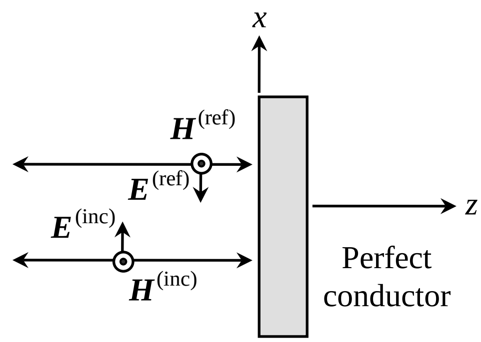
Spring 2015 Written Comprehensive Exam
Opti 501

System of units: MKSA

A monochromatic plane-wave of frequency ω traveling in free space is reflected at normal incidence from the flat surface of a perfect conductor. Denoting the speed of light in vacuum by $c=1/\sqrt{\mu_0\varepsilon_0}$ and the impedance of free space by $Z_0=\sqrt{\mu_0/\varepsilon_0}$, the incident and reflected E- and H-fields are given by

$$\begin{cases} \boldsymbol{E}^{(\mathrm{inc})}(\boldsymbol{r},t) = E_0\hat{\boldsymbol{x}}\cos[(\omega/c)\,z-\omega t], \\ \boldsymbol{H}^{(\mathrm{inc})}(\boldsymbol{r},t) = (E_0/Z_0)\hat{\boldsymbol{y}}\cos[(\omega/c)\,z-\omega t]. \end{cases}$$

$$\begin{cases} \boldsymbol{E}^{(\mathrm{ref})}(\boldsymbol{r},t) = -E_0\hat{\boldsymbol{x}}\cos[(\omega/c)\,z+\omega t], \\ \boldsymbol{H}^{(\mathrm{ref})}(\boldsymbol{r},t) = (E_0/Z_0)\hat{\boldsymbol{y}}\cos[(\omega/c)\,z+\omega t]. \end{cases}$$



2 Pts a) Write expressions for the total E-field and total H-field amplitudes in the half-space $z\le 0$.

Hint: $\cos a+\cos b = 2\cos[(a+b)/2]\cos[(a-b)/2]$; $\cos a-\cos b = -2\sin[(a+b)/2]\sin[(a-b)/2]$.

2 Pts b) Identify locations along the z-axis where the E-field is exactly equal to zero, and also locations where the H-field is exactly zero.

3 Pts c) Determine the local energy densities of the E- and H-fields in the half-space $z\le 0$.

3 Pts d) Find the total Poynting vector $\boldsymbol{S}(\boldsymbol{r},t)$ in the half-space $z\le 0$, and explain the behavior of the electromagnetic energy as a function of time by analyzing the time-dependence of the Poynting vector in relation to the local energy densities of the E- and H-fields.

Hint: $2\sin(a)\cos(a)=\sin(2a)$.

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OPTI 502

Consider the optical system depicted in Fig. 1. The system consists of a negative thin lens (L1) and a positive thin lens (L2) having an equal absolute value of focal length $|f|$. The spacing between L1 and L2 lenses is $|f|$.

1) (2 pts) Calculate system focal length.
2) (2 pts) Identify the locations of the 1st and 2nd principal planes with respect to the first lens element.
3) (2 pts) Identify the location of an aperture stop such that the system operates as a telecentric system.
4) (2 pts) For the location of the aperture stop identified from 3), sketch the trajectories of an upper marginal ray and a chief ray. Assume the object is at –Infinity.
5) (2 pts) For the location of the aperture stop identified from 3), identify the diameter of the lens element L2 in mm so that no vignetting occurs on lens element L2. Assume $|f|$=10mm, F/#=5, the object is at –Infinity, and the tangent of object angle is 0.1.

