# WRITTEN PRELIM EXAM – FIRST DAY

## Fall 2003

September 23, 2003
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

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The following are some helpful items:

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$e = 1.6 \times 10^{-19}\ \mathrm{C}$

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$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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$\sin 2A = 2 \sin A \cos A$

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$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

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$\sinh x = \frac{1}{2}(e^x - e^{-x})$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

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$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

(a) Write down the macroscopic Maxwell equations in the MKSA system of units including contributions from both bound and free charges. (2 points)

For the remainder of this question consider a plane-wave electromagnetic field $\vec{E}(\vec{r},t)$ of frequency $\omega$ that is propagating along the optic-axis of a uniaxial crystal that is chosen to coincide with the z-axis. The electric field of the propagating field can be written as

$$\vec{E}(\vec{r},t) = \frac{1}{2}\left[\vec{x}\mathcal{E}_x e^{i((\omega/c)n_x z - \omega t)} e^{-\alpha_x z/2} + \vec{y}\mathcal{E}_y e^{i((\omega/c)n_y z - \omega t)} e^{-\alpha_y z/2} + c.c\right],$$

where $\vec{x}, \vec{y}$ are unit vectors along the x and y axes, $\mathcal{E}_x, \mathcal{E}_y$ are the incident complex electric field amplitudes along the x and y directions just inside the crystal, $n_x$ and $\alpha_x$ are the refractive-index and absorption coefficient for a field polarized along the x-axis, and $n_y$ and $\alpha_y$ are the corresponding values for a y-polarized field.

(b) First, consider the case of a lossless crystal and an incident field that is polarized at 45° with respect to the x-axis, $\mathcal{E}_x = \mathcal{E}_y$. Using the above electric field solution derive an expression for the length of crystal needed to convert the incident linearly polarized field into either a left- or right-handed circularly polarized field. (3 points)

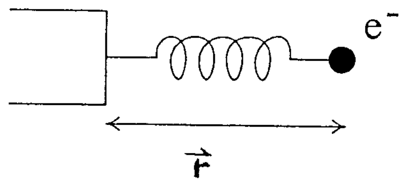
(c) Next, consider the same situation as part (b) except that absorption is present with the absorption coefficient $\alpha_x$ slightly larger than $\alpha_y$. Explain how you would adjust the input linear polarization state so that the output would still be circularly polarized even in the presence of absorption. (3 points)

(d) Finally, consider the case that $n_x = n_y$ but the absorption coefficients along the x and y axes are different $\alpha_x \neq \alpha_y$. Explain whether or not this crystal could be used to convert an input linearly polarized field into a circularly polarized state. (2 points)

Question #2

FALL 2003

(a) Many optical phenomena can be modeled by the "electron-on-a-spring" classical oscillator:




Write out the equation of motion for such an oscillator having a resonant frequency $\omega_0$, a damping rate $\gamma = 1/\tau$, and interacting with a monochromatic electromagnetic field via the Lorentz force. Your equation must include the complete, exact Lorentz force. (15%)

(b) Describe any approximations that are normally made to the Lorentz force oscillator-field interaction that make the equation of motion easily soluble, and indicate why they should be valid for a classical oscillator interacting with a typical optical field. (10%)

(c) The steady-state solution of the equation of motion in part (b) for a linearly polarized plane wave optical field $E = E_0\, e^{-i\omega t}$ is

$$x(t) = \frac{eE}{m} \frac{1}{(\omega^2 - \omega_0^2) + i\omega/\tau} = \frac{eE}{m} \frac{(\omega^2 - \omega_0^2) - i\omega/\tau}{(\omega^2 - \omega_0^2)^2 + \omega^2/\tau^2}$$

Consider a medium made up of identical classical oscillators where the density of oscillators is $N$ oscillators/cm$^3$. Assuming that the medium is optically thin, derive analytic formulas for the index of refraction and the absorption coefficient of this medium and sketch the behavior of the two quantities versus frequency in the spectral region around $\omega_0$. (35%)

(d) What modification(s) need to be made to the expression for $x(t)$ in part (c) for it to apply to the conduction electrons in a metal? (10%)

