WRITTEN PRELIM EXAM – FIRST DAY

Spring 2002

February 19, 2002
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

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$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

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$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

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$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

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$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Question # 1 SPRING 2002

1. Given a surface radius of 10 mm and a refractive index of 1.5, locate the cardinal points of the following biconvex lenses with respect to the center of the lens; a thin lens, a lens with a thickness of 10 mm, and a lens with a thickness of 20 mm.

(50%)

2. Derive a general expression for the distance from the center of a biconvex lens to the rear principal point as a function of t/R where R is the (fixed) radius of the surfaces, n is the refractive index, and t is the thickness of the lens. Check your results with the values obtained in part 1.

(50%)

Question #2 **Spring 2002**

Consider a solid that can be modeled with Lorentz oscillators. Let the density of the Lorentz oscillators be N. For simplicity, assume that all damping effects can be completely neglected. The dielectric function is then given by

$$\varepsilon(\omega) = 1 - \frac{\omega_{pl}^2}{\omega^2 - \omega_0^2}$$

where ω_{pl} is the so-called plasma frequency and ω_0 is the eigenfrequency of the oscillator.

(a) Sketch the frequency dependence of $\varepsilon(\omega)$ for $\omega \geq 0$. Completely label all special points in your plot (i.e., specify $\varepsilon(0)$, $\varepsilon(\infty)$ and any other relevant points). Also, specify in your sketch the frequency interval in which the material would exhibit a normal-incidence reflectivity of $R = 1$. Briefly justify your answer in two steps. First, discuss whether or not the refractive index n is given by $n = \sqrt{\varepsilon}$ in the frequency region under consideration (there's no derivation requested for this answer). Secondly, specify the normal-incidence reflectivity in terms of the refractive index $n(\omega)$ (again, you hopefully know this relation from memory, you don't have to derive it here).

(50 %)

(b) Specify the length, $\Delta\omega$, of the frequency interval determined in (a) in terms of the plasma frequency and the oscillator's eigenfrequency. In addition to the general formula for $\Delta\omega$, derive an approximate formula in which $\Delta\omega$ is linearized in ω_{pl}^2/ω_0^2. To this end, you have to assume that the plasma frequency is much smaller than the eigenfrequency. The linearization is based on a simple first-order Taylor expansion. In this case, how does $\Delta\omega$ vary with the dipole density N? (No formal derivation is requested here. However, if you don't know the answer, you probably have enough time to derive it. It's not a difficult derivation.)

(50 %)

(a) Consider a one-dimensional harmonic oscillator consisting of a negatively charged particle bound by a linear restoring force to a positively charged particle fixed at the origin. Suppose that the oscillator is excited by an optical field into a coherent superposition of eigenstates $\Psi_n(x,t)$ and $\Psi_k(x,t)$. Starting from the definition of $\langle d \rangle$, the expectation value of the electric dipole moment, derive an explicit expression for $\langle d \rangle$, simplifying the result as much as possible. Do not assume that the probability amplitudes for the two eigenstates in this superposition are equal. (30%)

(b) If a plane wave optical field polarized in the $\hat{\mathbf{x}}$ direction and propagating along the Z axis acts on the harmonic oscillator, what new term(s) must be added to the Hamiltonian to describe the interaction of the oscillator with the field (assuming that the dipole approximation has been made)? (10%)

(c) The probability of making an optical transition from state ψ_n to state ψ_{n+1} for the harmonic oscillator depends on the value of $\wp_{n\,n+1}$, the transition dipole matrix element. Starting from the definition of $\wp_{n\,n+1}$, calculate an explicit expression for this transition dipole matrix element. (30%)

(d) What are the selection rules for dipole-allowed transitions in the 1-D harmonic oscillator? (10%)

(e) If a strong, near-resonant optical field is incident on a sample of 1-D harmonic oscillators that are initially in a mixture of various initial eigenstates $\Psi_m(x,t)$, would it be appropriate to treat the oscillator-field interaction using the two-level atom approximation? Explain why or why not. (20%)

