Optical Sciences PhD Qualifying Exam: Instructions and Equation Sheet

Answer all four questions on the following pages. Start each answer on a new sheet of paper.

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$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

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$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

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$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

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$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla\cdot\mathbf{F}) + \mathbf{F}\cdot\nabla\phi$

$\nabla \cdot (\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla\times\mathbf{F}) + \nabla\phi\times\mathbf{F}$

$\nabla \times (\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\cdot\mathbf{G}) - \mathbf{G}(\nabla\cdot\mathbf{F}) + (\mathbf{G}\cdot\nabla)\mathbf{F} - (\mathbf{F}\cdot\nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}}(\mathbf{F}\cdot\mathbf{n})\,da = \int_{\mathrm{V}}(\nabla\cdot\mathbf{F})\,d^3x$

$\oint_{\mathrm{C}}\mathbf{F}\cdot d\ell = \int_{\mathrm{S}}(\nabla\times\mathbf{F})\cdot\mathbf{n}\,da$

$\oint_{\mathrm{S}}\phi\mathbf{n}\,da = \int_{\mathrm{V}}\nabla\phi\,d^3x$

$\oint_{\mathrm{S}}\mathbf{F}(\mathbf{G}\cdot\mathbf{n})\,da = \int_{\mathrm{V}}[\mathbf{F}(\nabla\cdot\mathbf{G}) + (\mathbf{G}\cdot\nabla)\mathbf{F}]\,d^3x$

$\oint_{\mathrm{S}}(\mathbf{n}\times\mathbf{F})\,da = \int_{\mathrm{V}}(\nabla\times\mathbf{F})\,d^3x$

PhD Qualifying Exam, August 2024

Opti 501, Day 1

System of units: SI (or MKSA)

The electric field of a homogeneous plane-wave propagating in free space is given by $\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0 \exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)]$. Here, $\boldsymbol{E}_0 = \boldsymbol{E}_0' + \mathrm{i}\boldsymbol{E}_0''$, where $\boldsymbol{E}_0'$ and $\boldsymbol{E}_0''$ are real-valued but otherwise arbitrary vectors, and the k-vector and the frequency ω are real-valued.

a) What is the dispersion relation in free space, and what does it have to say about the vector $\boldsymbol{k}$?

b) Invoke Maxwell's equation $\boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r},t) = \rho_{\text{free}}(\boldsymbol{r},t)$ to show that $\boldsymbol{k}\cdot\boldsymbol{E}_0' = \boldsymbol{k}\cdot\boldsymbol{E}_0'' = 0$. (Recall that the plane-wave is in empty space, where no sources of the electromagnetic field reside.)

c) Invoke Maxwell's equation $\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r},t) = -\partial\boldsymbol{B}(\boldsymbol{r},t)/\partial t$ to find the magnetic H-field of the plane-wave.

d) Use the formula $\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle = ½\mathrm{Re}(\boldsymbol{E}\times\boldsymbol{H}^*)$ to evaluate the time-averaged Poynting vector. Explain (in words) the meaning and the significance of this time-averaged Poynting vector, specifically, its units and its relation to the electromagnetic energy of the plane-wave.

e) Write complete expressions for the real parts of the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields. Proceed to write the complete expression for the (real-valued) Poynting vector $\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t)\times\boldsymbol{H}(\boldsymbol{r},t)$.

f) Verify that the time-averaged value of $\boldsymbol{S}(\boldsymbol{r},t)$ obtained in (e) agrees with the result of part (d).

Hint: $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c}) = (\boldsymbol{a}\cdot\boldsymbol{c})\boldsymbol{b} - (\boldsymbol{a}\cdot\boldsymbol{b})\boldsymbol{c}$; $\cos^2\varphi = ½[1+\cos(2\varphi)]$;

$\sin^2\varphi = ½[1-\cos(2\varphi)]$; $\sin(2\varphi) = 2\sin\varphi\cos\varphi$.

