WRITTEN PRELIM EXAM – FIRST DAY

FALL 1999

October 4, 1999
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

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$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

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$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

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$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

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$\nabla \times \nabla\phi = 0$

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$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

I

Fall 1999

Consider an electromagnetic field whose space and time dependence is given by

$\vec{E} = \vec{E}(\vec{r} \cdot \hat{s} + ct)$ and $\vec{H} = \vec{H}(\vec{r} \cdot \hat{s} + ct)$

where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ and $\hat{s}$ is a unit vector.

- Determine by use of Maxwell's equations whether $\vec{E}$, $\vec{H}$ and the direction of propagation form a Right-hand or Left-hand triad of mutually perpendicular vectors. (75%)
- Show that the Poynting vector in this case is proportional to the field square and give the proportionality constant. (25%)

Use MKSA units.

II

Consider the equation of motion of an electron

$$e\mathbf{E}' = m\ddot{\mathbf{r}} + g\dot{\mathbf{r}} + q\mathbf{r}$$

Here, **E**', m, e, q, g, and **r** are the incident electromagnetic field, mass and charge of the electron, spring constant, damping factor, and electron position, respectively. Assume a harmonic field with angular frequency ω. Now, let there be N atoms, each having a single electron such that the total polarization is given by $\mathbf{P} = N\, e\, \mathbf{r} = N\, \boldsymbol{\alpha}\, \mathbf{E}'$, where α is the mean polarizability.

(a) Solve for $\mathbf{r} = \mathbf{r}(\omega)$ in the steady state and find the dependence of ω on q and m. (25%)
(b) Plot the dispersion curve with and without damping. (25%)
(c) What are the normal and anomalous dispersion regions and why are they coined as such? (25%)
(d) What is the effect of damping on the resonance frequency? (25%)

Do a first-order design of a telephoto lens (two thin lenses) with the following specifications:

Focal length:	200 mm
Unvignetted field of view:	+/- 10 deg
F/#:	f/4
Stop location:	Front element
Telephoto ratio*:	0.75
Back focal distance:	75 mm

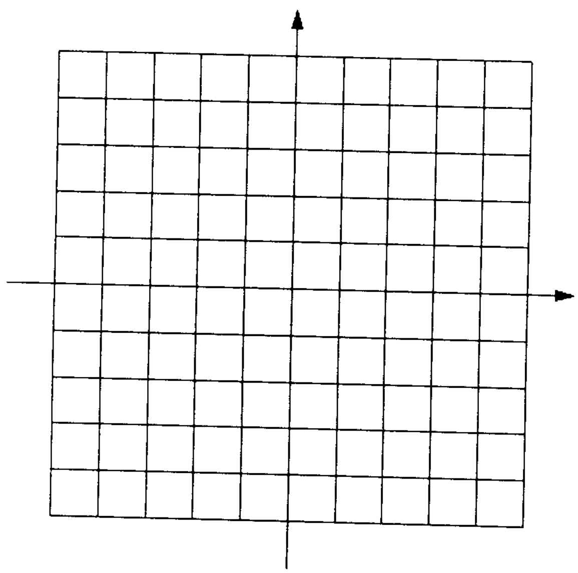
Provide the required element focal lengths, diameters and spacing.

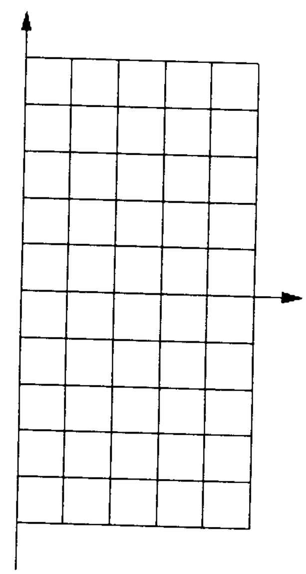
*The telephoto ratio is the ratio of the system length (front element to image plane) to the focal length.

IV

Fall1999

Draw to scale the ray fans at the edge of the field in an f/10 system where each of the third-order aberrations has a wave aberration value of 1 micrometer. All of the aberrations are present. Using an appropriate scale factor, label the plots.