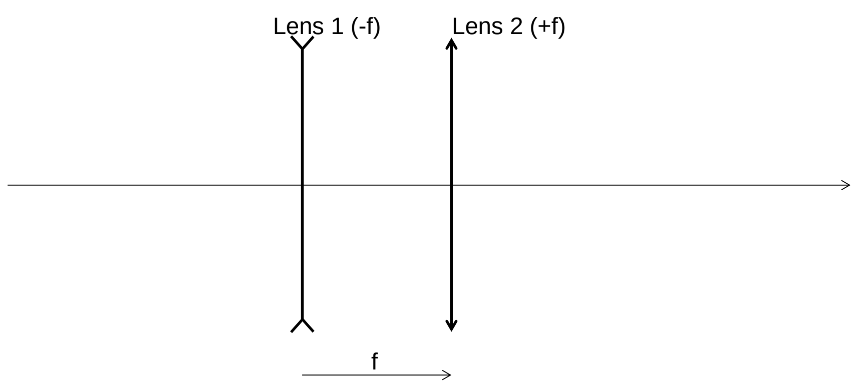


Fig. 1

Spring 2015 Written Comprehensive Exam

OPTI 503

An F/# = 10 N-BK7 lens has W_{040} = 1 wave of spherical aberration. The working wavelength is 0.5876 µm. The focal length f_d = 100 mm. The refractive index of N-BK7 is n_d = 1.5168 and Abbe value is

$V_d = \frac{n_d - 1}{n_F - n_C}$ =64.17.

1. Calculate the amount of the transverse ray aberration ε_y in mm for an upper marginal ray? (1 pt)

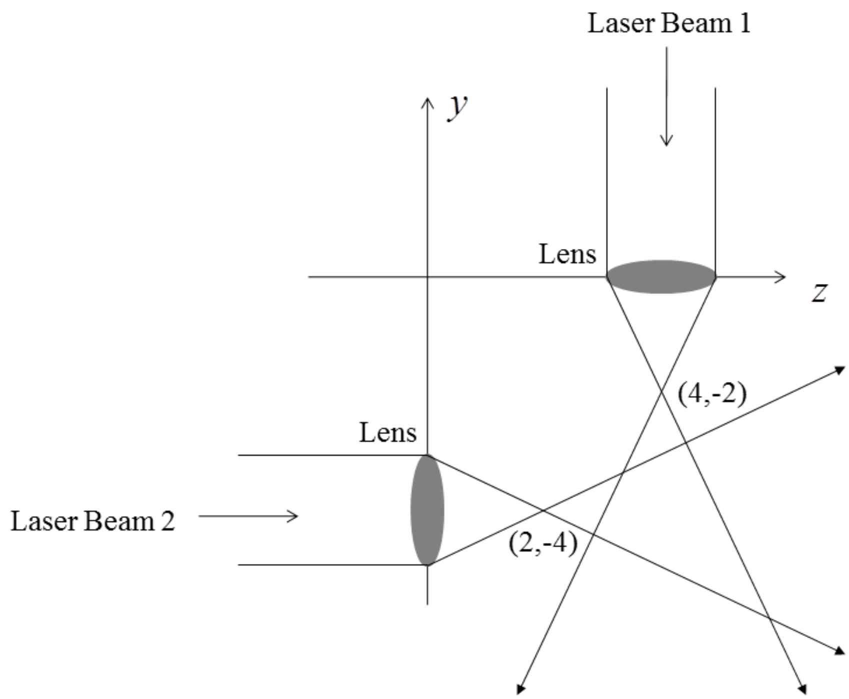
2. To cancel out the transverse aberration of the marginal ray, what amount of defocus in mm should be introduced? How far should the image plane be moved from the paraxial image plane? (4 pts)

3. What amount of defocus should be introduced to minimize the wavefront variance (HINT: mid-focus)? (1 pt)

4. If the size of the lens aperture is increased so that the F/# = 5, what is the new spherical aberration value W_{040} and what is the new ray aberration ε_y? (2 pts)

5. What is the longitudinal chromatic aberration between *F*- and *C*- wavelengths (0.486 µm and 0.6563 µm)? (2 pts)

Spring 2015 Written Comprehensive Exam
Opti505R



Hypothetical Laser Beam 1 and Laser Beam 2 are coherent and of equal phase and equal amplitude. Each beam is focused by a lens. Locations of foci are shown (units of wavelength). Polarization is out of the plane of the drawing. Ignore diffraction effects.

a.) (6pts) Sketch the fringe pattern by showing locations of bright fringe centers. Label each fringe with *m*, which is the corresponding OPD in waves.
b.) (2pts) What is the geometric shape of the fringe centers?
b.) (2pts) Approximately how many fringes are visible in this cross section?

