

Introduction

For centuries, telescopes operating at frequencies across the electromagnetic spectrum have been used to probe our solar system and beyond. Radiation collected from different spectrums inform us about specific properties of the composition and dynamics of space and celestial bodies. Radio frequencies provide unique information about the dynamics of the universe that other spectrums cannot, as well operates as a communication frequency between Earth and space probes.

In this paper, we discuss a surface metrology method that supports research that enables lower cost radio telescope dishes (antennas) through a method known as thermoforming. The metrology technique is known as fringe-projection profilometry (FPP), and leverages concepts of photogrammetry.

Why Radio Telescopes?

The technology required to observe each electromagnetic spectrum varies greatly due to atmospheric absorption (*Figure 1*) and instrument sophistication. The main contributors to our atmospheric absorption on Earth are water vapor (mid-IR and microwave), carbon dioxide (mid and far IR), and ozone (UV). Frequencies higher than the UV are absorbed by almost all materials due to the UV band-edge. The main sources of attenuation in the atmosphere are absorption and scattering. Absorption is dependent on how the electrons in a material respond to a given frequency, and how the reemitted light interferes with the original incident light. Scattering is dependent on size of particles in the atmosphere relative to the electromagnetic frequency. With a combination of these attenuations, many bands of the electromagnetic spectrum, especially terahertz frequencies, cannot be observed from the ground and require a space-based telescope, driving up cost, complexity, reliability, and observation lifetime.

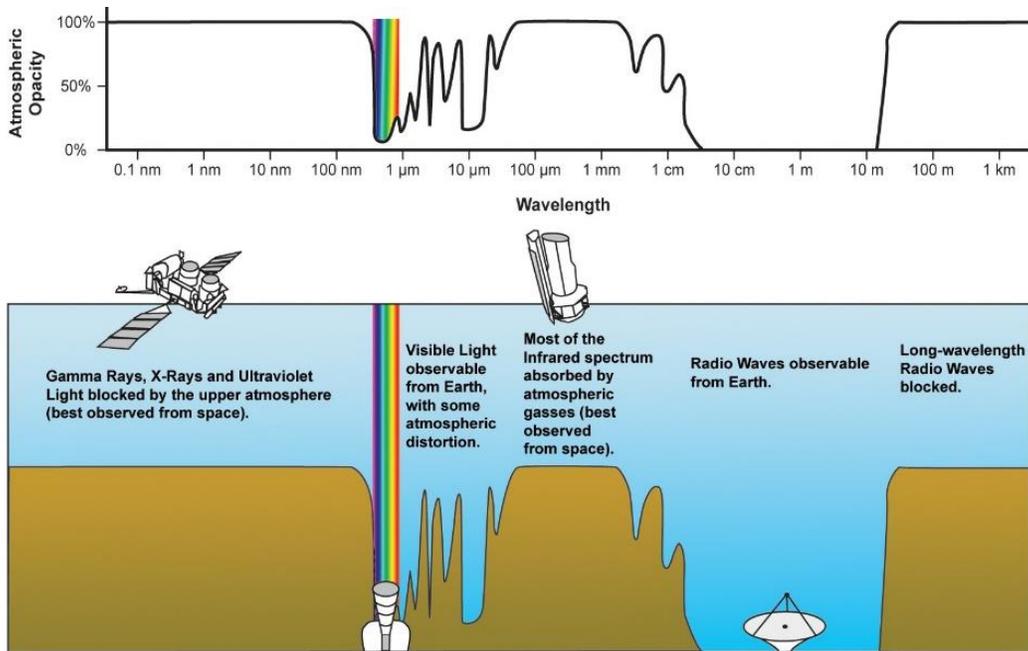


Figure 1: Typical atmospheric transmission windows, and the respective telescopes.¹

Fortunately, radio frequencies (20 kHz to 300 GHz or 1 cm to 15 m wavelength) have minimal to no interaction with the electron resonant properties of molecules in our atmosphere nor scattering, opening a wide transmission window allowing for effective observations to be made on the ground without paying for heavy equipment to be launched into orbit. However, even with the high efficiency transmission of the atmosphere, the radio signal from space is extremely weak.