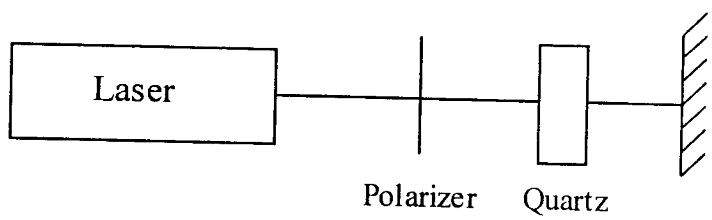




a) A quartz plate in the setup shown below has indices of refraction $n_o=1.544$ and $n_e=1.553$. The laser wavelength is 633 nm.

i) (1 Pt) What is the optimum relationship between the transmission axis of the polarizer and the axes of the quartz for a minimum amount of light being reflected back to the laser? Explain.

ii) (2 Pts) What is the minimum quartz thickness for having no light reflected back to the laser?

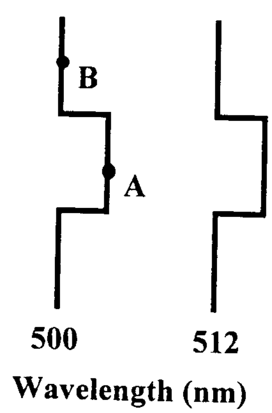


b) (2 Pts) A Michelson Stellar Interferometer working at a wavelength of 500 nm is used to measure the separation of binary stars. What is the separation of two stars, in seconds of arc, if the first minimum of fringe visibility is obtained with a mirror separation of 2 meters?

c) The following interferogram is obtained using a FECO interferometer to test a nearly flat mirror.

i) (2 Pts) Sketch the interferometer setup showing the important components.

ii) (3 Pts) What is the approximate surface height difference between point A and point B?

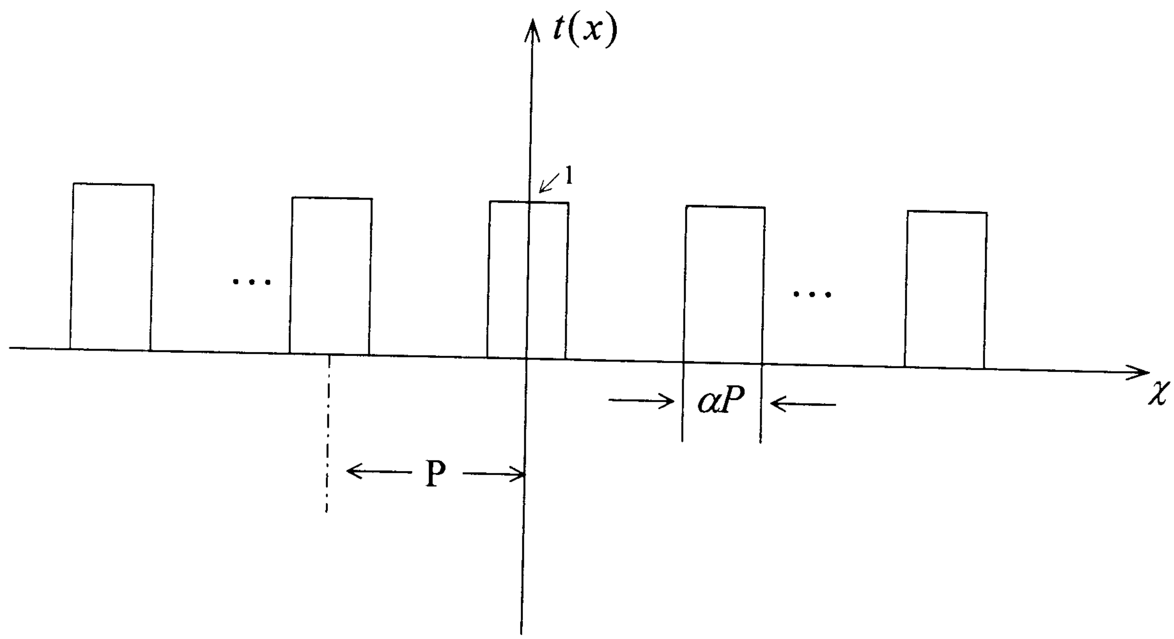


VI

A transmission diffraction grating has period P, duty cycle α and a total number of line-pairs N. The amplitude transmission function $t(x)$ is shown in the figure, where the apertures of width αP are fully transparent and the space between the apertures is fully opaque.

