ_0 = 1550nm is along **z** (out of the plane of the paper). The cladding materials can be assumed to extend to y = +/- ∞, and all materials are assumed to extend to x = +/- ∞.

a) Make a simple diagram to indicate the electric field polarization direction for **TE** and **TM** guided modes; you can ignore longitudinal field components. (2 points)

b) If n_2 = 1.4440, n_1^x = 1.4584 and n_1^y = 1.4657, what is the numerical aperture for **TM** modes? (2 points)

c) Assuming the same indices given in Part b), we would like to make the waveguide single-mode for **TE** propagation. What is the thickness d_{mm} at which the waveguide will become multimode for **TE** propagation at λ_0 = 1550nm? (3 points)

d) A properly designed birefringent waveguide can be used as a waveguide polarizer, which is a device that has a guided mode for a given polarization but does not have a guided mode for the orthogonal polarization. Explain why the waveguide as shown cannot be used as waveguide polarizer regardless of the values of the refractive indices and thickness, and indicate generally what could be changed so that this is not the case. (3 points)

Spring 2013 Written Comprehensive Exam
OPTI 507

Consider a cubic crystal with lattice constant a, shaped in the form of a cube of volume $V = L^3$ (where L is the edge length of the cube). Denote the number of unit cell in each Cartesian direction by N. Write down the three primitive translation vectors for this crystal. Also, write down a Bloch wave function and show that, under periodic boundary conditions, the Bloch wave functions yield a condition on the wave vectors restricting them to form a discrete set for each Cartesian coordinate. Using that set, show that a sum over wave vectors can approximately be written as an integral:

$$\frac{1}{V}\sum_{\vec{k}} \sim \text{const.} \int d^3k$$

Determine the constant in front of the integral.

(10 points)

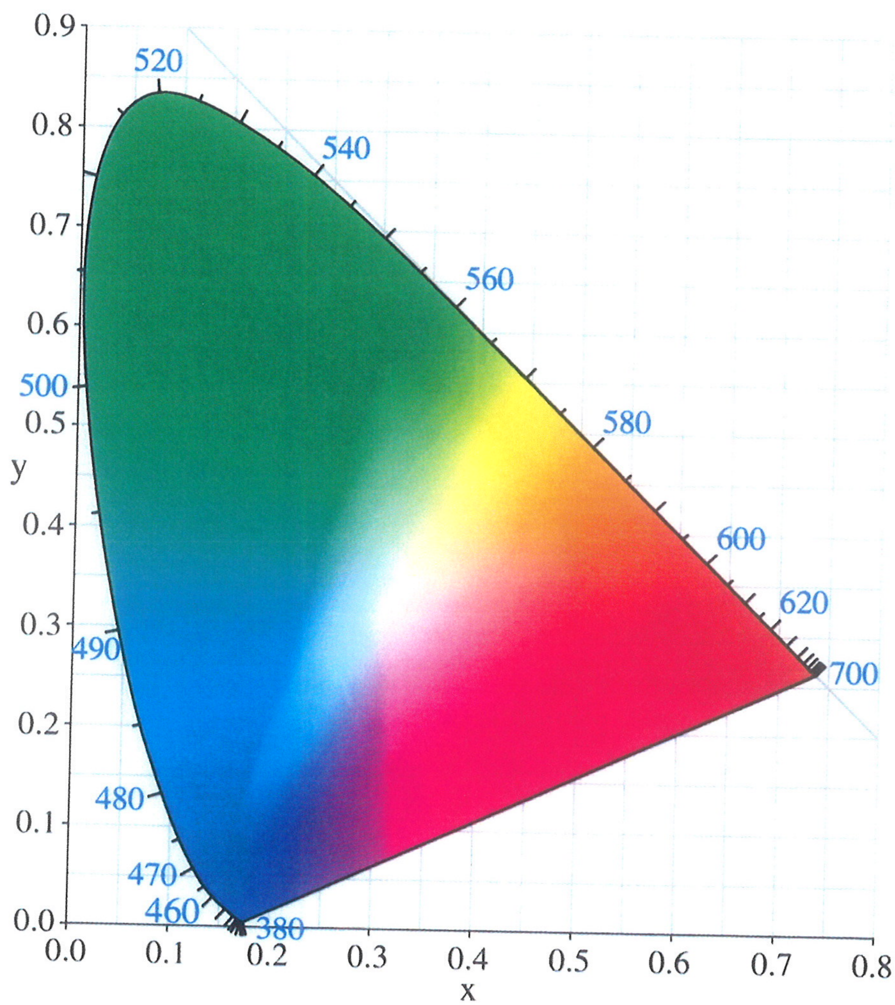
The CIE xy chromaticity diagram is shown below. The xy-chromaticity coordinates and luminance values of two LEDs are:

