

OPTI 501, Electromagnetic Waves (3)

Vector fields, Maxwell's equations, electromagnetic field energy, wave equations, free-space solutions, box modes, Fresnel equations, scalar and vector potentials, gauge transformations, Lorentz model for dielectric media, metal optics, crystal optics, dipole radiation, mathematical formalism of polarized light.

Prereqs: PHYS 241, MATH 223

Grading: Homework: 10%, Midterms (in-class): $2 \times 25\% = 50\%$, Final (in-class): 40%.

Textbooks (Reference Only):

- 1) R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures in Physics*, Addison-Wesley, Reading, MA (1964).
- 2) J. D. Jackson, *Classical Electrodynamics*, Wiley, New York, 1975.
- 3) M. Mansuripur, *Classical Optics and its Applications*, Cambridge, London, 2002.
- 4) M. Born and E. Wolf, *Principles of Optics*, Pergamon, Oxford, 1983.
- 5) Jin Au Kong, *Electromagnetic Wave Theory*, EMW Publishing, Cambridge, MA, 2000.
- 6) M. V. Klein, *Optics*, Wiley, New York, 1970.

Academic Integrity

According to the Arizona Code of Academic Integrity, <http://web.arizona.edu/~studpubs/policies/cacaint.htm>, "Integrity is expected of every student in all academic work. The guiding principle of academic integrity is that a student's submitted work must be the student's own." Unless otherwise noted by the instructor, work for all assignments in this course must be conducted independently by each student. **CO-AUTHORED WORK OF ANY KIND IS UNACCEPTABLE.** Misappropriation of exams before or after they are given will be considered academic misconduct. Misconduct of any kind will be prosecuted and may result in any or all of the following:

- Reduction of grade
- Failing grade
- Referral to the Dean of Students for consideration of additional penalty, i.e. notation on a student's transcript re. academic integrity violation, etc.

Students with a Learning Disability

If a student is registered with the Disability Resource Center, he/she must submit appropriate documentation to the instructor if he/she is requesting reasonable accommodations. <http://drc.arizona.edu/instructor/syllabus-statement.shtml>.

Course Outline

The information contained in this syllabus, other than the grade and absence policies, may be subject to change with reasonable advance notice, as deemed appropriate by the instructor.

1. Vector analysis

- Scalars.
- Vectors: Cartesian coordinates, basis vectors and notation, cylindrical polar and spherical polar coordinate systems.
- Tensors: Basic idea, zero, first, and second rank tensors, dyadic tensors.
- Basic vector algebra: Addition of vectors, parallelogram law, product of scalars and vectors, dot and cross products, vector identities, coordinate inversion, polar vectors and pseudo or axial vectors.
- Scalar fields: Gradient operator as a vector operator and its interpretation, Laplacian operator as a scalar operator, grad and Laplacian operators in different coordinate systems.
- Vector fields: div and curl operators, form in different coordinate systems, variety of second order derivatives of vector fields, transverse (solenoidal) and longitudinal (irrotational) fields, Helmholtz theorem.
- Vector integration: Line, surface, and volume integrals, divergence of a vector field, flux of a vector field, the divergence theorem, curl of a vector field, circulation density, Stokes theorem, uniqueness theorem.

2. Maxwell's equations

- Macroscopic electrodynamics: Macroscopic Maxwell equations in the MKSA system of units, definitions, constitutive relations in media, bound and free charges, charge conservation and the continuity equation, symmetries under space inversion and time reversal.
- Electromagnetic field energy and momentum: EM field energy, Poynting vector, non-uniqueness of the Poynting vector, Lorentz force on charges, mechanical energy exchanged with charges, radiation pressure force on atoms and mirrors, expression for the EM field momentum and momentum density.

3. Wave equation and fundamental solutions

- Wave equation: Maxwell's wave equation in linear isotropic media, one-dimensional case, counter-propagating waves, refractive-index and the speed of light, temporal Fourier transform and the complex representation for a monochromatic field, Helmholtz equation, dispersion relation for plane-waves, linearly and circularly polarized fields, complex basis vectors, phase and group velocities.
- Plane-wave propagation in conducting media: Telegrapher's equation, absorption coefficient and Beer's law.
- Spatial Fourier transform: Maxwell's equations and the wave equation in reciprocal space, electric and magnetic fields for plane-waves, Poynting vector and time averaged intensity.