(a) Write the expression for the far-field diffraction pattern of this grating in terms of P, α, N, and λ, where λ is the wavelength of the normally incident beam. (70%)

(b) What is the diffraction efficiency (i.e., fraction of optical power) in the 0^{th} order and $\pm$ 1st orders of diffraction? (30%)



WRITTEN PRELIM EXAM – SECOND DAY

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VII **Fall 1999**

a) State the Van Cittert-Zernike theorem. 30%

b) Use the complex degree of coherence function as a starting point and derive the Van Cittert-Zernike theorem. 70%

VIII.

Fall 1999

The central star ⊗ in the Shoemaker system in the distant Wyant galaxy radiates as a 4500K blackbody and subtends a total angle of 1 degree as seen from the nearby planet Baily.

a) (20%) What is the radiance of ⊗?

b) (20%) What is the Ra constant (irradiance at mean ⊗-Baily distance)?

c) (5%) The eccentricity of Baily's orbit around ⊗ is about ±3%. How does this influence the ⊗ constant?

d) (30%) If the average reflectance of Baily in the range 0.3-3 microns is 0.2 and the average emittance of Baily is 0.9 for wavelengths longer than 3 microns, what is the equilibrium temperature of Baily?

e) (5%) How is the equilibrium temperature of Baily influenced by the eccentricity of its orbit about ⊗?

f) (10%) What is the radiance of Baily with no external sources (i.e., nighttime)?

g) (10%) What is the radiance of Baily when irradiated by ⊗ at noon?

Consider the two bandstructures shown in Figs. 1 and 2.

(a)
Which of the two bandstructure figures shows the bandstructure of AlAs and which one shows that of InAs? Use the information given in Fig. 3 to clearly justify your answer. (Note: the values of the data shown in Fig. 3 may differ slightly from those shown in Figs. 1 and 2; this, however, is inconsequential for the answer to the present question.)

Determine the mole fraction x that would give you the appropriate alloy for a bulk semiconductor laser which lases roughly at a wavelength of $\lambda = 1.23\mu m$. Assume the gain band width to be essentially zero and neglect band renormalization effects due to the electron-hole plasma
(Note: $\hbar c = 1.97 \times 10^{-5} eV cm$.)

Given the fact that a semiconductor laser can only operate on a direct transition, determine the largest frequency (in units of eV) you can achieve with $Al_xIn_{1-x}As$. Specify also the corresponding mole fraction.

(60 %)

(b)
Consider the lowest conduction band in AlAs. Which effective mass is larger, that at the Γ-point or that at the X-point?

Consider the highest valence band in Fig. 1. The Δ-line connecting Γ_{15} and X_5 can be approximately written as

$$\varepsilon_k^{\Delta} = A\frac{1}{2}[\cos(\pi k/k_X) - 1]$$

State the general relationship between the velocity $\vec{v}_{\vec{k}}$ and the bandstructure $\varepsilon_{\vec{k}}$. Estimate the maximum velocity (in units of m/s) along Δ, assuming a lattice constant of $a = 5\text{Å}$ and using an estimate for the bandwidth of the Δ-line based on the figure. Note that the wave vector at X is $\vec{k}_X = \frac{2\pi}{a}(1,0,0)$. (Hint: if you don't remember the general relationship between the velocity $\vec{v}_{\vec{k}}$ and the bandstructure $\varepsilon_{\vec{k}}$, recall the simple case of parabolic bands.)
(Note: $\hbar = 6.58 \times 10^{-16} eVs$.)

(40 %)

Note: All answers need to include a brief discussion in order to justify the answer.