Spring 2015 Comprehensive Exam
OPTI 507

Assume you have a crystal with the following primitive translation vectors in reciprocal space: $\vec{A} = \frac{2\pi}{a}\hat{x}$, $\vec{B} = \frac{2\pi}{a}\hat{y}$ and $\vec{C} = \frac{2\pi}{c}\hat{z}$ with $a \neq c$. Determine the size of the primitive unit cell (call it V_r) in reciprocal space, the primitive translation vectors in real space, the size of the Wigner-Seitz cell, and a vector that is normal to the plane (341) [just state the vector and its Cartesian components, it is not requested that you prove that your vector is indeed normal to the plane (341)].

(10 points)

Spring 2015 Written Comprehensive Exam
OPTI 511

Consider an atom interacting with a monochromatic optical plane wave of angular frequency ω, polarization $\hat{\epsilon}$, and electric field amplitude $\mathcal{E}_0$. In the 2-level atom approximation, the atom is treated as having a ground state $|\psi_g\rangle$ and a single excited state $|\psi_e\rangle$, which are associated with internal-state energies of E_g and E_e (respectively). For the following questions, we consider the semiclassical model of atom-light interaction, and neglect spontaneous emission effects.

(a - 1 point.) In terms of the quantities given above, write an expression for the electric dipole matrix element associated with the transition between the ground and excited states. Define any additional quantities that you use if they are not already defined above. You do not need to (nor can you) solve for the dipole matrix element. You may write the dipole matrix element as either a vector or scalar quantity, as long as it is consistent with your answers to the questions below.

(b - 4 points.) Assume that the atom is in the ground state at time $t = 0$. Write a mathematical expression for the probability $P_e(t)$ of finding the atom in the excited state as a function of t, assuming the electric field also turns on at $t = 0$. Name and define each of the new quantities and parameters that appear in your expression in terms of the physical parameters and quantities defined at the introduction to this problem. For example, the bare Rabi frequency should appear in your expression, and you should define this in terms of the quantities given above. You do not need to redefine the dipole matrix element. If you do not remember the formula, make a sketch of $P_e(t)$ vs t and for partial credit, give your best guess at the correct mathematical expression.

(c - 2 points.) In addition to the two-level atom and semi-classical approximations, what two other approximations are involved in obtaining the probability $P_e(t)$ that you gave or sketched above? Name and briefly describe these approximations.

(d - 2 points.) Suppose that the light is tuned to the atomic resonance, and that the bare Rabi frequency for the atom-light coupling is 10^6 rad/s. The light is turned on at $t = 0$, when the atom is in the ground state, and is turned off at a time that corresponds to a π pulse. Give a number for the duration of time that the light is on, and indicate the state of the atom at the end of the π pulse.

(e - 1 point.) Suppose that the atom is initially in the ground state, then interacts with on-resonant light for a time corresponding to a $\pi/2$ pulse. Give an expression for $\langle E \rangle$, the expectation value of the internal energy of the atom, at the moment that the light turns off.

Spring 2015 Written Comprehensive Examination
OPTI 537

Answer the following questions related to radiative transport. The radiative transport equation (RTE), or Boltzmann equation, describes the time evolution of a phase-space distribution function w in terms of four processes: absorption, emission, propagation, and scatter

$$\frac{dw}{dt} = \left[\frac{\partial w}{\partial t}\right]_{abs} + \left[\frac{\partial w}{\partial t}\right]_{emiss} + \left[\frac{\partial w}{\partial t}\right]_{prop} + \left[\frac{\partial w}{\partial t}\right]_{scat} \qquad (1)$$

(a) (1 point) What is w a function of and what are its units? What does it describe?

(b) (2 points) In class, we derived forms for each of the terms in the RTE and wrote an overall spatio-temporal-integro-differential transport equation of the form:

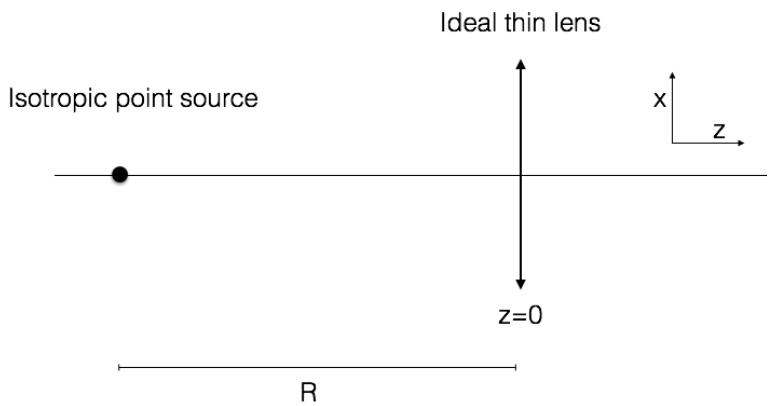
$$\frac{dw}{dt} = -c_m \mu_{total} w + \Xi_{p,E} - c_m \hat{\mathbf{s}} \cdot \nabla w + \boldsymbol{K} w \qquad (2)$$

where ***K*** is an integral operator. Write a general expression for the *K* operator for both inelastic collisions and elastic collisions**.**

