WRITTEN PRELIM EXAM – SECOND DAY

SPRING 2000

February 23, 2000
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

I

Spring 2000

Consider a plane parallel, lossless, transparent plate having a refractive index n' and a thickness h surrounded by air. Let a monochromatic plane wave with wavelength λ_0 be incident upon the plate at an angle θ.

1. Calculate the phase difference δ associated with two successive reflections inside the plate? (20%)
2. By counting multiple reflections, calculate the reflectivity $\mathcal{R}$ and transmissivity $\mathcal{T}$ through the plate. (20%)
3. Calculate the corresponding reflected and transmitted light intensities $I^{(r)}$ and $I^{(t)}$. (20%)
4. Plot $I^{(r)}$ and $I^{(t)}$ as a function of δ for three values of $\mathcal{R}$. (20%)
5. Explain what is the finesse $\mathcal{F}$, and how does such a system act as an interferometer. (20%)

(a)

Consider a plane wave of the form

$$E(z,t) = E_0 \cos(\omega t - kz + \phi)$$

where all quantities are real. Sketch the wave for fixed $z = 0$ as function of t for both $\phi = 0$ and $\phi = \pi/2$. Assuming $\phi = 0$ and $k = n\frac{\omega}{c}$, derive an expression for $z(t)$ such that the phase of the wave is a constant. Also, use the resulting expression to specify the phase velocity.

(30 %)

(b)

A wave packet consists of many waves of the form given in (a). For simplicity, consider a wave packet that contains only two waves with equal amplitude, one wave with frequency $\omega - \frac{\Delta\omega}{2}$ and wave vector $k - \frac{\Delta k}{2}$, and the other with frequency $\omega + \frac{\Delta\omega}{2}$ and wave vector $k + \frac{\Delta k}{2}$. Both individual waves should be "cosine waves," (i.e., both waves have $\phi = 0$). Derive an expression for the total "wave packet." The final expression should be given in terms of real quantities (i.e., it should not contain complex exponentials which you may have used in the derivation). Identify in your final expression two contributions: (i) the so-called carrier wave and (ii) the so-called pulse train envelope. Under the conditions made in this problem, is the carrier wave necessarily a cosine wave? Specify the "phase velocity" of the pulse train envelope (more correctly, this "phase velocity" would have to be called "group velocity"). Show that the resulting expression for the group velocity coincides with the standard definition of group velocity if both $\Delta\omega$ and Δk approach zero. Also, specify the maximum amplitude of this wave packet.

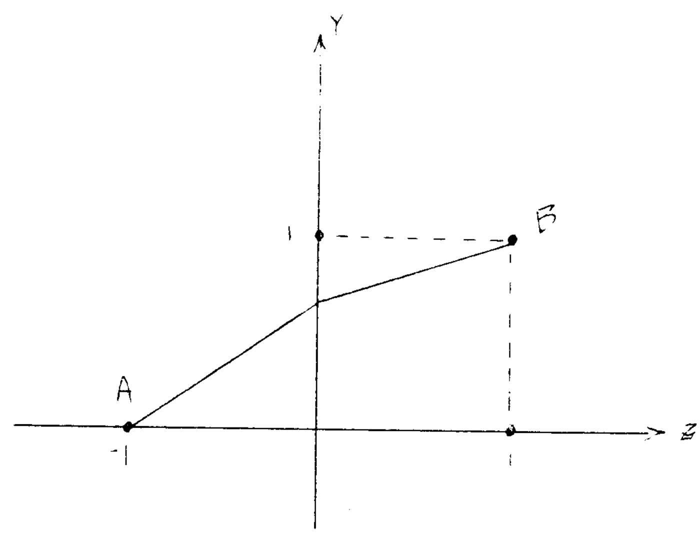
(70 %)

III

Spring 2000

1) State Fermat's Principle.

2) Derive Snell's Law from Fermat's Principle using the following diagram.



IV

Spring 2000

a) (20 %) What is the minimum number of phase steps (measurements) required in phase-shifting interferometry? Why?

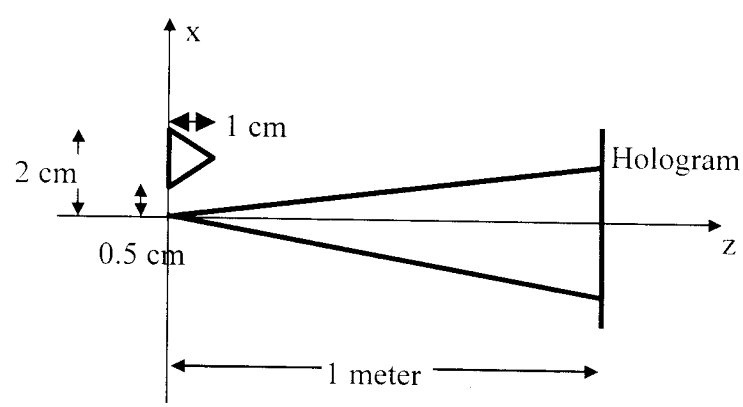
b) (20 %) The integrating-bucket version of phase-shifting interferometry is used with a Mach-Zehnder interferometer. How much is the contrast of the signal reduced if the detector integrates the light falling on it as the phase changes by 90 degrees?