(e) Use the appropriately modified result for $x(t)$ to derive an approximate formula for the dielectric constant of a metal at frequencies such that $\omega \gg 1/\tau$ ? (30%)

For a particular experiment, light from a circular exit pupil 2 cm in diameter must be transmitted through two cylindrical cells in sequence. The first cell is 2 cm in diameter and 4 cm long. The second cell is 1 cm in diameter and 1 cm long. Both are filled with gasses with n=1.0. The cell walls are black and absorbing.

a) What is the largest throughput (etendué) beam which can be transmitted through cell 1? Cell 2? (2 points)
b) What solid angle of light from the exit pupil can be coupled through the cells? (2 points)
c) Design an optical system with thin lenses to couple this maximum solid angle from the exit pupil through the first cell, then the second cell, without vignetting. Let the exit pupil be located at z=0. Specify the positions of the lenses and cells and the effective focal lengths of the lenses. Describe the first order properties of the different sections of your solution. (6 points)

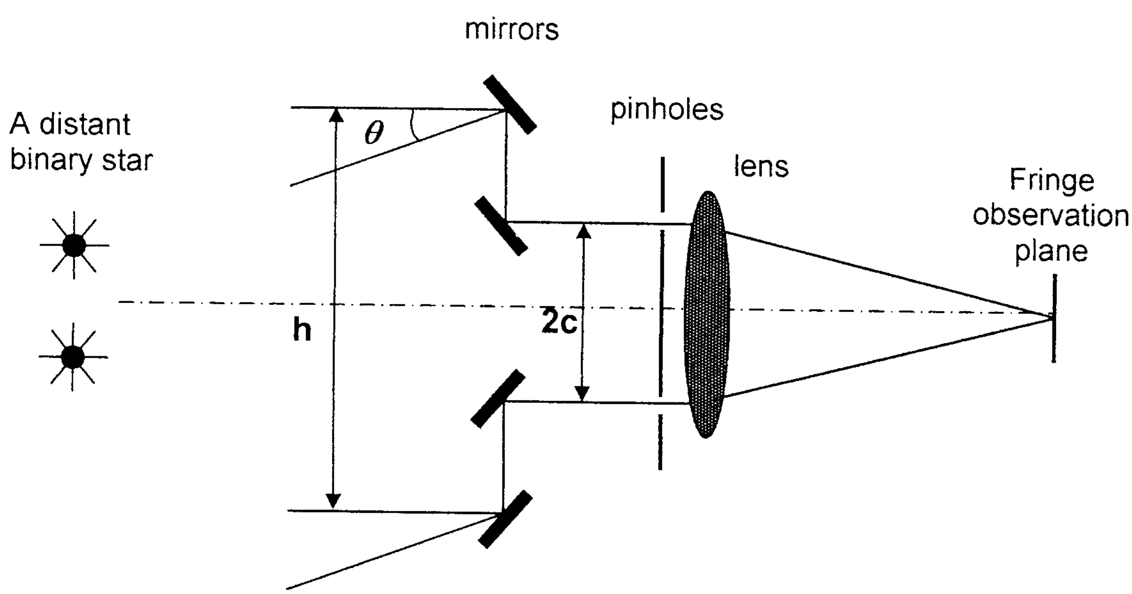
1. Define each of the five third order transverse ray aberrations in terms of object (field) coordinate, h, and pupil coordinates ($\rho$, $\theta$). (2 points)
2. Explain the relationship between wavefront aberrations and transverse ray aberrations. (2 points)
3. Describe the effect of each aberration individually on the paraxial image. Draw the spot diagrams for a point source at the edge of the field. (2 points)
4. Which aberrations can be partially compensated with defocus? (1 point)
5. Can any third order aberrations be partially compensated with tilt? Why? (1 point)
6. Why is it usually more important to compensate for coma than astigmatism in an astronomical telescope? (2 points)

Question #5 FALL 2003

The Michelson Stellar interferometer shown below is used to measure the separation of a particular binary star. The radiance distribution is

$$a^2(\theta) = I_0\{\delta(\theta+\Delta\theta)+\delta(\theta-\Delta\theta)\},$$

where +/- $\Delta\theta$ are the angles between the optical axis and the ray coming from each star, and $I_0$ is the radiance of each star. Assume that a narrow-band optical filter is used to obtain quasimonochromatic illumination at $\bar{\lambda}$ = 500 nm. If the first zero in fringe visibility occurs at a mirror separation of $h$ = 15 meters, what is the angular separation of the stars?