In general, radio astronomy probes the sky for different molecular and atomic spectral-line emissions emitted due to energy level transitions. One of the most important transitions in radio astronomy is the spin-flip transition of hydrogen. This transition is sourced by considering the hyperfine structure of the hydrogen atom, which includes the angular momentum produced by both electron and proton spin momentum in calculations of available energy levels. When a hydrogen atoms proton and electron spin orientations swap from parallel (excited state $F = 1$) to anti parallel (ground state $F = 0$), the energy difference produces an emitted photon of just $5.87 \mu\text{eV}$, which corresponds to a 21.106 cm wavelength, which is commonly known as the 21-cm line. Assuming that hydrogen is equally distributed throughout the galaxy, it is expected that this transition can be detected along any line-of-sight in the

sky. By measuring the amplitude of this frequency, information about the concentrations of hydrogen gas can be extracted. By reading variations in frequency of this signal due to doppler shift, velocities of celestial objects relative to earth can be measured, giving key insight into the gravitational dynamics of galaxies and the universe.

Unfortunately, this transition is considered highly forbidden, and therefore is very weak. The Einstein coefficient is approximately 10^{-15} s^{-1} , or 10 million years. This transition is very unlikely, but due to the incredibly high number of total hydrogen atoms, this still results in enough emitted photons to be measured on Earth. This low emission probability also applies for many other atomic/molecular spectral emissions in radio frequencies (water vapor, ammonia, carbon dioxide etc.), which can be used to identify the presence of an atom or molecule. A very large collection area is required to collect enough photons to detect this weak signal and have a high signal to noise ratio, hence the reason why radio telescopes are often many meters in diameter and make use of multiple apertures.

Even with weak signals in the radio frequency, radio astronomy is still very attractive to astronomers due to the relatively low cost of creating large aperture telescopes. The long wavelength of radio waves means that common surface quality requirements such as $\lambda/8$ root-mean-square (RMS) that are on the nanometer scale for visible telescopes, are in the 10's or 100's of microns scale for even the highest frequency emission lines that are important to radio astronomy. This relaxes requirements on the manufacturing of the focusing optics and the cost of surface metrology, as well as makes the instrument more tolerant to errors.

Weak signal is not the only reason for multiple aperture telescope systems. Angular resolution of a telescope is dependent on the wavelength and the aperture diameter:

$$\theta_{res} = \frac{1.22\lambda}{D}$$

This means that for large λ (~in the centimeters), the angular resolution is poor. Increasing the aperture diameter D offsets the reduction in angular resolution due to the long wavelength of radio, but even with a 10 m aperture the angular resolution is many orders of magnitude worse than that of even an amateur visible telescope, which are typically less than an arcsecond.

A unique property of radio telescopes is aperture synthesis. The frequency of radio waves is slow enough that the phase of the signal can be directly sensed and recorded and combined later with data collected from radio telescopes in different locations. Since the phase can be directly measured, the signals can be combined in post-processing. This vastly increases the effective aperture diameter, which directly improves angular resolution. By making the telescopes hundreds of meters or even many kilometers apart, an angular resolution comparable or even better than visible telescopes is produced. Another implication of direct phase measurement is the ability to do frequency analysis over a large bandwidth in post-processing. A simple Fourier transform on the received signal isolates the frequency content of a signal, allowing observation of many spectral emission lines at once. These properties make radio telescopes a crucial instrument for astronomers.

Radio Antenna Design Fundamentals

Detectors in the radio frequency are simply antennas that output a single binned signal, as opposed to a CCD or CMOS detector for optical frequencies that sample many field-of-views simultaneously. They are often cryogenically cooled to reduce Johnson-Nyquist noise, which is proportional to temperature. Considering the reflector design, parabolic primary reflectors produce diffraction limited performance on axis with no spherical aberration, so a radio telescope could be built with just a single reflector and a receiver at the prime focus. Radio telescope complexity can vary from as simple as this single reflector to a much more complicated system. For example, a scanning mirror later in the beam path could be used to scan the sky near the pointing direction. These advanced systems require a secondary mirror for many purposes:

- Two-reflector Cassegrain designs dramatically improve off-axis performance for elevation/azimuthal scanning due to reduction in coma and astigmatism
- A convex secondary mirror increases the $f/\#$ of the telescope to match more common feed-horn shapes, used for isolating different frequency channels.
- The feed antenna is directed toward the sky rather than back at the primary dish. This way, the receiver is pointed at the cold sky rather than the warm ground, reducing background noise.