(c) (2 points) For the rest of the questions, please consider the propagation of photons is limited to one plane (problem is in 2 dimensions and not 3).

Consider a point source emitting incoherent light at a constant rate isotropically in all directions in air. Assume that scattering in air **cannot** be neglected but that absorption can. Further assume that when a scatter occurs, it is elastic and the angle of scatter is isotropic in all directions. Write (but do not solve) an RTE that fully describes this system. If you introduce terms, please describe what they are.

(d) (3 points) We now add an ideal thin lens to the setup in part c). A distance R away from the point source is an ideal thin lens of infinite extent (see picture). Again, scattering in air **cannot** be neglected but that absorption can. Write (but do not solve) an RTE that fully describes this system. If you introduce terms, please describe what they are.



(HINT 1) It may be helpful to remember that the ABCD matrix for an ideal thin lens is $\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$.

(HINT 2) An ideal thin lens changes the direction of a ray. The only term in the RTE that changes ray direction is the scattering kernel.

(e) (2 points) What terms would need to be modified in your RTE for part to include chromatic aberration in the thin lens.

Spring 2015 Written Comprehensive Exam
OPTI 544

Consider an atom interacting with a monochromatic optical plane wave of angular frequency ω, polarization $\hat{\epsilon}$, and electric field amplitude $\mathcal{E}_0$. In the 2-level atom approximation, the atom is treated as having a ground state $|\psi_g\rangle$ and a single excited state $|\psi_e\rangle$, which are associated with internal-state energies of E_g and E_e (respectively). For the following questions, we consider the semiclassical model of atom-light interaction, and neglect spontaneous emission effects.

(a - 1 point.) In terms of the quantities given above, write an expression for the electric dipole matrix element associated with the transition between the ground and excited states. Define any additional quantities that you use if they are not already defined above. You do not need to (nor can you) solve for the dipole matrix element. You may write the dipole matrix element as either a vector or scalar quantity, as long as it is consistent with your answers to the questions below.

(b - 4 points.) Assume that the atom is in the ground state at time $t = 0$. Write a mathematical expression for the probability $P_e(t)$ of finding the atom in the excited state as a function of t, assuming the electric field also turns on at $t = 0$. Name and define each of the new quantities and parameters that appear in your expression in terms of the physical parameters and quantities defined at the introduction to this problem. For example, the bare Rabi frequency should appear in your expression, and you should define this in terms of the quantities given above. You do not need to redefine the dipole matrix element. If you do not remember the formula, make a sketch of $P_e(t)$ vs t and for partial credit, give your best guess at the correct mathematical expression.

(c - 2 points.) In addition to the two-level atom and semi-classical approximations, what two other approximations are involved in obtaining the probability $P_e(t)$ that you gave or sketched above? Name and briefly describe these approximations.

(d - 2 points.) Suppose that the light is tuned to the atomic resonance, and that the bare Rabi frequency for the atom-light coupling is 10^6 rad/s. The light is turned on at $t = 0$, when the atom is in the ground state, and is turned off at a time that corresponds to a π pulse. Give a number for the duration of time that the light is on, and indicate the state of the atom at the end of the π pulse.

(e - 1 point.) Suppose that the atom is initially in the ground state, then interacts with on-resonant light for a time corresponding to a $\pi/2$ pulse. Give an expression for $\langle E\rangle$, the expectation value of the internal energy of the atom, at the moment that the light turns off.

Spring 2015 Written Comprehensive Exam
Opti 546

This problem explores the ray and Gaussian beam optics of an optical resonator. The ABCD law for the complex beam parameter $\frac{1}{q} = \frac{1}{R} + \frac{2i}{kw^2}$ for a single round trip of the resonator is given by

$$\frac{1}{q_{n+1}} = \frac{C + D/q_n}{A + B/q_n}, \qquad M_L = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \qquad M_R = \begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix},$$

M_L and M_R being the ray transfer matrices for free-space and a mirror.