Question #6 Fall 2003

each

4 points part A, 2 points ^ parts B, C, and D

A laboratory calibration is being performed based on a spherical integrating source with an exit aperture that is 20 cm in diameter. The output of the sphere source at the exit aperture is 10 $W/(m^2 sr \mu m)$ at a wavelength of 900 nm.

The radiometer being calibrated is a simple tube radiometer with a 1 $mm^2$ silicon detector at the base of a 10 cm long tube that is 2 cm in diameter. The radiometer views the sphere's exit port from a distance of 2 m (as measured to the detector) and at an angle of 45 degrees from the axis normal to the exit port. A 10-nm wide spectral filter centered at 900 nm with an average transmittance of 0.7 is placed in front of the detector for spectral selection.

A) What is the radiant flux (power) through the detector?

B) What is the spectral radiance on the detector?

C) It is determined that all of the geometric information (distance, aperture sizes, view geometry, etc.) have uncertainties of 1% in the knowledge of that parameter. Show which parameter will be the dominant uncertainty source.

D) A lens is placed at the front aperture of the radiometer. Describe what happens to the radiant flux through the detector.

Be sure to state all assumptions.

# WRITTEN PRELIM EXAM – SECOND DAY

## Fall 2003

September 24, 2003
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You have a lens assembly made up of two identical thin lenses separated by 25 mm, and the focal length of each thin lens is 50 mm. Fixed object and image planes are separated by 150 mm. What are the two possible image magnifications that can be obtained with your lens assembly?

(a) The crystal structure of a crystalline solid is determined by the Bravais lattice and the basis. A Bravais lattice can be characterized by the infinite set of lattice vectors

$$\mathbf{R_n} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$$

where the $n_i$'s are arbitrary (positive or negative) integers and the $\mathbf{a}_i$'s are primitive translation vectors.
Specify which (if any) of the following statements are correct:

1. The $\mathbf{a}_i$'s are mutually orthogonal.
2. The $\mathbf{a}_i$'s form an orthonormal set.
3. The unit of the modulus, $|\mathbf{a}_i|$, is cubic meter.
4. The $\mathbf{a}_i$'s are linearly independent.

Also, show that the sum of two arbitrary lattice vectors is a lattice vector.

(30 %)

(b) To any lattice in real space, defined by the primitive translation vectors $\mathbf{a}_i$, one can define a reciprocal lattice with primitive translation vectors in reciprocal space,

$$\mathbf{b}_1 = \frac{2\pi}{V_c}\left(\mathbf{a}_2 \times \mathbf{a}_3\right), \qquad \mathbf{b}_2 = \frac{2\pi}{V_c}\left(\mathbf{a}_3 \times \mathbf{a}_1\right), \qquad \mathbf{b}_3 = \frac{2\pi}{V_c}\left(\mathbf{a}_1 \times \mathbf{a}_2\right),$$

where $V_c$ is the volume of the unit cell in real space.
Consider a simple cubic lattice with lattice constant $a$. Determine the primitive translation vectors in reciprocal space, classify the reciprocal space (i.e., what kind of lattice is it?), and determine the volumes $V_c$ and $V_c^b$ of the primitive unit cell of the original simple cubic lattice in real space and that of the primitive unit cell of the corresponding lattice in reciprocal space, respectively.

(70 %)

Question #9 Fall 2003

A random variable x obeys an exponential probability density with mean value $a$ over the positive interval $(0, \infty)$.

(a) 10% Define this probability density.