Another purpose behind a two-mirror antenna is to direct the beam into an internal structure that can continue to manipulate it (swap feed horns/detectors easily, protect the elements from weather). Often the same designs for radio telescopes are also used for communication. A schematic for an advanced radio telescope or communications downlink is below:

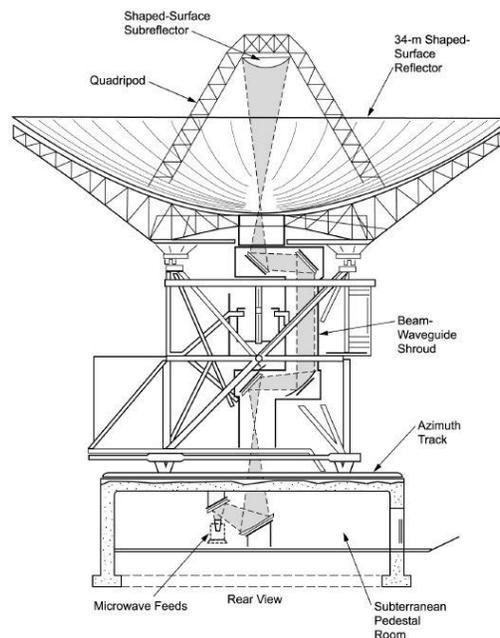


Figure 2: 34-meter beam waveguide antenna for the NASA Deep Space Network.²

Antenna Manufacturing – Thermoforming

To make such a large, precisely shaped primary reflector as in *Figure 2*, the surface must be split up into many much smaller segments to ease the manufacturing requirements. Most common methods

of producing segments (or panels) are both time consuming and costly. With hundreds of panels needed for just one antenna, the reflecting surface becomes a significant portion of the total cost of the system.

The Steward Observatory Solar Lab (SOSL) has been developing technology to rapidly produce precise panels with a method called thermoforming. The basic concept is to create a mold with the desired shape, then heat up a sheet of aluminum, which will “fall” into the shape of the mold under its own weight as it softens with higher temperatures. Taking this one step further, an adjustable steel mold made of hexagonal tiles connected with blade flexures (*Figure 3*) was developed to dynamically change the mold shape, so that many different panel shapes could be made without machining a unique mold for every panel.

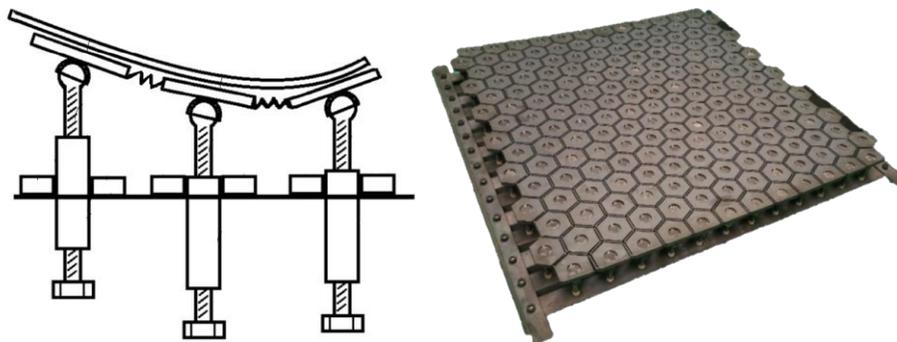


Figure 3: Concept drawing for adjustable mold with screw actuators (left) and actual 0.5 m square mold (right)³.

There are a few main sources of error when it comes to antenna manufacturing. Just like optical wavelength telescope design, wavefront error is important for radio telescopes, however much of the challenge comes from environmental and manufacturing errors than it does from the design. Wavefront errors are double the surface height error. This results in phase errors at the detector plane, which makes combining signals through aperture synthesis more challenging and less precise, reducing angular resolution. These errors ultimately come from self-weight deflections, temperature gradients, panels misalignments, and the deviation from the ideal shape for each individual panel.⁴ To allow room in the error budget for these imperfections, it is important to have panels that are as close to the ideal shape

as possible. We have been using a surface RMS error requirement of $\lambda/25$ and for the potentially shortest observation wavelength of 1mm, this requirement is $40 \mu m$.

