WRITTEN PRELIM EXAM – FIRST DAY

Spring 2003

February 18, 2003
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

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$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

$\cosh x = \frac{1}{2}(e^x + e^{-x})$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

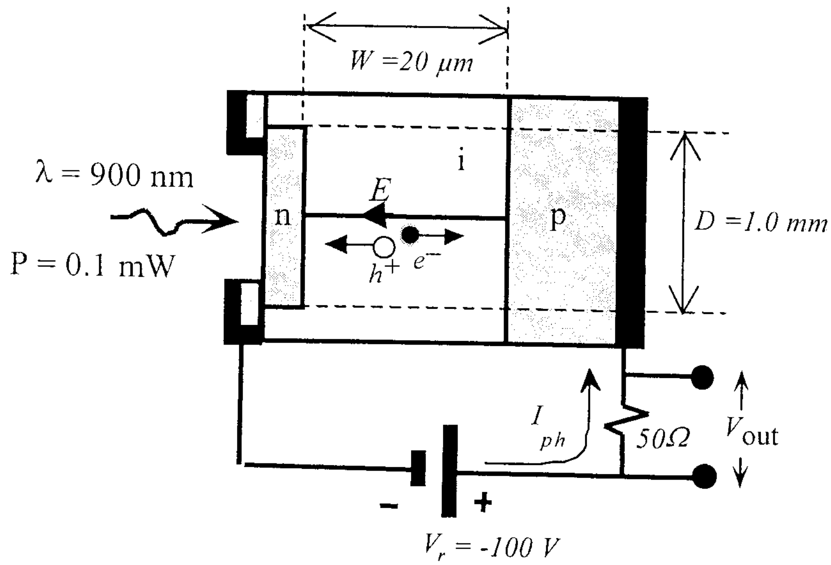
$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$



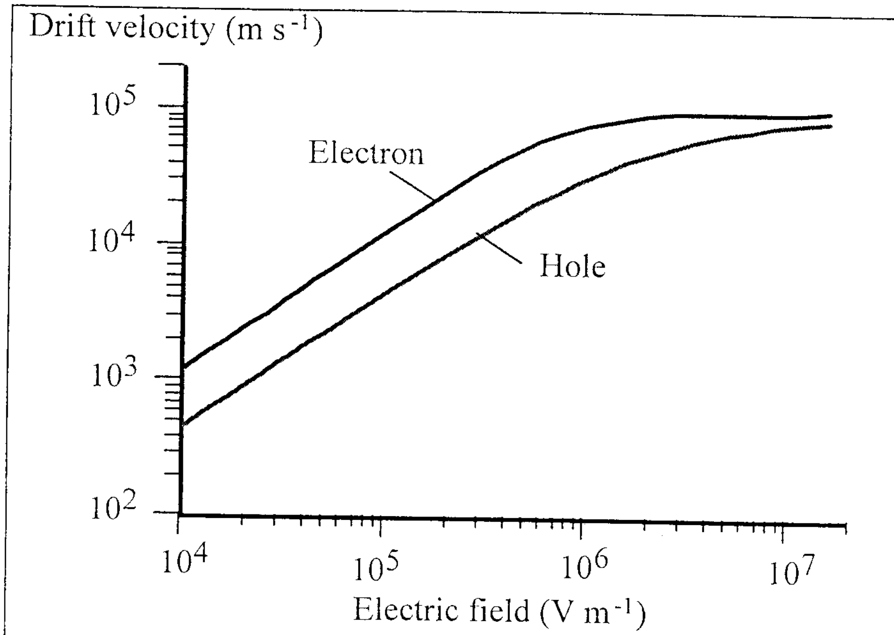
a) The p-i-n photodiode shown above is fabricated from n-type, p-type and i-type (undoped) silicon. The front surface has been anti-reflection coated so that all incident light enters the semiconductor. The absorption coefficient for silicon is approximately 300 cm^{-1} for light with a wavelength of 900 nm. The n-type layer is 0.1 microns thick – sufficiently thin so that light absorption is negligible for this layer. Calculate the quantum efficiency for the photodiode at 900 nm, assuming the any light that passes through the i–layer into the p-layer is lost and does not contribute to photocurrent.

b) Find the responsivity for the photodiode at 900 nm in units of Amps/Watt. (Planck's constant is $h = 6.6261 \times 10^{-34}$ J s).

c) Find the voltage across the external 50-ohm load if the optical power on the photodiode is 0.1 mW at a wavelength of 900 nm.

d) The photodiode has a reverse bias of 100 volts that appears entirely across the undoped i-layer of width 20 microns. Find the transit time for an electron, and for a hole, across the entire 20-micron length of the i-layer, using the drift velocity curves on the next page.