Qualifying Exam Question OPTI 502: Day 1

August 2024

Assume the lens in one of the cameras of your cell phone is a single thin lens. The image height above the optical axis is 3 mm, the focal length is 4 mm, and the lens is 2 mm in physical diameter. The stop aperture is at the thin lens and is also 2 mm in diameter and the object is at infinity.

A. Determine the lens full field of view in degrees.
B. Determine the f-number of the lens. Draw a drawing showing the lens, the optical axis, and the chief and marginal rays when the object is at infinity.
C. If the stop is shifted along the optical axis so that chief ray intersection with the thin lens is 3 mm above the optical axis, where is the stop aperture located? What is the f-number of the lens after the stop has been shifted?
D. Determine whether after stop shifting in part C there is light vignetting for the off-axis beam at the edge of the field, i.e. the field corner at 3 mm. If so, estimate how much light is lost for that off-axis beam.
E. Determine the hyperfocal distance assuming a spot size of 0.0027 mm.
F. Determine the image position along the optical axis when the object is at 0.5 meters from the thin lens.
G. Determine the Lagrange invariant for this thin lens optical system.
H. The lens is now made twice as big, i.e. scaled up by a factor of two in every linear dimension. Is an image of the Moon formed by the lens brighter, dimmer, or no change in brightness, explain.
I. Use two thin lenses as given in the preamble to the question and design an afocal relay system with a transverse magnification of m= -1, working f-number of 4, and an image height of 0.5 mm. The stop aperture is midway between the thin lenses. Make a drawing that includes the thin lenses, the optical axis, the aperture stop, and the chief and marginal rays. Where is the exit pupil? If the object moves 1 mm closer to the relay, how much and where the image moves with respect to the relay?

Ph.D. Qualifying Exam, August 2024

Day 1

OPTI 505R

I am looking through a window screen made up of a square wire mesh of wires at a point source a distance of 10 meters from the screen.

a) (5pts) If the mesh has wires 0.2 mm apart and the wavelength is 500 nm, what is the spacing of the square arrangement of bright spots that I see located about the point source?

b) (5pts) Qualitatively, what determines the relative intensities of the bright spots?

August 2024 Qualifying Exam
Day 1
OPTI 511R / 570/ 544
(10 points total)

This problem deals with a one-dimensional quantum harmonic oscillator of frequency ω describing a particle of mass m moving along the x-axis. The operators for the position and momentum are denoted $\hat{x} \equiv x$ and $\hat{p} = -i\hbar\frac{\partial}{\partial x}$. The Hamiltonian for the system is $\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2$, with energy eigenvalues $E_n = \hbar\omega(n+1/2)$, $n = 0, 1, 2, \ldots$, and associated energy eigenstates $|n\rangle$ in Dirac notation. In both the position and momentum representations the energy eigenstates $\psi_n(x)$ and $\tilde{\psi}_n(p)$ (where the tilde denotes Fourier transform) have the same Hermite-Gaussian functional form with respect to x or p, respectively.

(a - 2pts) Starting from the above definitions evaluate the commutator $[\hat{x}, \hat{p}]$ by considering its action on a test function $f(x)$. Do not just state the result.

(b - 2pts) Based on the information given above provide an argument for why both the position and momentum expectation values $\langle n|\hat{x}|n\rangle = \langle n|\hat{p}|n\rangle = 0$ for the energy eigenstates $|n\rangle$.

(c - 2pts) Obtain an uncertainty relation for the product of the quantum mechanical uncertainties σ_p in momentum and σ_x in position for an arbitrary quantum state $|\phi\rangle$. These uncertainties are defined in the usual way as standard deviations. (In this problem, σ_x **does not** denote the harmonic oscillator length $\sqrt{\hbar/m\omega}$.)

(d - 2pts) Obtain an uncertainty relation for the product of the quantum mechanical uncertainties σ_H in the energy and σ_x in position for an arbitrary quantum state $|\phi\rangle$, making sure to simplify your expression as far as possible.

(e - 1pt) Evaluate your expression for the product $\sigma_H\sigma_x$ from part (d) for an energy eigenstate $|n\rangle$.

(f - 1pt) Evaluate the quantum mechanical uncertainty in energy σ_H for an energy eigenstate $|n\rangle$.