There are many metrology methods to achieve this precision and accuracy (profilometers, laser trackers etc.), but many of them have sparse sampling and require a long duration measurement. When manufacturing medium-volume, high-precision panels, this is simply not an option. The following sections describe a low-cost, accurate method to measure depth known as stereo vision, and an extension of this method for antenna panel metrology known as fringe-projection profilometry.

Stereo Vision Photogrammetry Basics

Stereo vision (SV) is the extraction of 3D information from two 2D sets of images from two cameras. If the locations of the cameras is known, and the same object can be identified in both images, then simple line intersection can be used to identify the 3D location of the object. However, there are many sources of error to account for. Error in the known locations of the cameras, error in identifying the same object on both cameras, and error in focal length will result in an incorrect or uncertain 3D location. The uncertainties of the accuracy of these values can be realized by visualizing the solution as an intersection of cones rather than lines, as in *Figure 4*, producing an uncertainty volume. For objects that are near flat, the uncertainty in x and y is much less relevant than in z (depth).

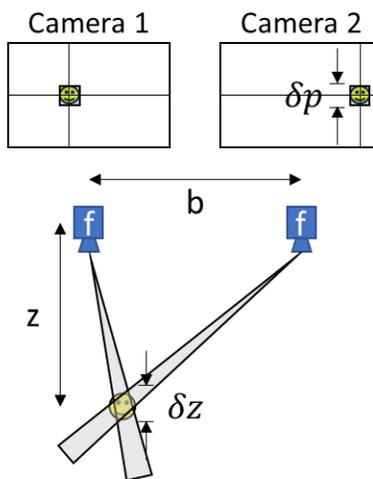


Figure 4: Basics of Stereo Camera depth measurement for identifying the location of a smiley face.

These errors result in an uncertainty in the depth, or depth resolution, of the object δz , and is given by:

$$\delta z = \frac{z^2}{bf} \delta p$$

Where z is the object's perpendicular distance from the camera baseline distance b , f is the focal length of the cameras, and δp is the error in identifying the disparity of the object between the two images, in number of pixels multiplied by the pixel size.⁵ All of these parameters can be tuned to optimize the depth resolution of the system to meet the $40 \mu m$ requirement. Parameters z , f , and b can be all adjusted via different configurations of the cameras. A large baseline, short object distance, and long focal length, achieves the best results. The object distance scales the depth resolution quadratically, so it is the most important factor to maintain. However, there are still some practical limitations. If the object that needs to be measured is large, a shorter focal length must be used to increase the FOV, but this reduces depth resolution. If the required baseline is large relative to the object distance, the angle of incidence to the object may become too high, decreasing spatial resolution.

Fringe Projection Profilometry

One way to improve depth resolution is to decrease the object correlation uncertainty δp . This can be done either by simply decreasing the pixel size or improving the method by which objects are correlated. Fringe projection profilometry (FPP) is a method of correlating object points by projecting a series of orthogonal phase-shifted sinusoidal patterns onto the unit under test (UUT).

By using the 4-step phase shifting equation, 4 phase maps are produced, one horizontal and one vertical phase, for each camera. This encodes each point on the object with a unique X and Y phase, which can be used to match pixel locations on one camera to pixel locations on the other.

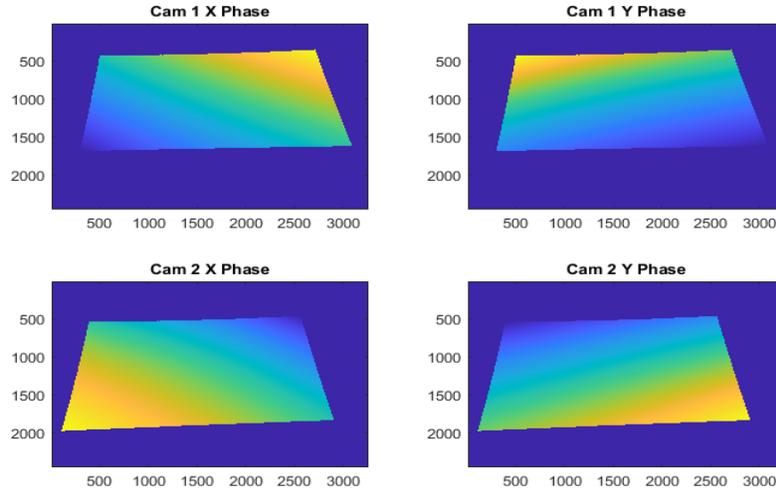


Figure 5: X and Y phase for each camera when measuring a square panel.