Each such value of $x$ is input to a nonlinear processor that produces an output

$$y = x + \frac{1}{x}.$$

(b) 10% Sketch this function $y(x)$, indicating as well the minimum value of $y$.

Find the probability density $p_Y(y)$.

In doing so, answer the following questions:

(c) 10% What is the equation giving the root(s) $x$ for each value of $y$?

(d) 10% Is this a one-root, a two-root, or etc., problem?

(e) 10% What particular form does the Jacobian approach take for this root-case?

(f) 50% Combine all of the preceding to find the probablility density.

Question #10 Fall 2003

Let $f(t;a,b)$ be the stochastic process

$$f(t;a,b) = a\cos(\omega_0 t) + b\sin(\omega_0 t), \tag{1}$$

where $a$ and $b$ are independent random variables, and $\omega_0$ is a positive constant.

1. (30%) What conditions must be satisfied by the random variables $a$ and $b$ in order for $f(t;a,b)$ to be a wide-sense stationary (WSS) random process? (Note the trig. identities at the bottom of this page)

2. (20%) Define and derive the power spectrum $S_f(\omega)$ of this stochastic process using the conditions derived in part 1.

3. (30%) Let us define a new stochastic process,

$$g(t;a,b) = H\left[f(t;a,b)\right], \tag{2}$$

where $H[]$ is a linear operator with optical transfer function (OTF) given by,

$$OTF(\omega) = \exp\left(-\frac{\alpha^2\omega^2}{2}\right), \tag{3}$$

where $\alpha$ is a positive, real constant. Determine both the power spectrum and the autocorrelation for this new stochastic process $g(t;a,b)$.

4. (20%) Now consider the addition of a white-noise process, *i.e.*,

$$\begin{aligned} g'(t;a,b) &= H\left[f(t;a,b)\right] + n(t) & (4)\\ &= g(t;a,b) + n(t), & (5) \end{aligned}$$

where $n(t)$ is a zero-mean, white-noise stochastic process with variance 1. The process $n(t)$ is independent of $f(t;a,b)$ and $g(t;a,b)$. How does this white noise process effect both the power spectrum and the autocorrelation functions of $g(t;a,b)$ you derived in part 3?

Useful trigonometric identities:

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

(a) (2 points) Which one of the following statements correctly describes the gain medium of a simple CW laser that can be converted into a mode-locked laser? (Assume that laser operation is restricted to the $TEM_{00}$ transverse mode.) **Choose one of the following, and in a sentence or two explain your reasoning.**

(i) The gain medium must be homogeneously broadened.
(ii) The gain medium must be inhomogeneously broadened.
(iii) The gain medium can be *either* homogeneously or inhomogeneously broadened.
(iv) The gain medium can be *neither* homogeneously nor inhomogeneously broadened.

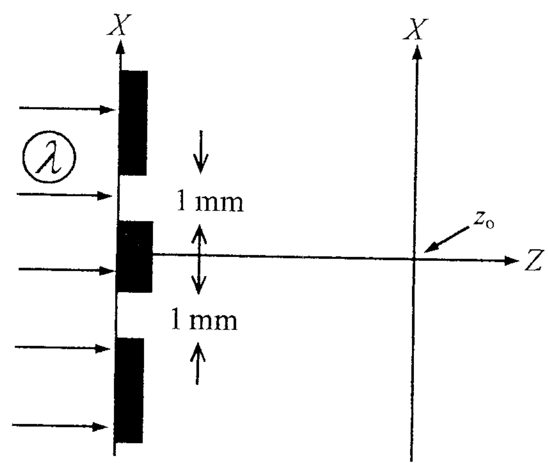
(b) (4 points) Suppose that a homogeneously broadened laser with a 30 cm long confocal resonator is operating in a single mode characterized by the axial mode number $q = 10^6$ and the transverse mode numbers $n = m = 0$ (i.e., $TEM_{00}$). **Answer each of the following questions:**

(i) What is the wavelength of the laser light produced?
(ii) What is the free spectral range of this laser resonator?
(iii) If a hair falls onto a resonator mirror, causing the transverse mode of the laser to suddenly jump from the $TEM_{00}$ mode to the $TEM_{01}$ mode (with the axial mode number remaining unchanged), what is the associated change in the optical frequency of the laser light?
(iv) Is this new frequency higher or lower than the frequency of the original $TEM_{00}$ laser light?