The following results may be useful:

$$\hat{a} \equiv \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,x + \sqrt{\frac{\hbar}{m\omega}}\,\frac{d}{dx}\right) \qquad \text{(lowering operator)}$$

$$\hat{a}^\dagger \equiv \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,x - \sqrt{\frac{\hbar}{m\omega}}\,\frac{d}{dx}\right) \qquad \text{(raising operator)}$$

$$\hat{a}\,\psi_n = \sqrt{n}\,\psi_{n-1}$$

$$\hat{a}^\dagger\,\psi_n = \sqrt{n+1}\,\psi_{n+1}$$

Question #4

Spring 2002

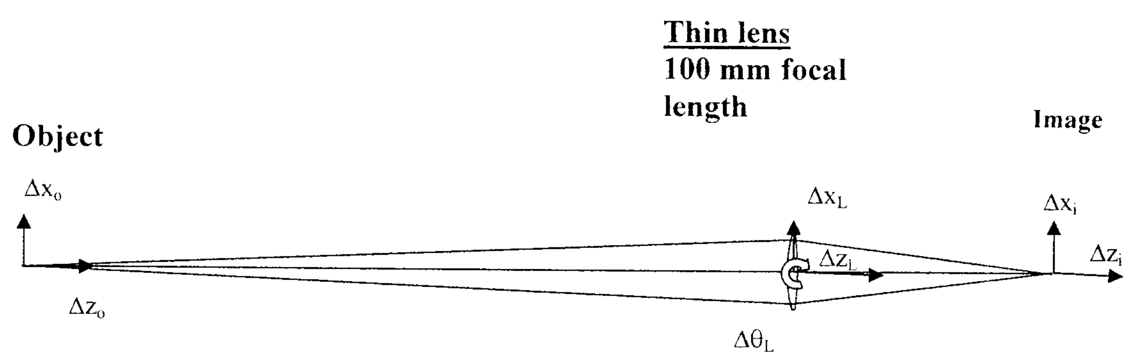
Consider a simple imaging system using an ideal thin positive lens with focal length f, creating a real image with magnification m (define as image height/object height) .

A) Give an expression for the object and image locations relative to the lens. (15%)

B) How would the answer change if a negative lens was used instead of the positive lens. (10%)

C) For the case of the ideal positive thin lens, give an expression for motion of the image, Δx_i and Δz_i when small motions are made for the each of the following cases: (75%)

Motion of the object Δx_o (assume lens is fixed)
Motion of the object Δz_o (assume lens is fixed)
Motion of the lens Δx_L (assume object is fixed)
Motion of the lens Δz_L (assume object is fixed)
Tilt of the lens $\Delta\theta_L$ about its y axis (assume object is fixed)



Question #5 SPRING 2002

1. Discuss the local symmetry of the images for each of the third-order aberrations both in the image plane and through focus.

   50%

2. Describe for each of the third-order aberrations the appearance (size and orientation) in the image field of a concentric circular array of images including the center of the image field and four equally spaced zones about the center.

   50%

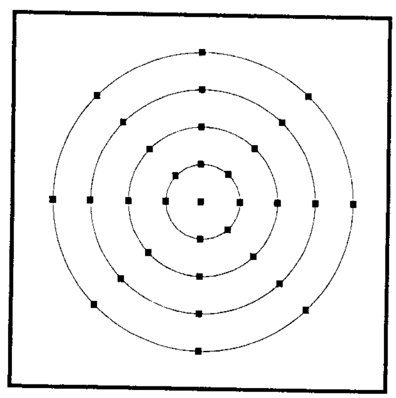
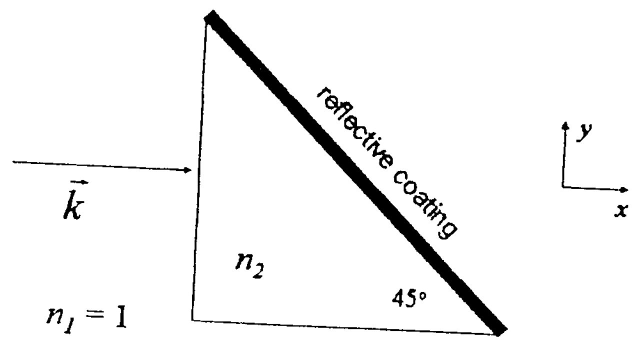