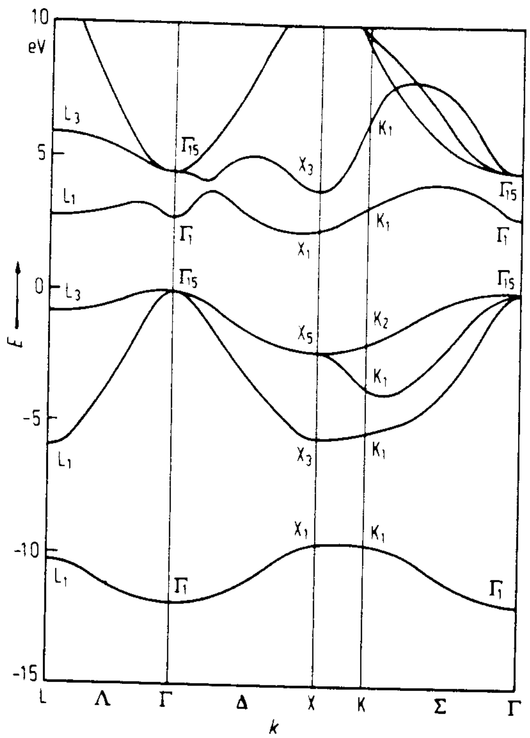


Fig 1

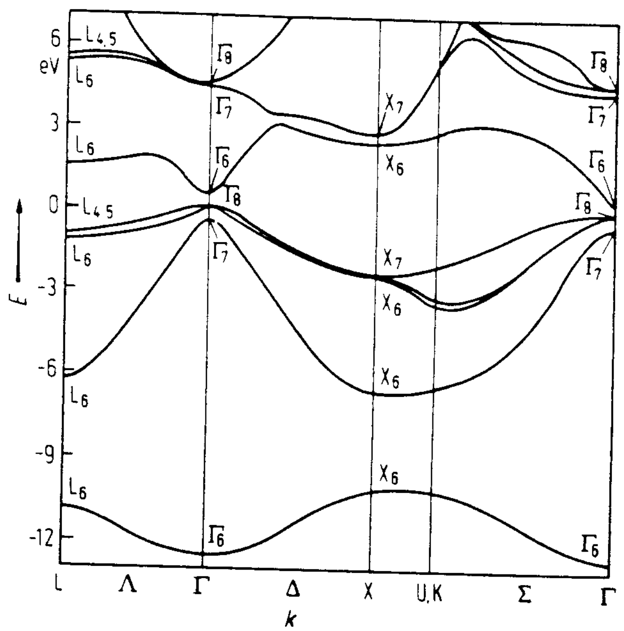


Fig. 2

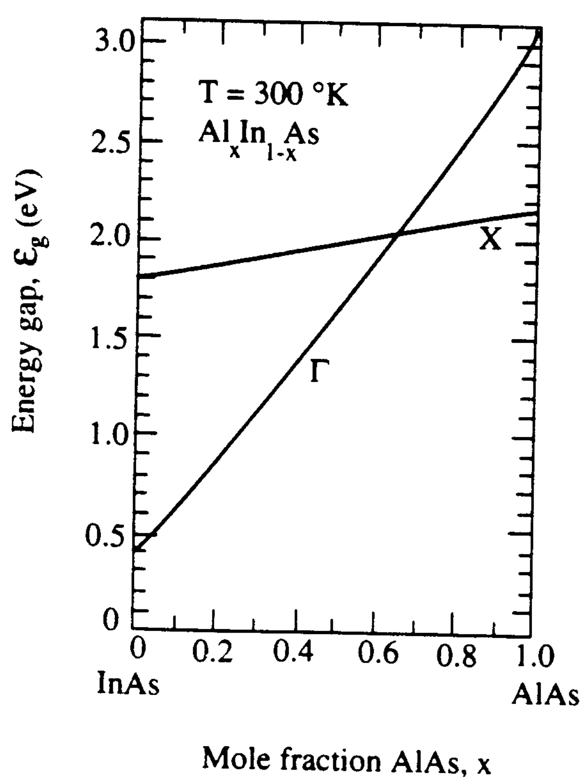


Figure 3. Compositional dependence of the direct-energy gap Γ and the indirect energy gap X for $Al_xIn_{1-x}As$

X

Fall 1999

(a) Define the entropy H for a random variable y obeying a probability density law $p(y|a)$ where a is some (fixed) parameter of the law. (20%)