(c) (2 points) **Give the *maximum* number of frequency components** (or peaks in a power vs. frequency measurement) that can exist in the laser light of a simple gas laser having the following characteristics: (i) the laser resonator is 30 cm long, (ii) the Doppler width of the gas is 3.2 GHz, (iii) the on-resonance small-signal gain is twice the threshold gain, and (iv) the laser operates in only the $TEM_{00}$ transverse mode.

(d) (2 points) An optical resonator is to be constructed with two mirrors, one of which is concave, having a radius of curvature of 50 cm, and the other is convex, having a radius of curvature of -100 cm. **For what range of cavity lengths will this resonator be stable?**

A plane, monochromatic beam (wavelength = $\lambda$) is incident on a pair of apertures in a thin mask, as shown in the figure. (This is a two-dimensional problem in which the $Y$-dependence should be ignored.) The apertures are rectangular slits, each 1mm wide, with a 2mm center-to-center spacing. Assuming $\lambda = 0.5\mu m$, determine the (complex) light amplitude distribution in the far-field at $Z = z_o$.



# MINOR WRITTEN PRELIM EXAM

## Fall 2003

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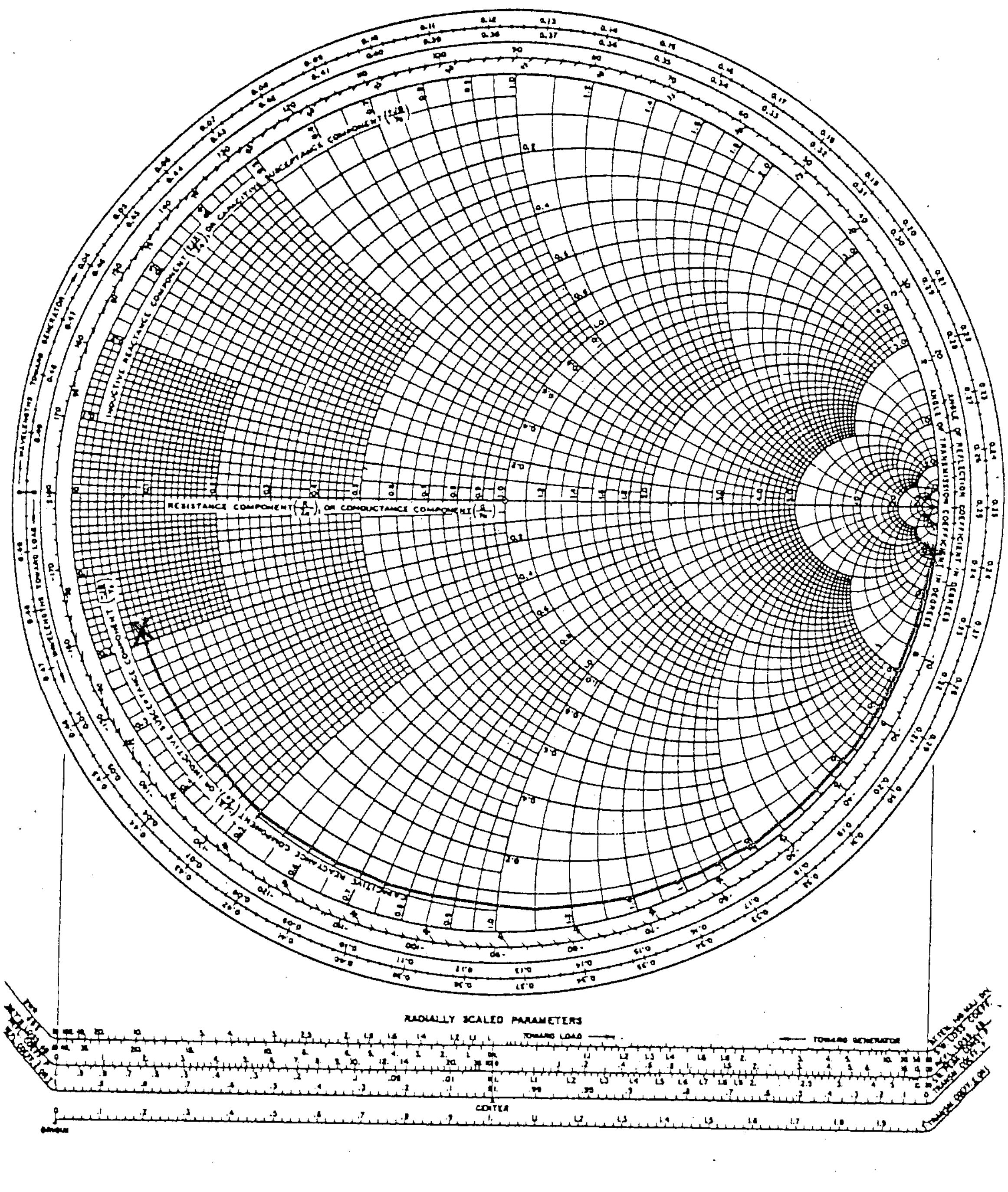
# Prelim Question for
# OPTI 582 High Bandwidth Photonics