Useful formulas

Heisenberg's Uncertainty Principle

For any two physical quantities (observables) A and B and their associated linear operators $\hat{A}$ and $\hat{B}$,

$$\sigma_A^2\sigma_B^2 \geq \left|\frac{1}{2i}\langle[\hat{A}, \hat{B}]\rangle\right|^2.$$

Commutation relations
For any three linear operators $\hat{A}$, $\hat{B}$, and $\hat{C}$

$$[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B}.$$

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$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

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$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

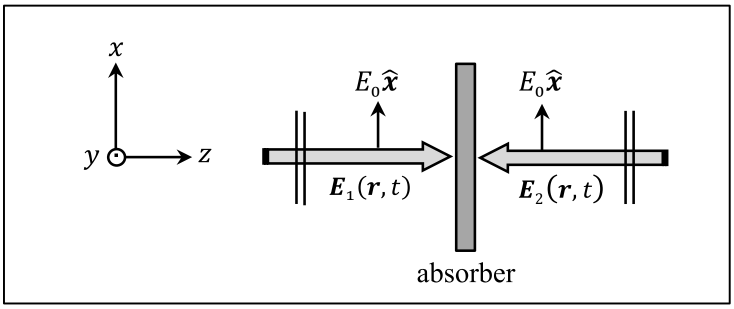
$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

PhD Qualifying Exam, August 2024

Opti 501, Day 2

System of units: SI (or MKSA)

The figure shows two counter-propagating plane-waves in free space arriving at a thin sheet of absorbing material located in the xy-plane at $z = 0$. In the half-space $z < 0$, the E-field of the right-propagating plane-wave is $\boldsymbol{E}_1(\boldsymbol{r}, t) = E_0\hat{\boldsymbol{x}}\cos[(\omega/c)(z - ct)]$, whereas in the half-space $z > 0$, the E-field of the left-propagating plane-wave is $\boldsymbol{E}_2(\boldsymbol{r}, t) = E_0\hat{\boldsymbol{x}}\cos[(\omega/c)(z + ct)]$.



a) Identify the oscillation frequency, the k-vector, and the polarization state of each plane-wave.

b) Find the magnetic fields $\boldsymbol{H}_1(\boldsymbol{r}, t)$ and $\boldsymbol{H}_2(\boldsymbol{r}, t)$ of both plane-waves, each in its respective half-space.

c) Find the Poynting vectors $\boldsymbol{S}_1(\boldsymbol{r}, t)$ and $\boldsymbol{S}_2(\boldsymbol{r}, t)$ of both plane-waves, each in its respective half-space.

d) Use the formula $\langle \boldsymbol{S}(\boldsymbol{r}, t)\rangle = ½\mathrm{Re}(\boldsymbol{E} \times \boldsymbol{H}^*)$ to calculate the *time-averaged* Poynting vector for each plane-wave. Confirm that the results thus obtained agree with the results of part (c).

Note: In part (d), $\boldsymbol{E}$ and $\boldsymbol{H}$ refer to the complex version of the fields.

e) Invoke Maxwell's boundary conditions at the front and back facets of the thin-sheet absorber (located at $z = 0^+$ and $z = 0^-$) to determine the parallel component $\boldsymbol{E}_\parallel$ of the E-field as well as the perpendicular components $\boldsymbol{D}_\perp$ and $\boldsymbol{B}_\perp$ of the $\boldsymbol{D}$ and $\boldsymbol{B}$ fields inside the absorber.

f) Invoke Maxwell's boundary condition for the parallel component $\boldsymbol{H}_\parallel$ of the H-field to determine the electric current-density $\boldsymbol{J}_s$ inside the absorber.

Note: Since the absorber is assumed to be infinitesimally thin, its current-density can be treated as a surface-current-density, having the units of ampere/meter.

g) Confirm that the rate $\boldsymbol{E} \cdot \boldsymbol{J}_s$ at which electromagnetic energy is taken up by the absorber equals the rate of delivery of electromagnetic energy by the incoming pair of plane-waves.