- Other solutions of the Helmholtz equation: Spherical waves, Bessel beams solutions in cylindrical coordinates, standing wave modes in an electromagnetic box.

4. Dielectric interfaces

- Electromagnetic boundary conditions: General boundary conditions, application to a dielectric interface.
- Plane-wave relations: Plane-wave incident on a dielectric interface, reflected and refracted fields, energy and momentum relations, law of reflection and Snell's law of refraction, admittance, external and internal reflection.
- Reflected and refracted fields: Helmholtz equations for the electric and magnetic fields and boundary conditions, TE or s-polarization and TM or p-polarization, reflection and transmission coefficients, normal incidence.
- Fresnel's equations: Forms of Fresnel's equations, reflectivity for s- and p-polarizations, reflection of unpolarized light, critical angle for internal reflection and Brewster's angle, Brewster windows, critical angle and total internal reflection (TIR), TIR and optical waveguiding in fibers, phase of reflected field in TIR and the Goos-Haenchen shift for confined beams, evanescent waves in TIR and frustrated TIR, transmissivity and conservation of energy.

5. Scalar and vector potentials

- Maxwell-Lorentz equations: Microscopic Maxwell equations for a system of point charges, Newton-Lorentz equations for the motion of the charges, conservation laws for energy and momentum for the Maxwell-Lorentz system.
- Scalar and vector potentials: Vector and scalar potentials and their relation to the physical fields, wave equations for the potentials, nonuniqueness of potentials and gauge transformations, Lorentz gauge and Coulomb (or radiation) gauge and associated wave equations for the potentials, separation into transverse and longitudinal fields in the Coulomb gauge.
- Classical electrodynamics in the Coulomb gauge: Electric versus magnetic field contributions to the Lorentz force, Coulomb interaction energy, transverse optical polarization, equations of classical electrodynamics in the Coulomb gauge.

6. Classical theory of dispersion

- Classical model for dielectrics: Classical atoms for a charge neutral system, gas of atoms as a model for a dielectric medium, center-of-mass (COM) motion and relative motion for a single classical atom treated in the Coulomb gauge, classical bound states of the relative motion and the failure of classical theory to predict stable atoms, Lorentz electron oscillator model for the relative motion of a bound electron-ion pair.
- Dipole approximation: Newton-Lorentz equations for a bound electron-ion pair in an applied light field in the Coulomb gauge, COM motion and light forces, dipole approximation for the relative motion of a bound electron-ion pair, electronic polarizability.

- Optical polarization for Lorentz oscillators: Macroscopic spatial averages for the electromagnetic field and polarization, wave equation for the electric field and oscillator equation for the polarization, local fields in dense media, Lorentz-Lorenz or Clausius-Mossotti equation, formal solution for the polarization in the time domain, linear response function and causality, response function for the oscillator model, solution for the polarization in the frequency domain, linear optical susceptibility, linear susceptibilities for the Lorentz oscillator model and an instantaneously responding medium.
- Optical properties of dielectrics: Helmholtz equation and plane-wave monochromatic solutions for propagation in a dielectric, refractive-index and absorption spectra for a Lorentz oscillator and their relation to the linear susceptibility, absorption resonance and normal and anomalous dispersion, optical spectra of dielectrics using multiple Lorentz oscillators and the oscillator strength, transparency region of dielectrics, Sellmeier and Cauchy formulae for the refractive-index, Kramers-Kronig relations.
- Magneto-optics: Electron motion in the presence of a static magnetic field, optical polarization, susceptibility and dielectric tensors, circularly polarized eigenstates, circular electron motion in a magnetic field, magneto-optical Faraday effect, Faraday rotation, description of optical activity.

7. Metal optics

- Classical electrodynamics for a metal: Wave equation for the transverse electric field, current density in a metal, spatial averaging and the Drude model for the current density, linear response function for the current density, frequency dependent current density and the frequency dependent conductivity.
- Optical properties of metals: Helmholtz equation for the electric field in a metal, complex refractive-index for a metal, plasma frequency, dielectric constant of metals and limiting cases, skin depth, generic frequency dependence of optical properties of metals.
- Plasma-electron oscillations: Charge density variations in an electron plasma, longitudinal electric field and the Lorentz restoring force, plasma-electron oscillations and the plasma frequency, plasma oscillations as a collective excitation of the metal.
- Reflection from metals: Plane-wave reflection from an absorbing medium (metal) for normal incidence, derivation of the intensity reflectivity, transmitted field and the skin depth, Hagen-Rubens formula, comments on reflection from an absorbing interface in general, plasma shutter.