1. The impedance of a high frequency laser diode is shown on the Smith chart. The measurements were made in a system with a 50Ω characteristic impedance and measured from 45 MHz to 50 GHz. Using a simple high frequency laser model that is composed of a resistor and capacitor in series, determine the resistance of the laser diode.



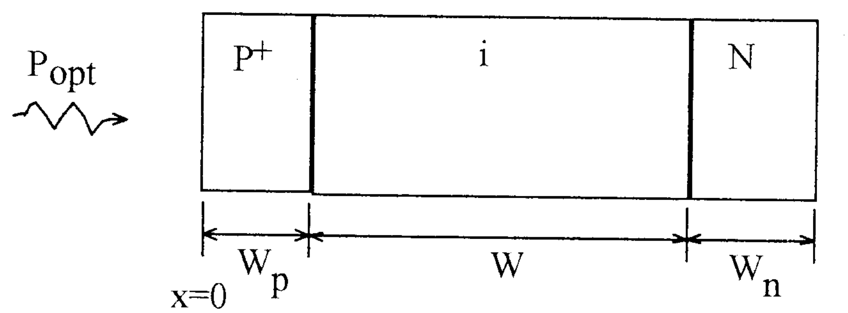
Question #2 Fall 2003

**Prelim Question for**
**OPTI 580 Microphotonics**

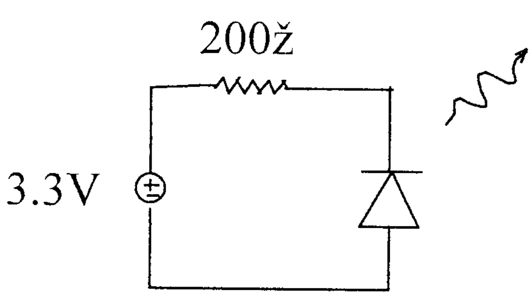
1. The p-layer of a pin photodiode is much thinner than $1/\alpha$ and the pin photodiode is operating in the reverse bias region. The generation rate for electron-hole pairs is given by

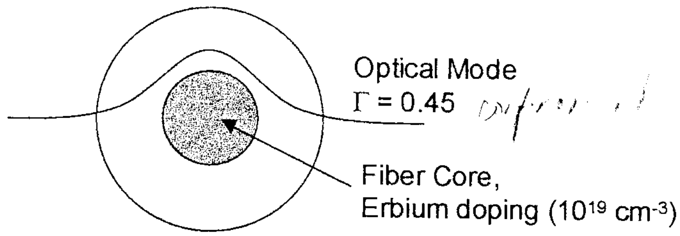
$$G(x,\lambda) = \alpha(1-R)\Phi e^{-\alpha x}$$

Calculate the drift current in the diode.