Qualifying Exam Question OPTI 502 **August 2024 Day 2**

1) In this problem, you will use a singlet lens to focus light. The glass you will use is N-SF2, for which the properties are shown below.

N-SF2:
$n_F = 1.661$ $n_C = 1.642$
$n_d = 1.648$ P = 0.292

a. Using the tables below and <u>ray tracing methods,</u> locate the focal point for light of the C line and F line for a singlet biconvex lens made of N-SF2, which is 5 mm thick, 25 mm in diameter, with R1 = 100 mm and R2 = -100 mm. I have set up the ray trace tables below with the lens thickness and radii. List the distance behind the second surface that the focal point is located (the back focal distance, BFD), and the longitudinal chromatic aberration between the F and C lines. ** Solutions using non-ray trace methods will not receive credit.

			Surface 1		Surface 2						
R			100 mm		-100 mm						
t				5 mm							
n		1.0				1.0					
-ϕ											
t/n											
y											
nu											
u											
y											
nu											
u											

			Surface 1		Surface 2						
R			100 mm		-100 mm						
t				5 mm							
n		1.0				1.0					
-ϕ											
t/n											
y											
nu											
u											
y											
nu											
u											

Focal point is located ____________ mm behind the second surface for the F Line (BFD)

Focal point is located ____________ mm behind the second surface for the C Line (BFD)

Longitudinal chromatic aberration δf_{FC} = _____________________ mm

b. Verify the back focal distance for the F Line using Gaussian Reduction.

Back focal distance (BFD) for the F Line = _______________________ mm

c. Now, assuming the first surface is the aperture stop, trace a chief ray through the system for a field of view of $\bar{u} = 0.1$. For an object at infinity, what is the lateral chromatic aberration in microns between F and C wavelengths at the image plane as defined by the focus of the C line?

Lateral Chromatic Aberration = ________________________ microns

d. Treating the system as a thin lens (e.g. assume the thickness is zero), calculate the system focal length for the d wavelength and the Abbe number. Use these to approximate the longitudinal chromatic aberration as $\delta f_{FC} = f_d/\nu$.

Focal length for the d Line: _______________ mm

$\nu =$ ___________

$\delta f_{FC} =$ ___________________ mm

e. Design an achromatic doublet with the system focal length f=75 mm (close to the singlet lens in problem a) using N-BaK4 and N-SF2. Use N-BaK4 as the first lens, N-SF2 as the second lens. You can treat this doublet as a thin lens. What are the powers of the two lenses for the achromatic doublet?

N-BaK4 Glass Code: 569560 P = 0.303

$f_1 =$ _______________ mm $f_2 =$ ___________________ mm

f. What is the secondary chromatic aberration of this doublet?

$\delta f_{Cd} =$ ____________ mm

If you use this page for any parts of your solution, staple together this page and all other pages that contain your solutions to this problem.