(a - 2pts) We consider a hemi-confocal optical resonator composed of one flat mirror and one concave curved mirror of radius of curvature R separated by a distance $L = R/2$. Calculate the ray transfer matrix M_1 for a single round trip of this resonator taking the flat mirror as the reference plane.

(b - 2pts) For the ray optics approach consider an initial ray vector $\binom{x_0}{x_0'}$. By examining the ray vector $\binom{x_1}{x_1'}$ after one round trip argue that a single round trip of the hemi-confocal resonator corresponds to an optical system that produces a Fourier transform.

(c - 2pts) Building on part (b), show that for an initial on-axis ray ($x_0 = 0$) the ray also crosses the axis after two and four round trips, $x_2 = x_4 = 0$. (Hint: You need to calculate M_2 and M_4 for two and four round trips.)

(d - 2pts) Turning now to the Gaussian beam optics of the hemi-confocal resonator, consider an initial Gaussian beam at the flat mirror of infinite radius of curvature and spot size w_0. Show that the Gaussian beam spot sizes w_2 and w_4 after two and four round trips of the resonator, respectively, are both equal to the initial spot size $w_2 = w_4 = w_0$. Thus, the initial field is reproduced in spot size after every two round trips of the resonator, irrespective of the value of w_0, analogous to the ray crossing in part (c).

(e - 2pts) Obtain an expression for the spot size w_1 after one round trip of the resonator given the initial spot size w_0. By demanding that $w_1 = w_0$, so that the spot size repeats after every round trip, obtain an expression for the stable Gaussian mode size w_0 of the hemi-confocal optical resonator in terms of k and L.

Spring 2015 Written Comprehensive Exam
Opti 501

<u>System of units: MKSA</u>

Inside a homogeneous, isotropic, non-magnetic, dielectric medium of refractive index $n(\omega)$, a monochromatic, homogeneous plane-wave propagates along the z-axis. The plane-wave is linearly-polarized along the x-axis, and the medium is transparent, that is, $n(\omega)$ is real and positive.

4 Pts a) Write expressions for the plane-wave's electric and magnetic fields, $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{H}(\boldsymbol{r},t)$, in terms of the E-field amplitude E_0, the angular frequency ω, the refractive index $n(\omega)$, the speed of light in vacuum c, and the impedance of free space Z_0.

2 Pts b) Express the dielectric function $\varepsilon(\omega)$ and the electric susceptibility $\chi(\omega)$ as functions of the refractive index $n(\omega)$.

4 Pts c) Write an expression for the polarization distribution $\boldsymbol{P}(\boldsymbol{r},t)$ in terms of E_0, ω, c, ε_0 and $n(\omega)$. What are the distributions of the electric bound-charge and bound-current densities, $\rho_{\text{bound}}(\boldsymbol{r},t)$ and $\boldsymbol{J}_{\text{bound}}(\boldsymbol{r},t)$, in the medium?

Spring 2015 Written Comprehensive Exam

OPTI 502

You are building a f/10 camera system to be used with a CMOS sensor. The focal length of the objective or imaging lens is f=100 mm. To construct this lens system, you have available only two positive lenses with focal lengths of 50 mm and 100 mm, but you have an unlimited selection of negative lenses. All the lenses are ideal thin lenses, the object is at infinity, and the aperture stop is located at the lens element closest to the object.

1. When a single f=100 mm thin lens is used as the objective lens, what are the back working distance and the diameter of the aperture stop? (1 pt)
2. The CMOS sensor has 1000x1000 pixels, each pixel is 5x5 µm. What is the diagonal field of view in degrees? (1 pt)
3. Due to the space limitations, the distance from the last lens to the CMOS sensor should be 80 mm. Using one positive lens and one negative lens construct the objective lens with a system focal length f=100 mm. Which two lenses will you use? What is the required element spacing, and which lens is closer to the CMOS sensor? What are the exit pupil position and exit pupil diameter? (4 pts)
4. You now want to build an objective with the same focal length f=100 mm, but with a distance between the last lens element and to CMOS sensor larger than 100 mm. What is the focal length of the negative lens you will use? What is the element spacing, and what is the distance between the last lens and the CMOS sensor? (2 points)
5. In either of the situations described in parts 3 or 4, a 20mm thick beamsplitter cube (refractive index n=1.5) is placed between the second lens and the sensor. Discuss in detail what changes are required to obtain an in-focus image on the sensor. Assume the beamsplitter cube does not introduce any aberration. (2 pts)

Two RED and GREEN LEDs are provided. The special radiances of the two LEDs are listed in Table 1. The luminous efficiencies and corresponding xyz color matching functions are given in Table 2. The CIE xy chromaticity diagram is also shown below. The white point is located at (0.33,0.33).