2. Determine the ELECTRICAL power absorbed by the VCSEL using a piecewise linear model for the VCSEL where $V_{DO}$=1.4 V and $R_{DO}$=50Ω.

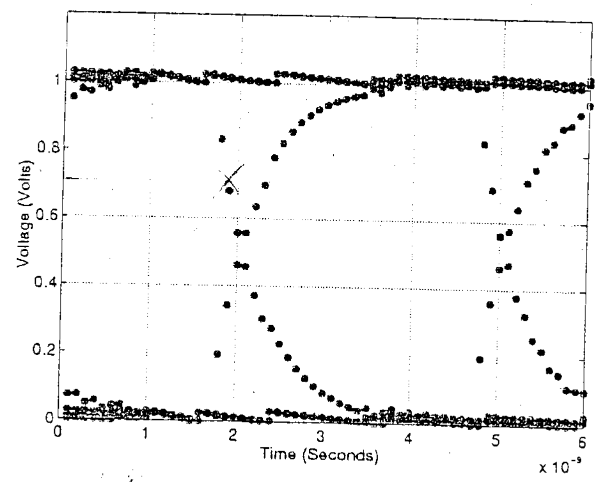




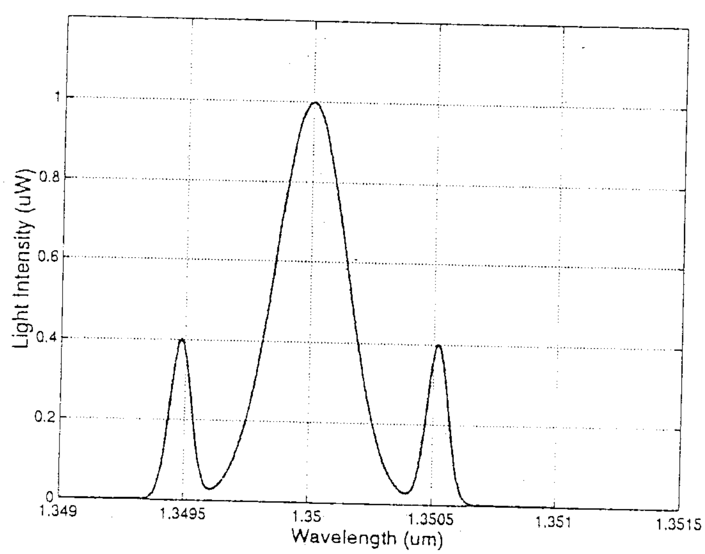

The core of an optical fiber has been doped with erbium ions at a density of $10^{19}$ ions per $cm^3$. For light of 1540 nm the emission cross-section for erbium in glass fiber is $6 \times 10^{-21}$ $cm^2$. Find the maximum optical gain $G = P_{out}/P_{in}$ that can be achieved with the fiber at 1540 nm, if the overlap of the optical mode with the doped fiber core is 0.45 and the fiber is 10 meters long.

You need to deposit a single-layer thin film coating in order to minimize the reflectance of normal-incidence light of wavelength(lambda)= 450 nm from a non-absorbing (k=0) substrate with index of refraction n=1.776. The only material you have for making the coating has n=1.5, k=0.0013. Approximately how thick should you make your coating?

2. The eye diagram was taken using an InGaAs photodetector. Assume that the bandwidth of the eye diagram is limited by the photodetector, and calculate the 3dB frequency point for the photodetector. Clearly indicate how you obtain your answer.




3. The optical spectrum from the output of a modulator is shown below. Determine the modulation frequency of the modulator.



Question #5

## Opti-677 Minor, Fall 2003

**Question 1:** Pattern resolution in a photoresist is affected by several effects such as standing waves and swing. We consider a photoresist of index n2 and thickness D deposited on a substrate of index n3.

1- Describe the standing wave in a photoresist. (2 points)
2- What is the period of the standing wave? (2 points)
3- What is the resulting effect on pattern resolution and uniformity? (2 points)
4- Propose two techniques to minimize the effect of standing waves? (2 points)

**Question 2:** What is the advantage of MBE over MOCVD? (2 points)