c) A light source having two wavelengths, 500 nm and 510 nm, is used in a Young's double slit experiment. Each wavelength by itself would give unity contrast fringes.

i) (20 %) How many bright fringes do we have for the 500 nm wavelength before the fringe visibility becomes a minimum?

ii) (20 %) What is the ratio of the intensities of the two wavelengths if the minimum fringe visibility is 0.4?

d) (20 %) A hologram is made using a point reference source and a triangular shaped object as shown below. The hologram is reconstructed using a point source 2 m from the hologram. Give the x and z coordinates of the primary and conjugate images. Give a sketch to show what the images look like.



V

Spring 2000

All questions have the same percentage weight.

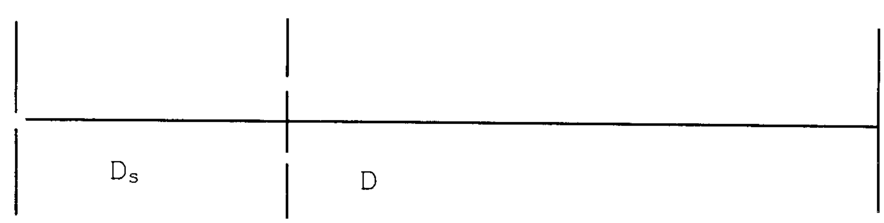
1) Explain what a linear system is. Can light propagation be modeled as a linear system?
2) If so, write the equations that relate the complex amplitude of an arbitrary field before and after propagation of a distance z. Write the equations in both the spatial and Fourier domains.
3) In general what is the transfer function for propagation in free space? What is the corresponding impulse response? How are these functions related ? Write the actual equations.
4) In the context of imaging systems explain what are the optical transfer function and the modulation transfer functions.
5) What does the MTF cutoff frequency mean?
6) Calculate the optical transfer function for a perfect imaging system that has a square aperture. Make a sketch of this function along one direction of your choice.

The figure shows an arrangement for a Young's interference experiment. A point source is placed in front of the two-aperture plane at a distance D_s = 0.5 m. The observation plane is behind the two-aperture plane at a distance D = 1.0 m. The source is quasi-monochromatic of mean wavelength 500.0 nm. It was observed that the fringe period is 1.0 mm.

Now keep the distances D_s and D fixed.

1. When the point source is replaced by an incoherent slit source of width 0.5 mm (mean wavelength 500 nm), no fringes are seen.
2. When the slit source is of width 1.0 mm no fringes are seen.
3. However, when the slit source of size 0.5 mm is replaced by an incoherent circular source of diameter 0.5 mm (mean wavelength 500 nm) fringes are seen.

<u>Explain</u> all these findings by <u>using</u> <u>considerations</u> <u>of</u> <u>spatial</u> <u>coherence</u> of the light from the respective sources. Assume that the sources have constant radiant exitance.



VII. Spring 2000

A Scud missile is entering the atmosphere at 100 km altitude. The missile is cylindrical in shape with a cross-sectional diameter of 1 m with a length of 4 m. Its temperature is 1000 K with an emissivity of 0.8 and it can be considered to be a lambertian radiator. You have a sensor with a 10^{-3} radians × 10^{-3} radians instantaneous field of view and a 10^{-2} radians × 10^{-2} radians full field of view on the ground used to track the Scud. The atmospheric transmission is 85%.

a) (10%) Prove that the Scud is either a point source or extended source for your system.
b) (30%) Calculate the irradiance from the Scud (E_e) at the entrance pupil of an infrared sensor for the two spectral regions of 3-5 μm and 3.8-4.0 μm.

In addition, we have a Patriot interceptor which is to impact this Scud at 100 km that needs to be measured by the same infrared sensor. The Patriot may be assumed to be lambertian. It is a cylinder 30 cm in diameter by 2 m long at a temperature of 300 K and reflectivity of 0.6. The Patriot is also illuminated by the sun which is assumed to be a blackbody at 6000 K with an angular extent of ½ degree.

c) (40%) Calculate the irradiance from the Patriot interceptor (E_e) at the entrance pupil of the infrared sensor in the spectral region from 4 to 5 μm.
d) (20%) If the radiant power needed to give a signal-to-noise ratio of 10 (s/n=10) for all cases is 10^{-10} watts at the entrance pupil, what diameter is required for the entrance pupil?