Answer the following questions:

(5 points) Compute the radiance, luminance, XYZ tristimulus values, and x-y chromaticity coordinates of the two LEDs. This calculation must make use of the spectral data provided in Table 1 and Table 2.

(3 points) The light of the two LEDs are combined to produce a new beam (assume beams are perfectly overlapped in space). Compute the total luminance, the XYZ tristimulus values, and chromaticiy coordinates of the new color resulted from the beam mixing.

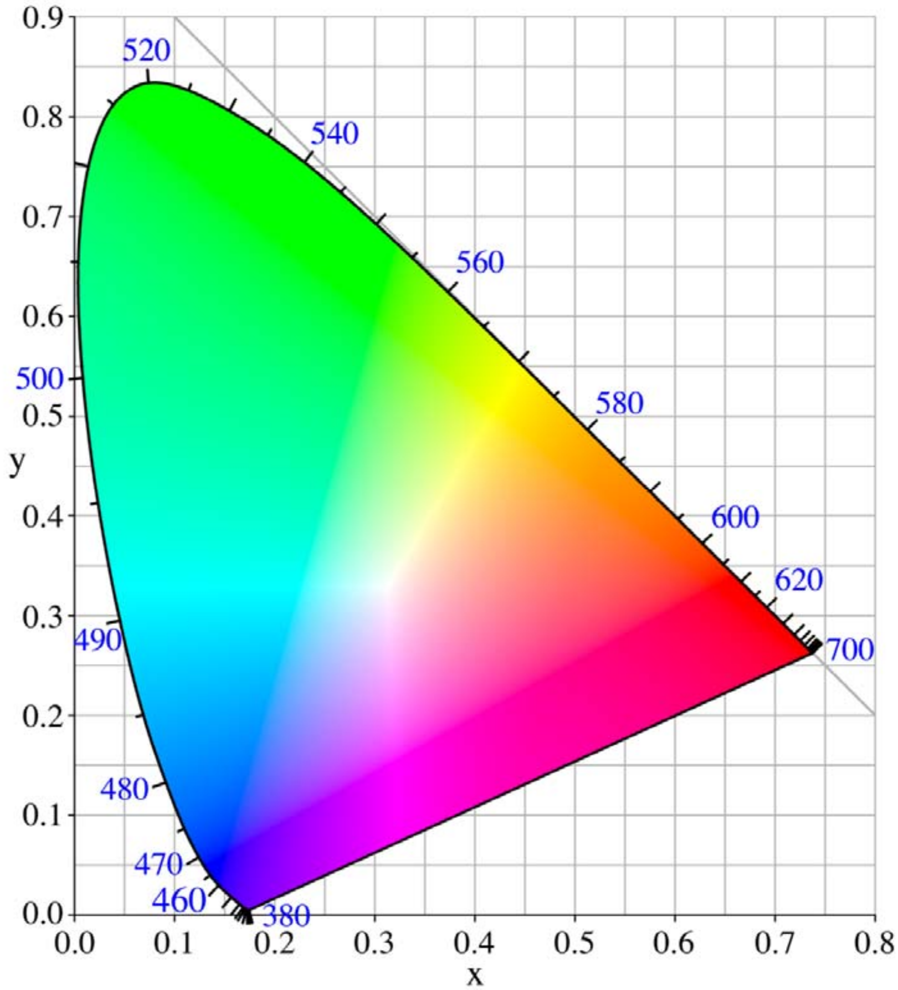
(2 points) Determine the dominant wavelength and color purity of the resulted color when the LEDs are combined.