OVER



Drift velocity vs. electric field for holes and electrons in Si.

QUESTION #2

SPRING 2003

An image $i(x)$ is formed as

$$i(x) = \sum_{n=1}^{N} o_n s(x - x_n) \qquad (1)$$

where the o_n are *known* intensities, with total intensity and squared intensities given respectively as

$$\sum_{n=1}^{N} o_n \equiv P, \quad \sum_{n=1}^{N} o_n^2 \equiv Q, \qquad (2)$$

P and Q known.

However, the positions x_n are *random variables* selected independently from the probability law

$$p(x) = \frac{1}{a}\mathrm{Rect}\left(\frac{x - a/2}{a}\right), \text{ where by definition} \qquad (3)$$

$$\mathrm{Rect}(y) \equiv 1 \text{ for } |y| \le 1/2, \ \mathrm{Rect}(y) \equiv 0 \text{ for } |y| > 1/2.$$

This model (1), (2), (3) describes a process consisting of the uniformly random scatter of star images in an image field, where the *n*th star image is of a known intensity o_n and is spread out spatially according to a known point spread function $s(x)$. The OTF is given as the deterministic function $T(\omega)$.

(10%) (a) What is the general name given to this kind of process?

(10%) (b) Sketch the probability law $p(x)$.

In general, the associated power spectrum of a function $f(x)$ is defined as the expectation

$$S_f(\omega) \equiv \left\langle |F(\omega)|^2 \right\rangle \text{ where the Fourier transform } F(\omega) \equiv \int dx f(x) e^{-j\omega x}, \ j \equiv \sqrt{-1}. \quad (4)$$

Denote the Fourier transform of $i(x)$ and $o(x)$ as $I(\omega)$ and $O(\omega)$, respectively.

(10%) (c) Express $I(\omega)$ in terms of o_n and the $s(x - x_n)$.

(20%) (d) Express the image power spectrum $S_i(\omega)$ as an expectation involving the o_n and the $s(x - x_n)$.

(50%) (e) Derive the expression relating $S_i(\omega)$ to $P, Q, T(\omega)$ and $\mathrm{sinc}^2(a\omega/2)$.

Note: $\mathrm{sinc}(x) \equiv \sin(x)/x$

QUESTION #3

Spring 2003

Determine the dispersion relation for the transverse electromagnetic eigenmodes inside a metal. Specifically, proceed in the following way.

(a) Using the appropriate Fourier transform, derive the general dispersion relation for the eigenmodes inside the material and present it in terms of the frequency ω, the magnitude of the wave vector k, the electric permittivity (or dielectric function) $\epsilon(\omega)$, and the speed of light c. The starting point of the derivation is the wave equation (in Gauss units)

$$\left\{\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right\}\vec{E}(\vec{r},t) = -\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\vec{P}(\vec{r},t)$$

For simplicity, we assume the following form for the linear optical response:

$$\vec{P}(\vec{r},t) = \int_{-\infty}^{\infty} dt'\chi(t-t')\vec{E}(\vec{r},t')$$

Assume that the "causality" is built into the (scalar) function $\chi(t-t')$. In other words, don't worry about causality here; the time integral goes indeed from $-\infty$ to ∞. Clearly, you need to express $\epsilon(\omega)$ in terms of $\chi(\omega)$ to derive the requested dispersion relation.

(50 %)

(b) Now consider the simple case of the optical response of an ideal metal:

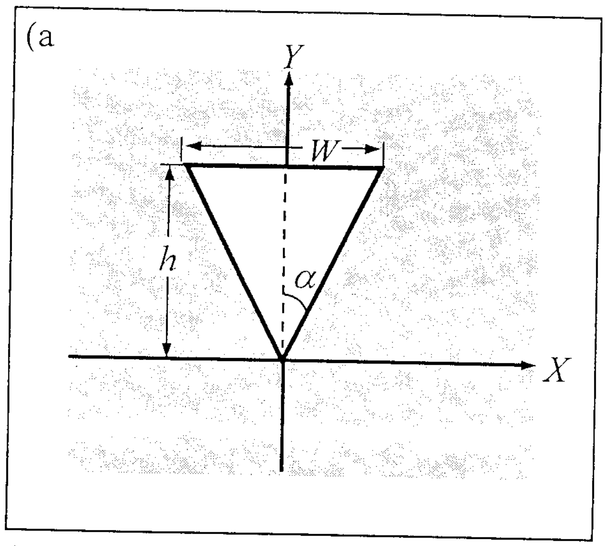
$$\varepsilon(\omega) = 1 - \frac{\omega_{pl}^2}{\omega^2}$$