Useful formulas:

$$\int_{3\,\mu m}^{5\,\mu m} L_e(\lambda,1000)d\lambda = 0.65 \text{ watts/cm}^2\text{-sr}$$

$$\int_{4\,\mu m}^{5\,\mu m} L_e(\lambda,1000)d\lambda = 0.3 \text{ watts/cm}^2\text{-sr}$$

$$\int_{3.8\,\mu m}^{4\,\mu m} L_e(\lambda,1000)d\lambda = 6.7\times10^{-2} \text{ watts/cm}^2\text{-sr}$$

$$\int_{4\,\mu m}^{5\,\mu m} L_e(\lambda,300)d\lambda = 1.6\times10^{-4} \text{ watts/cm}^2\text{-sr}$$

$$\int_{4\,\mu m}^{5\,\mu m} L_e(\lambda,6000)d\lambda = 9.5 \text{ watts/cm}^2\text{-sr}$$

We want to compare the performance of the human eye to a good photovoltaic (PV) detector. The source to be detected and relevant parameters are as follows:

SOURCE	Filtered tungsten lamp: filament temperature = 3200 K Circular source area with 3-mm diameter Emissivity=0.5 at λ=505 nm Spectrally filtered output using a narrow bandpass filter with a center wavelength of 505 nm, full-width at half maximum bandpass of 2 nm, and passband transmittance = 0.5 with rectangular shape and out-of-band transmittance = 0.0
ATMOSPHERE	Attenuation coefficient = 0 at λ=505 nm
HUMAN EYE	Noise-equivalent photon flux into the 8-mm diameter pupil = 100 s^{-1} at λ=505 nm
PV DETECTOR	$D^*=10^{14}$ cm-$Hz^{1/2}$-W^{-1} at λ=505 nm, diameter=8 mm

Generate appropriate equations using the above parameters to show the signal-to-noise ratio (SNR) as a function of distance between the source and detector. Determine the distance for each detector for a SNR=10. Which is better? (80%)

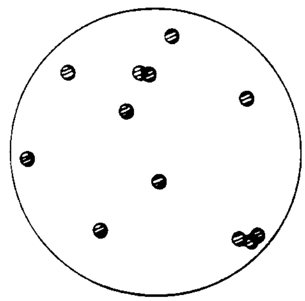
If the atmospheric attenuation coefficient is increased to 0.1/km, how are the SNR equations modified and what must be done to arrive at a solution to these modified SNR equations? (20%)

Probability density $p_X(x)$ is that of laser speckle intensity with mean intensity level a.

(a) Transformation to a new random variable y obeying $y = x^2$ is made. What is the probability density $p_Y(y)$ for the new random variable?

(b) Instead of the transformation in part (a), the laser speckle law $p_X(x)$ is transformed to a random variable z obeying $z = \exp(- x/a)$. What is the new probability density $p_Z(z)$?

The grains of a uniformly exposed photographic emulsion may be modelled as a random arrangement of 'poker chips', each of diameter a and a constant density level d_o. These are positioned with uniform randomness in coordinates x and y within the emulsion, resembling



An overlapping circular grain model of the emulsion. All grains have the same radius and photographic density

As pictured, the grains may generally overlap. The average number of grains/area is the constant value ν.

(a) The expression for the total density $d = d(\mathbf{r})$ at a given point $\mathbf{r} = (x,y)$ obeys a 'shot noise process'. Explain what *in general* such a process is, noting which quantities are known and deterministic, and which are random. Then express $d(\mathbf{r})$ as a specific shot noise process in terms of the given quantities of the photographic emulsion.

(b) Using the given parameters of the emulsion, determine the mean, variance and autocorrelation of density within the emulsion at a fixed point $\mathbf{r}_0$ that is not too close to the edge. These turn out to not depend explicitly upon $\mathbf{r}_0$. What kind of stationarity property does this signify?

(a) Name 3 different types of laser resonators (i.e., resonator configurations). For each resonator type that you've named, provide appropriate descriptions of the mirror curvatures and resonator lengths for that resonator type, and explain its advantages and disadvantages. (20%)

(b) The specifications for the output beam of most commercial lasers provide a value for M^2. What is M^2 and what does it tell you about the laser output? (15%)

(c) What is meant by the Rayleigh range of a laser beam? If a 1 millimeter diameter TEM_{00} beam from a He-Ne laser operating at 633 nm has a Rayleigh range of 0.5 meters, what would be the Rayleigh range if the beam diameter were increased to 1 cm? (15%)

(d) Most lasers (except for those whose resonator is formed from a waveguide or a fiber) can be forced into oscillating in a TEM_{00} mode by making appropriate modifications to the laser. What should you do in order to force a laser to oscillate in the TEM_{00} mode? Explain how the modifications you propose force the laser to oscillate in a TEM_{00} mode. What are the drawbacks of doing this? (25%)

(e) An inhomogeneously broadened laser can be forced to oscillate in a single axial mode by making appropriate modifications to its resonator. If changing the overall resonator length is <u>not</u> an option, what could you do in order to force a laser to oscillate in a single axial mode? Explain how the modifications you propose force the laser to oscillate in a single axial mode. (25%)

Obtain an analytical expression for the point spread function resulting from the indicated pupil function. Assume uniform amplitude for the elements in the pupil function, but treat the phase of the center element as a variable, assuming zero phase for the two side elements.