Table 1 Spectral radiance of LED emitters

	Wavelength (λ) (nm)	Spectral radiance (watts/(m^2.sr.um))
Red LED	λ <635nm	0
	635nm	30
	640nm	100
	645nm	30
	λ >645nm	0
Green LED	λ <505nm	0
	505nm	20
	510nm	80
	515nm	20
	λ >515nm	0

Table 2 Data for luminous sensitivity and color matching functions

Wavelength (λ) (nm)	Luminous efficiency v(λ)	x-bar (λ)	y-bar (λ)	z-bar (λ)
645	0.1382	0.3608	0.1382	0.00001
640	0.1750	0.4479	0.1750	0.00002
635	0.2170	0.5419	0.2170	0.00003
…				
515	0.6082	0.0291	0.6082	0.2123
510	0.503	0.0093	0.503	0.1582
505	0.4073	0.0024	0.4073	0.1117


520
540
560
580
600
620
700
500
490
480
470
460
380
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
y
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
x

Spring 2015 Written Comprehensive Exam
Opti505R

The photograph above was taken using a classical slit grating illuminated by a white-light point source. Observation is made in the Fraunhofer region of the grating.

a.) (5pts) Label the diffraction orders with respect to *m* representing the integer number of waves of OPD between adjacent slits. That is, label *m* as a positive, negative or zero-value integer above each diffracted order. The sign convention is ambiguous from the figure, so you may choose a convenient convention.
b.) (3pts) Explain the orientation of the colors you see in the diagram. Provide any relevant equations to justify your reasoning.
c.) (2pts) Assuming that the size of the point source is much smaller that the Fraunhofer diffraction limit, so that the source size does not influence the size of the spot in the Fraunhofer region, approximately how many periods of the grating are illuminated by the source?

Spring 2015 Comprehensive Exam
OPTI 507

Consider an exciton in a first-class dipole allowed semiconductor, such as GaAs. The exciton wave function is a product of two factors, one describing the relative electron-hole motion and the other the electron-hole pair's center-of-mass motion. Write down the time-independent Schrödinger equation of the center-of-mass wave function, the solution to the equation (omit normalization factors, they are not requested here), and find the center-of-mass energy (in unit of eV) for an exciton with a center-of-mass wave vector of 8 μm^{-1} (the electron and hole masses are $m_e = 0.067 m_0$ and $m_h = 0.1 m_0$).

(You may use $\hbar^2 / m_0 = 7.62$ eV Å^2.)

(10 points)

Spring 2015 Written Comprehensive Exam
Opti 511R

Consider a particle of mass m trapped in a 2-dimensional simple harmonic oscillator potential well given by $V(x,y)$. There is no force acting on the particle in the z direction. The potential well is characterized in the usual way, with oscillation frequencies ω_x and ω_y in the x and y directions, respectively. In this problem, $\omega_x = \omega_y$. The solutions to the time-independent Schrödinger equation may be written as the product state $\psi_{m,n}(x,y) = \psi_m(x) \cdot \psi_n(y)$. For example, the state $\psi_{10}(x,y) = a_{10} \cdot x \cdot e^{-(x^2+y^2)/(2\sigma^2)}$, where σ is a constant and a_{10} is the normalization coefficient.

a. (2 pts) Write an expression for the energy eigenvalues $E_{m,n}$ in terms of the quantities defined above.

Now assume the particle is in the following superposition state:

$$\phi(x,y) = \frac{1}{\sqrt{2}}(\psi_{01} + i\psi_{10}) \tag{1}$$

b. (1.5 pts) Is $\phi(x,y)$ an eigenstate of the energy operator? Please show your work to justify your answer, or explain your reasoning.

c. (5 pts) The operator for the z-component of angular momentum is given by $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$. Show that $\phi(x,y)$ is an eigenstate of $\hat{L}_z$ and determine the eigenvalue.

d. (1.5 pts) Is it possible to simultaneously determine both the exact energy and the z-component of angular momentum of the particle in state $\phi(x,y)$? Justify you answer.

Spring 2015 Written Comprehensive Examination
OPTI 537

Answer the following questions related to solid-state physics and semiconductor detectors. All parts weighted as indicated.

(a) (10%) Write the expression that defines the relationship between the direct (**R** = l**a** + m**b** + n**c**) and reciprocal (**G** = o**A** + p**B** + q**C**) lattice vectors for a perfect crystal. Define all terms.

(b) (10%) Explain what is meant by the 1st Brillouin zone, and write an expression for its volume for a cubic lattice with primitive direct lattice constants a = b = c = 5 Å.