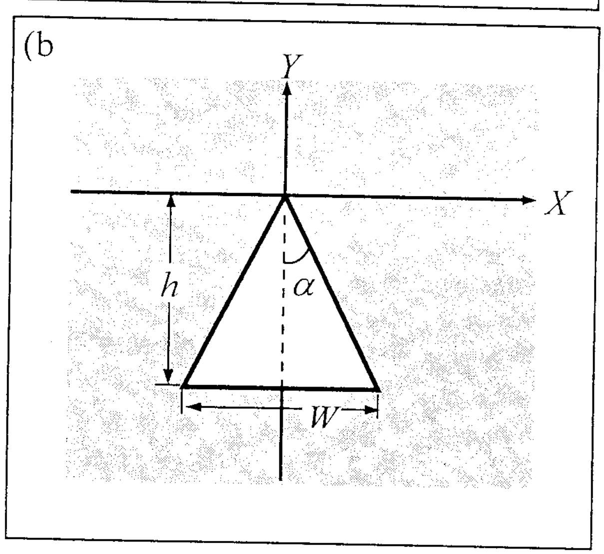
where ω_{pl} is the plasma frequency. Here we neglect, for simplicity, the contribution of inner-shell electrons as well as any damping effects. Using this model for $\varepsilon(\omega)$, derive an explicit expression for the dispersion relation $\omega(k)$ of the electromagnetic eigenmodes in the system (note that both ω and k are real). Also, give specific expressions for the dispersion in the following two limiting cases: (i) long-wavelength limit ($k \to 0$) and (ii) short-wavelength limit ($k \to \infty$). Finally, sketch $\omega(k)$. Completely label the axes with all relevant parameters and indicate explicitely the two limiting cases (i) and (ii).

(50 %)

A) Figure (a) shows a triangular aperture of width W, height h, and apex angle 2α in the XY-plane. Let the function $f^+(x, y)$ be defined such that within the aperture $f^+(x, y) = 1$, and outside the aperture $f^+(x, y) = 0$. Find the Fourier transform $F^+(\sigma_x, \sigma_y)$ of $f^+(x, y)$.

B) Let the aperture be flipped around the X-axis, as shown in Fig. (b). The functional form of the new aperture is $f^-(x, y) = f^+(x, -y)$. Without repeating the calculations of part (A), obtain the Fourier transform $F^-(\sigma_x, \sigma_y)$ of $f^-(x, y)$.





QUESTION #5

SPRING 2003

An air-spaced triplet objective is comprised of three thin lenses in air:

	Focal Length	Spacing
Lens 1	100 mm	25 mm
Lens 2	-50 mm	25 mm
Stop	-----	50 mm
Lens 3	100 mm	