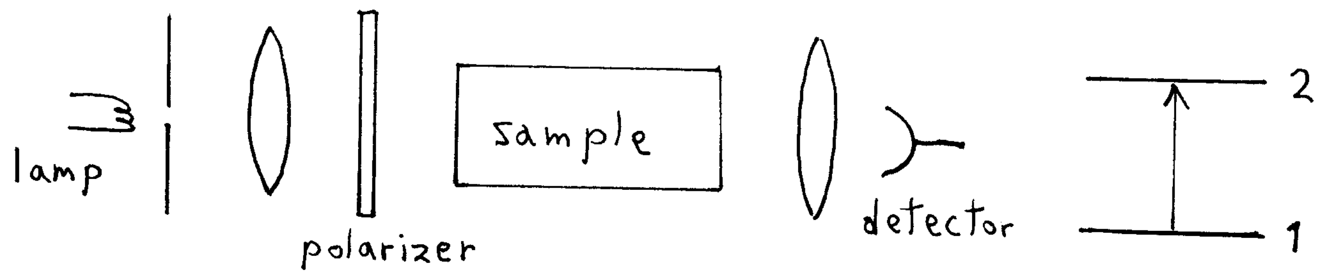
(b) Define the Fisher information I in the variable y about the parameter value a. (20%)

(c) Explain in words what the two information measures H and I measure. (20%)

(d) With a as above, suppose that $y = a + x$ where x is a Gaussian random variable with variance σ^2 . What is the form of the density law $p(y|a)$? (20%)

(e) Find both entropy H and information I for this case. (20%)

Consider a sample of an atomic gas with a constant amplitude, linearly polarized, collimated beam from a tungsten lamp passing through it and striking an optical detector:



(a) In thermal equilibrium, with the lamp turned off, the population of level 2 can be assumed to be negligible. When the lamp is turned on, its optical field causes transitions from some energy level "1" to a higher energy level "2" within the sample. Starting from the appropriate probability amplitude expression given below, derive an expression for the probability that an atom initially in level 1 will make a transition to level 2. (70%)

(b) Use your result from part (a) to calculate the absorbed power (Watts/cm^3) due to this transition, and hence the transition's absorption coefficient. (30%)

Some helpful results:

$$C_2 = i\frac{\mathcal{R}_0}{\mathcal{R}} e^{i(\omega_0-\omega)t/2} \sin(\mathcal{R}(t)/2)$$

$$C_1 = \frac{1}{2} e^{-i(\omega_0-\omega)t/2} \left\{ \left[1 + \frac{\omega_0-\omega}{\mathcal{R}}\right] e^{i\mathcal{R}t/2} + \left[1 - \frac{\omega_0-\omega}{\mathcal{R}}\right] e^{-i\mathcal{R}t/2} \right\}$$

where $\mathcal{R} = \sqrt{(\omega_0-\omega)^2 + (\wp_{12}E_0/\hbar)^2}$

$$C_2 = \frac{\wp_{12}E_0}{2\hbar} \left[\frac{e^{i(\omega_{21}-\omega)t} - 1}{(\omega_{21}-\omega)} \right]$$

$$C_1 = 1$$

Note also that $\displaystyle\int_0^\infty \frac{\sin^2 ax}{x^2}\, dx = \pi a$

Consider a He-Ne laser consisting of a gain tube with Brewster windows and a conventional 2-mirror resonator which is 30 cm in length. The lasing transition at 632 nm is inhomogeneously broadened with a full width (to the 1/e point) of 2.7 GHz (real frequency, not angular frequency).

(a) If the unsaturated gain at the peak of the gain curve is about a factor of three larger than the losses (which are assumed to be constant over the frequency region of interest), how many axial modes can lase? (20%)

(b) Assuming that there is no axial mode lying exactly at the center of the gain curve, make a sketch of gain versus frequency inside the gain medium for the situation where the laser is operating in steady state. (20%)

(c) Make a sketch of the behavior of the gain coefficient at one of the axial mode frequencies versus pump power supplied to the gain medium. Your sketch should show a range of pump power going from below the lasing threshold to well above it, and also indicate the pump power at which the lasing threshold is reached. (20%)

(d) How would you modify this laser in order to mode-lock it. (20%)

(e) If this laser is fully mode-locked, what is the approximate repetition rate and full width of the pulses produced by the laser. (20%)