(c) (20%) If a macroscopic crystal of the lattice structures in (b) above has dimensions of .5 mm x .5 mm x 1 mm in the $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$ directions (which correspond to the x, y, and z directions, respectively), and we invoke Bloch's theorem and Born-Von Karman boundary conditions, what is the spacing between the allowed k_x, k_y, and k_z states? (Express your answer in the customary units of cm^{-1}).

(d) (10%) How many electrons fit inside a single band in the 1st Brillouin zone of this crystal?

(e) (20%) Then consider a 2D square lattice with primitive lattice constants a = b = 5 Å. Use the Ewald sphere construction to work out how many allowed elastic reflections ($\mathbf{k}_{\mathbf{out}}$) there are if light has an incident **k** vector (in units of $Å^{-1}$)

$$\mathbf{k}_{inc} = \frac{3\pi}{5}\hat{\mathbf{A}} + \frac{\pi}{5}\hat{\mathbf{B}}$$

where $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are the unit vectors in reciprocal space corresponding to directions $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ in real space.

(f) (20%) Make a sketch of the basic structure of a CCD that shows how charge is stored following light absorption, and make a second diagram that explains how all of the data is read out from a 2D CCD imaging detector. Indicate the slow and fast clocks and where conversion to a voltage occurs.

(g) (10%) What charge transfer efficiency is required for a 2K $\times$ 2K CCD (remember that the symbol K generally stands for 1024 in detector/electronics contexts) read out on a single edge if in the **worst case** 90% of the original charge in a pixel is to be retained when the read out process is complete?

Spring 2015 Written Comprehensive Exam
Opti 544

Consider a particle of mass m trapped in a 2-dimensional simple harmonic oscillator potential well given by $V(x, y)$. There is no force acting on the particle in the z direction. The potential well is characterized in the usual way, with oscillation frequencies ω_x and ω_y in the x and y directions, respectively. In this problem, $\omega_x = \omega_y$. The solutions to the time-independent Schrödinger equation may be written as the product state $\psi_{m,n}(x, y) = \psi_m(x) \cdot \psi_n(y)$. For example, the state $\psi_{10}(x, y) = a_{10} \cdot x \cdot e^{-(x^2+y^2)/(2\sigma^2)}$, where σ is a constant and a_{10} is the normalization coefficient.

a. (2 pts) Write an expression for the energy eigenvalues $E_{m,n}$ in terms of the quantities defined above.

Now assume the particle is in the following superposition state:

$$\phi(x, y) = \frac{1}{\sqrt{2}}(\psi_{01} + i\psi_{10}) \tag{1}$$

b. (1.5 pts) Is $\phi(x, y)$ an eigenstate of the energy operator? Please show your work to justify your answer, or explain your reasoning.

c. (5 pts) The operator for the z-component of angular momentum is given by $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$. Show that $\phi(x, y)$ is an eigenstate of $\hat{L}_z$ and determine the eigenvalue.

d. (1.5 pts) Is it possible to simultaneously determine both the exact energy and the z-component of angular momentum of the particle in state $\phi(x, y)$? Justify you answer.

Spring 2015 Written Comprehensive Exam
Opti 546

(a- 2pts) Name and sketch an example of a interferometer involving two or more beams that is based on division of amplitude.

(b - 1pt) Consider a two-arm interferometer with free-space path lengths L_1 and L_2 in the two arms that employs a single light source with coherence time $\tau = 3\ ps$. *Explain* under what conditions on the path lengths L_1 and L_2 the visibility of interference fringes at the output of the interferometer will become degraded. (Assume that in the limit $\tau \to \infty$ the interference fringes have unit visibility.)

(c - 2pts) Write down the Lorentz electron oscillator model in component form in the principal axis system as appropriate to a uniaxial crystal, and identify the terms that account for the presence of longitudinal and transverse electric fields.

(d - 1pt) Identify two approximations employed in obtaining the Lorentz oscillator model you gave in part (c).

(e - 2pts) *Explain* why the Lorentz oscillator model you gave in part (c) cannot describe the process of harmonic generation, and list the additional physics that must be added to allow for the treatment of harmonic generation in transparent dielectric crystals. You may consider the simplified one-dimensional model for your discussion.

(f - 2pts) For an isotropic medium give a description of the circular dichroism that can result from the magneto-optical Zeeman effect making sure to highlight the geometry involved, and list the physics that must be added to the Lorentz electron oscillator model in part (c) to describe this effect.