8. Crystal optics

- Anisotropic Lorentz model: Periodic arrangement of atoms in crystals and the unit cell, anisotropic Lorentz electron oscillator model for electrons in the anisotropic environment of the unit cell, force constant tensor and symmetry properties, tensor notation for the Lorentz model, crystal axes system that diagonalizes the force constant tensor.
- Optical Polarization in crystals: Macroscopic polarization for a dielectric crystal, linear susceptibility tensor, principal axes system, examples of an isotropic medium

and the susceptibility tensor in the principal axes, general properties of the linear susceptibility tensor for transparent crystals, optically isotropic crystals, uniaxial crystals, principal axes and the ordinary and extraordinary indices of refraction, negative and positive uniaxials, biaxial media, dielectric tensor and the displacement vector, longitudinal fields in anisotropic crystals.

- Optical propagation in crystals: Wave equation in crystals with respect to the principal axes, dispersion relation for plane-waves and field eigen-polarizations, wavevector or normal surface, intersecting shells and the optic axes, two field eigen-polarizations for each direction of propagation, orthogonality properties, intercept of the wavevector surface with the (x-z) plane, phase-velocity surface, Fresnel's equations of wave normals, examination of a uniaxial crystal, ray-velocity surface and the ray axes, the index ellipsoid or optical indicatrix, calculation of refractive-indices and displacement eigen-polarizations from the index ellipsoid, case of uniaxial crystals, c-axis for crystals, waveplates.
- Double refraction: Double refraction at the interface between vacuum and a uniaxial crystal, wavevector versus ray directions, polarizing prisms, discussion of double refraction in biaxial crystals, conical refraction.
- Nonlinear optics: Lorentz model and linear optics, higher-order nonlinear polarizations and their relation to anharmonic electron motions, second-order polarization and three-wave mixing, electro-optical Pockels effect, second-harmonic generation and optical rectification, phase-matching using uniaxial crystals, third-order polarization and four-wave mixing, electro-optical Kerr effect, optical phase-conjugation, nonlinear refractive-index and self-phase modulation in fibers, nonlinear self-focusing.

9. Polarization optics

- Polarization of light waves: Linear, circular, and elliptical polarization states of a monochromatic plane-wave propagating in free-space, real and complex field representations, polarization basis states and their orthonormality properties, two linearly independent polarization states for each direction of propagation, transverse nature of light, Poynting vector.
- Jones calculus: Jones vector representation for the positive-frequency component of a monochromatic plane-wave electric field, cases of linear, circular, and elliptical polarized fields, (2x2) Jones matrix representation for various optical elements acting on polarized fields, linear polarizers, waveplates, phase-shifters, optical rotators, Jones matrices for rotated optical elements, Jones matrices for concatenated optical elements, eigen-polarizations for optical systems, examples including free-space, and a Faraday isolator.
- Coherency matrix: Coherency matrix representation of fields, coherency matrix as a Hermitian (2x2) matrix representation of a tensor, examples of polarized and unpolarized fields, Poynting vector and the trace of the coherency matrix.
- Stokes parameters and the Poincare sphere: Stokes parameters and their relation to the coherency matrix, representation of states of polarization of light on the Poincare sphere, Mueller matrices, action of optical elements as motion on the Poincare sphere.

10. Dipole radiation and macroscopic electrodynamics

- Dipole radiation: Wave equations for the electromagnetic potentials in the Lorentz gauge, current density for an oscillating point dipole or Lorentz oscillator, Green's function solution for the vector potential, calculation of the physical electric and magnetic fields in the far field approximation, Poynting vector and the dipole radiation pattern, power radiated by an oscillating dipole, oscillator damping rate.
- Absorption and scattering of radiation: Incident and scattered fields, incident, scattered, and absorbed Poynting vectors, power loss due to scattering, scattering cross-section, Rayleigh, resonance, and Thomson scattering, Beer's law, angular variation of the absorbed Poynting vector, power absorption, absorption cross-section and Beer's law.
- Physical origin of the refractive-index: Radiation from a sheet of dipoles, phase-retardation and the refractive-index.