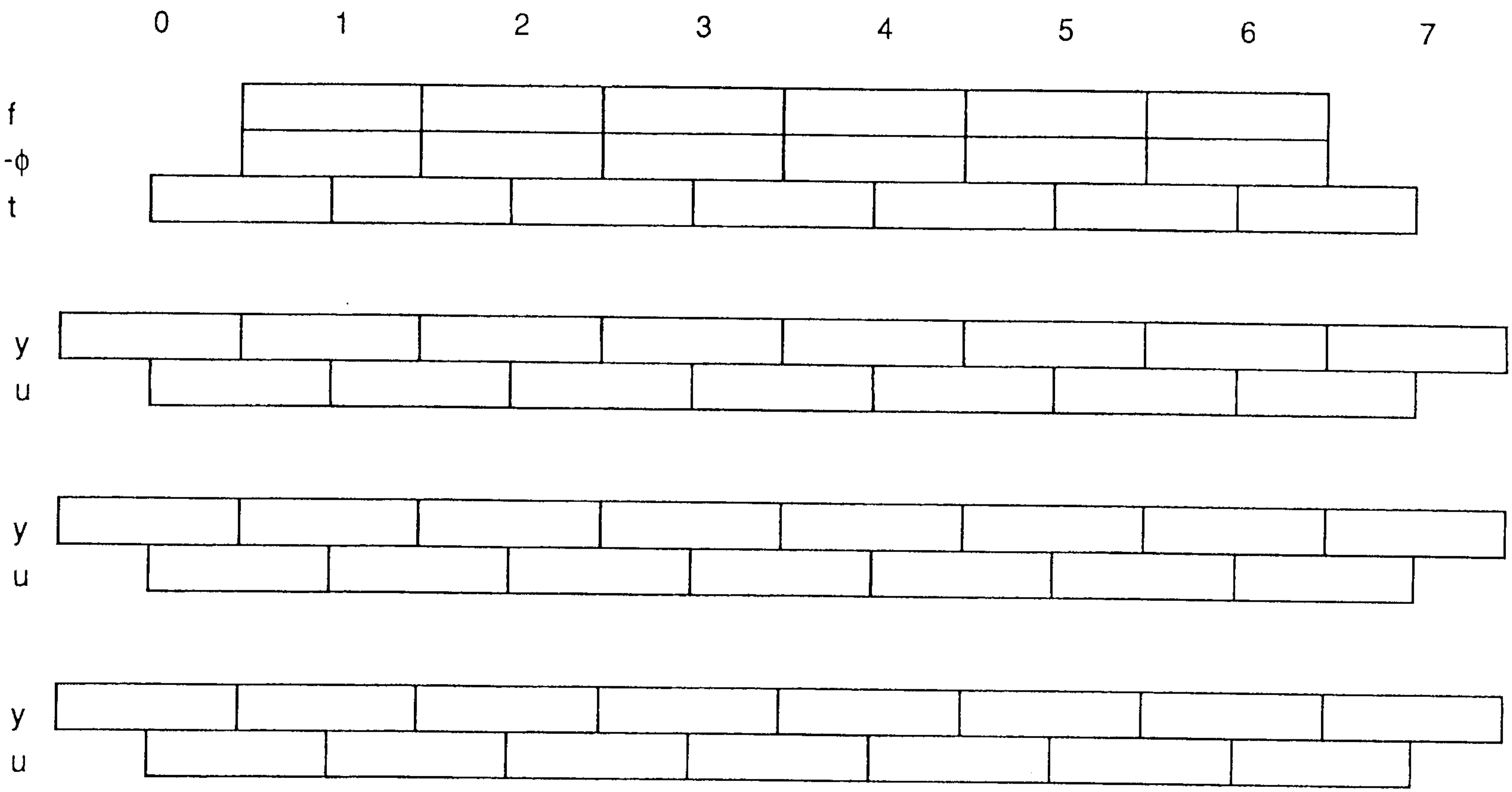
The object is at infinity, and the stop diameter is 20 mm. Use paraxial raytrace methods to determine the system focal length, the back focal distance, the entrance pupil location and size, and the exit pupil location and size.

A thin lens raytrace sheet is attached on the next page.

Do not use Gaussian methods for any portion of this problem.

OVER

Thin Lens YNU Method



WRITTEN PRELIM EXAM – SECOND DAY

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$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}}[\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