Ph.D. Qualifying Exam, August 2024

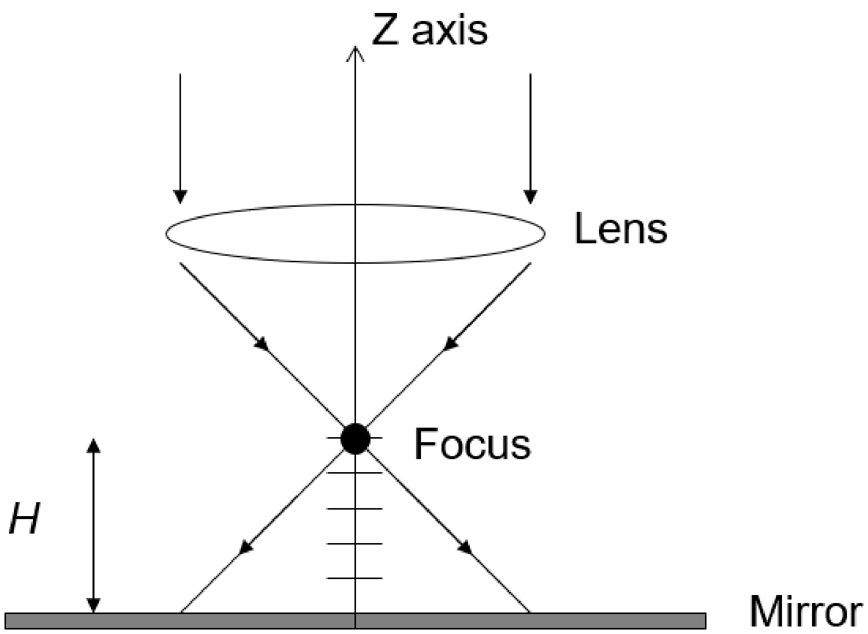
Day 2

OPTI 505R

An ideal lens is used to focus a $\lambda = 1\mu$m laser beam so that the focus point is $H = 2.5$ µm above a perfect electrical conductor (PEC) mirror. The full cone angle of the focused beam is 90°. Make a sketch of the interference pattern generated due to combination of the incident laser beam and the reflected beam from the mirror. Include only the space between the focus and the mirror. Do not include any interference that may occur above the focus toward the lens. Ignore any effects due to diffraction. State any assumptions that you make. The small horizontal lines are markers spaced by 0.5µm along the Z axis. The sketch should include:

a) (2pts) Positions of the bright fringe intercepts along the Z axis, as shown in the center of the diagram;
b) (2pts) Order numbers of the bright fringes;
c) (2pts) What is the geometrical shape of the fringes (planar, spherical, elliptical, hyperboloidal, etc)?

d) (2pts) Roughly sketch the fringe shapes on the drawing;
e) (2pts) If there is a fringe that is immediately next to the mirror surface, is it bright or dark? If no fringe, mark this answer with 'NO FRINGE'.



August 2024 Qualifying Exam Day 2
OPTI 511R / 570 / 544
(10 points total)

An electron at rest is placed in a uniform magnetic field along the $\hat{z}$-direction given by: $\vec{B} = B_o\hat{z}$, where B_o is a constant. The Hamiltonian describing the energy of the particle in the field given in terms of its gyromagnetic ratio γ is:

$$\hat{H} = -\gamma \vec{B} \cdot \hat{\vec{S}}$$

Recall that the spin "up" and "down" eigenstates along the z direction can be written in the basis of eigenstates of the $\hat{S}_x$ operator as:

$$\chi_+^{(z)} = a_o \chi_+^{(x)} + a_o \chi_-^{(x)}$$
$$\chi_-^{(z)} = a_o \chi_+^{(x)} - a_o \chi_-^{(x)}$$

where a_o is a real number. (Note that students who took OPTI 570 should recognize the $\chi_+^{(z)}$ as $|+\rangle_z$, $\chi_+^{(x)}$ as $|+\rangle_x$, etc for the the spin 1/2 electron.)

Suppose the electron at time $t = 0$ is in the spin "down" state along the x direction:

$$\chi(t = 0) = \chi_-^{(x)}$$

(a) [1 point] Write an expression for $\chi_-^{(x)}$ in terms of the eigenstates of the $\hat{S}_z$ operator and solve for the coefficient a_o.

(b) [2 points] Using the standard spin-1/2 matrix for $\hat{S}_x$ given below, show that your expression from part (a) is indeed an eigenstate of $\hat{S}_x$ with the correct eigenvalue.

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

(c) [1 point] If a measurement of the angular momentum about the x axis is to be made at time t=0, what are the possible values that could be measured and the probability for each?

(d) [1 point] If a measurement of the angular momentum about the z axis is to be made at time t=0, what are the possible values that could be measured and the probability for each?

(e) [2 points] Write an expression for the time dependent spinor state $\chi(t)$ for times $t > 0$. Be sure to express your answer using quantities defined in this problem (e.g. B_o and γ).

(f) [2 points] Calculate the expectation value, $\langle S_x \rangle$, of the electron spin angular momentum about the x axis.

(g) [1 point] Assume the magnetic field strength is such that γB_o =500 [s^{-1}]. Calculate the shortest time t required for the electron to change from the initial spin down state $\chi_-^{(x)}$ to the spin up state $\chi_+^{(x)}$ with certainty.