Once all unique phase pairs are found, the camera intrinsic parameters (translation, rotation, focal length, distortion) are used to triangulate a 3D point for each pair. ⁶

We use a configuration with a baseline distance of 3 m, a focal length of 12mm, an object distance of 0.8 m, and a pixel size of $2.4 \mu m$, and assume we can identify matching phase points to an accuracy of 0.5 pixels or better. This produces a theoretical depth resolution of about $21 \mu m$, which allows us to measure the panels with enough precision to qualify them for $\lambda/25$ RMS height error.

To obtain high accuracy, the system must also be carefully calibrated. This is typically done by imaging a known object, often a checkerboard, with a series of different orientations. Since the checkerboard square size is known, the 6 degrees of freedom (DOF) and focal lengths of both cameras can be solved for. Reprojection errors, or ray intersection errors with an ideal checkerboard, gives information about the accuracy of the calibration, and therefore any measurement made using that calibration. ⁷ We often measure reprojection errors to be around a quarter of a pixel, which keeps our 0.5 pixel point matching assumption valid for estimating system depth resolution accuracy.

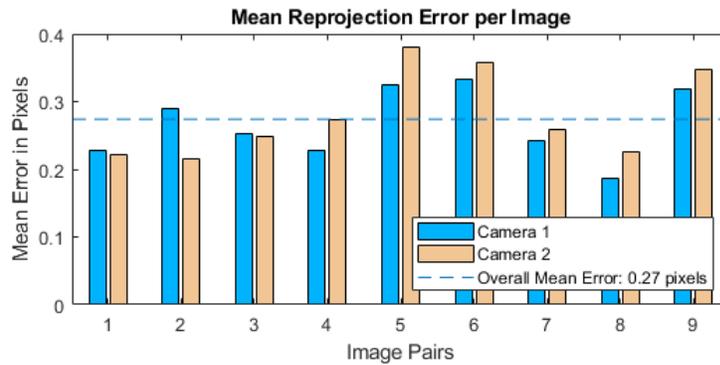


Figure 6: Reprojection errors (top) for each camera. Visualization of camera 2 and checkerboard locations relative to camera 1 (bottom).

Finally, the matched points combined with the calibrated camera geometry gives us point clouds that represent the object being measured. This can then be used in thermoforming for two purposes: closed loop feedback on how to adjust the flexure actuators (Figure 3) to achieve the desired shape and characterizing the discrepancies between a desired shape and the shape of the panel that is produced.

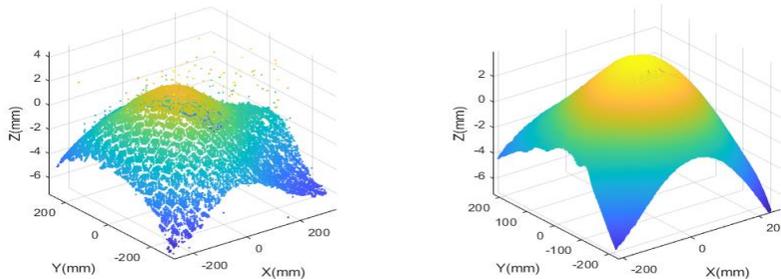


Figure 7: Point cloud of flexure shape (left) and point cloud of a formed panel (right). Each cloud has roughly 50,000 sampled points.

Conclusion

To summarize, radio antennas are crucial to the construction of both radio telescopes and space communication, which are the backbone of our exploration of space. Ultimately, this new panel manufacturing technology will enable lower cost and higher quality radio antennas. My research has been aimed at maintaining the theme of low-cost and flexibility in the metrology solution for the antenna segments, and FPP has proved to be a very promising candidate. Future research will likely consist of improved camera calibration and recalibration methods, as well as phase processing algorithms.

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