LED1:	x_1=0.65	y_1=0.3	L_1=100 lm/(m^{2}sr)
LED2:	x_2=0.1	y_2=0.7	L_2=200 lm/(m^{2}sr)

(3 points) Compute the XYZ-tristimulus values of the two LEDs individually.

(3 points) When the two LEDs are combined, what are the chromaticity coordinates and luminance of the resulted color?

(4 points) Determine the approximate dominant wavelength and the color purity of the resulted color when the LEDs are combined.



Spring 2013 Written Comprehensive Examination
OPTI 509

Jupiter is one of the brightest objects in the night sky. I'd like to take some pictures of it with a camera attached to a telescope. I need to estimate some of the radiometric properties of the image of Jupiter on the sensor of my camera. Some facts that you might need are:

The diameter of the Sun: $1.39 \cdot 10^9$ m
The diameter of Jupiter: $1.42 \cdot 10^8$ m
The diameter of Earth: $1.27 \cdot 10^7$ m
The distance between the Sun and Jupiter: $7.78 \cdot 10^{11}$ m
The distance between the Earth and Jupiter: $6.29 \cdot 10^{11}$ m
Radiance of the sun in the band the detector responds to: $1.42 \cdot 10^7$ W/m^2/sr
The reflectivity of Jupiter: Lambertian but only 20% of the power is reflected
Diameter of telescope mirror: 150 mm
Numerical aperture of telescope: 0.0625

You may assume that the telescope is an ideal, thin reflector with no central obscuration and 100% efficiency. For all of the questions, you may assume that there are no other sources of light besides Jupiter. You may neglect any effects of the atmosphere as well.

a. What is the irradiance from the sun on Jupiter?
b. What is the radiance of Jupiter?
c. What is the irradiance from Jupiter on the surface of the Earth?
d. What is the radiance in my image of Jupiter on the sensor?
e. What is the irradiance in my image of Jupiter on the sensor?
f. What is the total power in my image of Jupiter on the senor?
g. What changes could you make to the telescope design to increase the radiance in the image?
h. What changes could you make to the telescope design to increase the irradiance on the sensor while maintaining the total power?
i. What changes could you make to the telescope design to increase the total power while maintaining the irradiance on the sensor?

Spring 2013 Written Comprehensive Exam
OPTI 536

Each part of this question presents a verbal description of a specific linear imaging system. For each, explain how the abstract imaging equation $\boldsymbol{g} = \mathcal{H}\boldsymbol{f}$ applies. That is, give the physical meaning and typical units of the object **f** and the image $\boldsymbol{g}$. If $\boldsymbol{f}$ and/or $\boldsymbol{g}$ is a function, state its argument; if it is a discrete vector, explain the meaning of the components. Then give the mathematical form of the imaging operator in terms of the functions or vectors, stating any approximations needed for your equations to be valid. You do not need to give an explicit form for the kernel or matrix elements of $\mathcal{H}$. Classify each system as **CC**, **CD** or **DD** in the terminology used in class and in Barrett and Myers. Comment briefly on the conditions necessary for linearity to hold.