Image Field

Question #6

SPRING 2002



A λ= 500 nm plane wave traveling along the x axis illuminates a reflective surface oriented as shown in the diagram. Assume the coating provides a perfect reflection, without attenuation or phase shift. Determine the orientation of the fringes and the minimum fringe spacing for the following cases:

1. (20%) The plane wave is polarized in the z axis (out of the plane of the paper) and $n_2 = 1$.

2. (20%) The plane wave is polarized in the y axis and $n_2 = 1$.

3. (30%) The plane wave is polarized in the z axis (out of the plane of the paper) and $n_2 = 2$.

4. (30%) The plane wave is polarized in the y axis and $n_2 = 2$.

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I am looking through a window screen made up of a square mesh of wires at a point source 10 meters away.

a) (50%) If the mesh has wires 0.2 mm apart and the wavelength is 500 nm, what is the spacing of the square arrangement of bright spots that I see located about the point source?

b) (50%) What determines the relative intensities of the bright spots?

A planar, non-coherent, constant radiant exitance and quasi-monochromatic source of mean wavelength $\bar{\lambda} = 500.\,\text{nm}$ is covered with a mask given by

$$M(x) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi x}{40\bar{\lambda}}\right)\right].$$

This source is in the object plane of a converging image-forming (thin) lens working at unit magnification. The object distance is 20.0 cm from the lens.

- Obtain the spatial coherence function in the aperture plane of the lens.

The lens aperture is covered with a two pin-hole mask. Based on your spatial coherence function, give a qualitative description of what one would observe in the image plane if the

- pin-hole separation is 5.0 mm.
- pin-hole separation is 3.0 mm.

{Equal weight for all parts}

Question #9 SPRING 2002

1. What two physical effects are needed to obtain lasing? Give an example of how each is provided in practice.

   20%

2. A laser has a cavity of length L filled with a medium of refractive index n. What are the wavelengths and frequencies for the axial modes? What is the frequency difference between two modes?

   20%

3. What is modelocking used for and how is it achieved?

   20%

4. Consider a two-level system consisting of an upper state 2 and ground state 1. Write down the rate equations for this two-level system subjected to a coherent optical field E. Solve the equations and show whether or not population inversion can be achieved.

   40%

Question #10

SPRING 2002

For a photodetector that would be used for fiber optic receiver, a high quantum efficiency and a fast (quick) response is needed.

A) What is "quantum efficiency" in words and in an equation?

B) What detector material parameters affect the photo detector's quantum efficiency?

C) What photo detector material parameters affect the speed of response? Discuss each parameter's direct effect.

Question #11

SPRING 2002

a) What is the Huyghens wave envelope construction? Carefully explain this, including a diagram and all underlying assumptions.

25%

b) Based upon this construction, compute the amplitude function in the focal plane of a lens, for a slit source at infinity. Assume the presence of quasi-monochromatic light. Express the answer in terms of the pupil function of the lens. (For simplicity, work only with one coordinate – the one transverse to the slit.)

25%

c) Define the optical transfer function (OTF) of the lens, in the presence of quasi-monochromatic, incoherent light.

25%

d) Using the result in part (b), relate the OTF to the pupil function of the lens. Does an incoherent image obey sharp cutoff of frequencies? If so, what is the cutoff frequency?

Question #12 SPRING 2002

a) What is the Cauchy (or Lorentzian) probability density law? 20%

b) What is its second moment; its variance? 20%

c) Suppose that the arithmetic average of N independent and identically distributed Cauchy random variables is taken. Regard N as finite. What is the functional form of the resulting probability density law?

20%

d) What does the central limit theorem of statistics state? 20%

e) What does the probability law that is the answer to part (c) approach as $N \to \infty$? Why?

20%