A. An ideal thin lens imaging an incoherent planar object onto a viewing screen.

B. A lens with coma and astigmatism imaging an incoherent planar object onto a viewing screen.

C. An ideal thin lens imaging a coherently illuminated planar transparency onto a plane.

D. A cellphone camera imaging a portrait in an art gallery.

E. A fluorescent microscope imaging the 3D distribution of a fluorescent protein onto a CCD camera.

F. A computer simulation of the microscope in part D.

Spring 2013 Written Comprehensive Examination
OPTI 506

Jupiter is one of the brightest objects in the night sky. I'd like to take some pictures of it with a camera attached to a telescope. I need to estimate some of the radiometric properties of the image of Jupiter on the sensor of my camera. Some facts that you might need are:

The diameter of the Sun: $1.39 \cdot 10^9$ m
The diameter of Jupiter: $1.42 \cdot 10^8$ m
The diameter of Earth: $1.27 \cdot 10^7$ m
The distance between the Sun and Jupiter: $7.78 \cdot 10^{11}$ m
The distance between the Earth and Jupiter: $6.29 \cdot 10^{11}$ m
Radiance of the sun in the band the detector responds to: $1.42 \cdot 10^7$ W/m^2/sr
The reflectivity of Jupiter: Lambertian but only 20% of the power is reflected
Diameter of telescope mirror: 150 mm
Numerical aperture of telescope: 0.0625

You may assume that the telescope is an ideal, thin reflector with no central obscuration and 100% efficiency. For all of the questions, you may assume that there are no other sources of light besides Jupiter. You may neglect any effects of the atmosphere as well.

a. What is the irradiance from the sun on Jupiter?
b. What is the radiance of Jupiter?
c. What is the irradiance from Jupiter on the surface of the Earth?
d. What is the radiance in my image of Jupiter on the sensor?
e. What is the irradiance in my image of Jupiter on the sensor?
f. What is the total power in my image of Jupiter on the senor?
g. What changes could you make to the telescope design to increase the radiance in the image?
h. What changes could you make to the telescope design to increase the irradiance on the sensor while maintaining the total power?
i. What changes could you make to the telescope design to increase the total power while maintaining the irradiance on the sensor?

Spring 2013 Written Comprehensive Examination
OPTI 537

Answer the following questions related to semiconductor detectors. All parts weighted as indicated.

(a) (10%) Write the expression that defines the relationship between the direct and reciprocal lattices for a perfect crystal. Define all terms.

(b) (15%) What is the 1st Brillouin zone and what is its volume for an orthorhombic crystal with primitive direct lattice constants a=3 Å, b=4 Å, and c=5 Å.

(c) (15%) What is the significance of the reciprocal lattice when, for example, one is writing a Fourier representation the periodic potential $V(\mathbf{r}) = V(\mathbf{r}+\mathbf{R})$. ($\mathbf{R}$ spans all equivalent locations in the direct Bravais lattice).

(d) (15%) Consider a translation operator T_R such that $T_{\mathbf{R}}\left[\psi(\mathbf{r})\right] = \psi(\mathbf{r}+\mathbf{R})$. If $\mathbf{R}$ is again the translation vector that spans equivalent locations in the periodic crystal and $\psi(\mathbf{r})$ is an eigenfunction of $T_{\mathbf{R}}$, use the properties of the fourier transform to work out the associated eigenvalue. What have you just derived?

(e) (15%) What is the Born von Karmann boundary condition? Use it together with the nearly-free electron model and the result of (d) above to work out and explain whether an orthorhombic crystal with L copies of the unit cell in each direction, and 1 valence electron per unit cell, is an insulator, semiconductor, or metal. (OK to work out in 1 dimension).

(f) (15%) Make a sketch of the physical structure of a CCD constructed on a wafer of P-type silicon. Make a separate sketch showing the potential structure that explains how the device functions as a photodetector, how the photoelectrons are stored, and how the readout mechanism works. Indicate relative concentrations of majority and minority carriers.

(g) (15%) Make a sketch of the basic circuit of unit cell of a CMOS imaging sensor, how the photoelectrons are stored, and how the readout mechanism works. Make a separate sketch showing the potential structure of the PN junction that explains how the device functions as a photodetector. Indicate the relative concentrations of majority and minority carriers in the N and P sides.