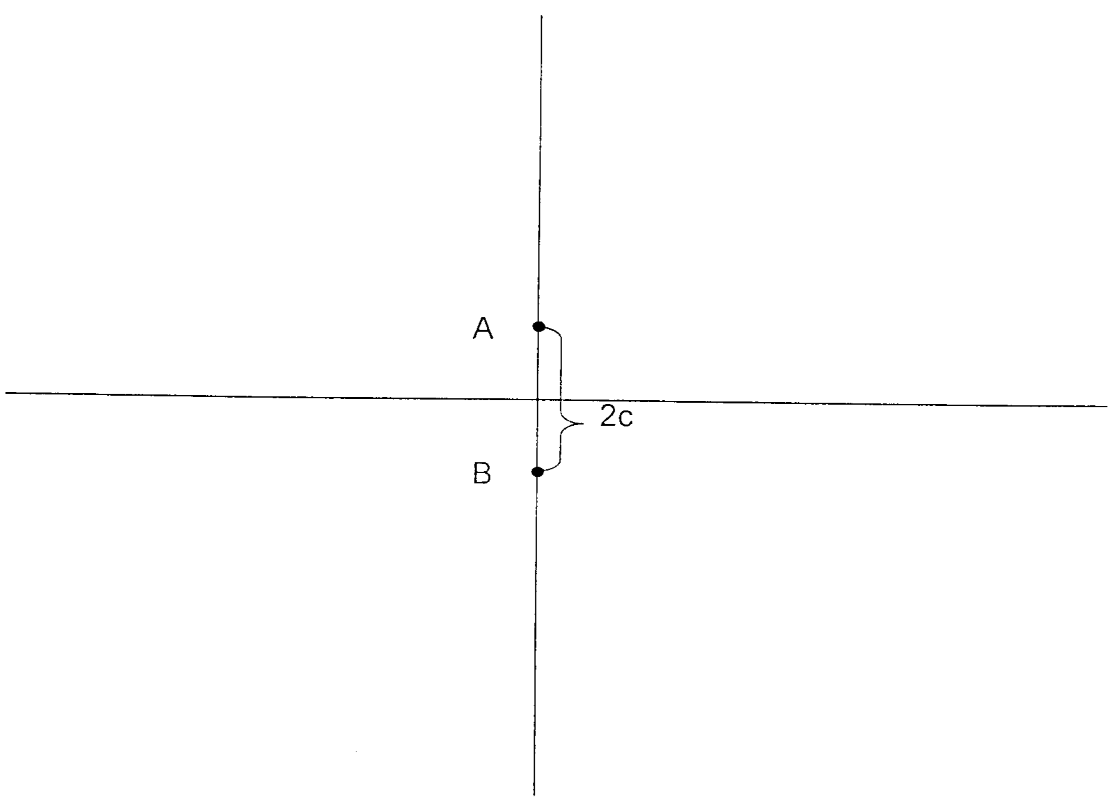
$\oint_{\mathrm{S}}(\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}}(\nabla \times \mathbf{F})\, d^3x$

QUESTION #6

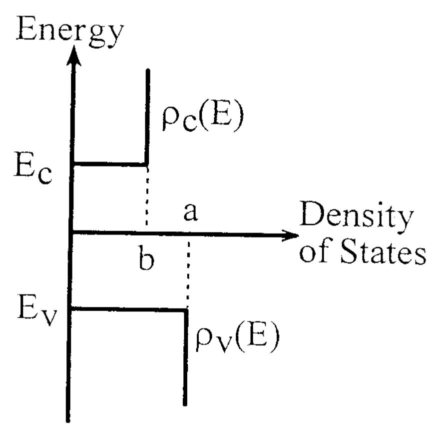
SPRING 2003

a) (50%) Two point sources A and B emit perfect spherical waves. The sources are in phase. Sketch the resulting fringe pattern in the plane of the paper if the distance 2c is two wavelengths. To indicate fringes, draw lines that indicate the points of maximum constructive interference.

b) (50%) How would your diagram change if source A is out of phase with source B by 180^0?



Consider the hypothetical density of states (D-O-S) function shown below. The constants "a" and "b" are assumed known. The conduction-band density of states is $\rho_C(E)$ and the valence-band density of states is $\rho_V(E)$.



a) (50%) Assuming the semiconductor is undoped, obtain an expression for the Fermi level E_F at room temperature. You may assume Boltzmann statistics.

b) (25%) For the D-O-S function, obtain an expression for the intrinsic carrier concentration (n_i).

c) (25%) Calculate the effective mass for an electron in the conduction band.

QUESTION #8

SPRING 2003

Consider an imaging system with 100 mm focal length and 20 mm diameter pupil. The system is focused on an object at infinity. Due to an offense against the sine condition, this system suffers from third-order coma, but no other aberrations. The amount of coma in the wavefront is given as

$$\Delta W = W_{131} H \rho^3 \cos\theta$$

where

$W_{131} = 10\ \mu\text{m}$

H = normalized field coordinate; a positive value implies a positive image height

ρ, θ = pupil coordinates

H is normalized to equal 0 on axis and equal 1 at the edge of the field at 10 mrad.
ρ is normalized to equal 0 on axis and equal 1 at the edge of the pupil at 10 mm

1. Sketch the aberrated image from a point source 5 mrad off axis. (Show only the geometric aberration and ignore diffraction effects.) Make sure to correctly show :

(20%) Shape of the image

(20%) Size of the image

(20%) Position of the image, relative to the axis

(20%) Orientation of the image relative to the axis

2. How does the shape of the image change through focus? Sketch the shape of the image at positions:

(10%) 0.5 mm inside of focus

(10%) 0.5 mm outside of focus

QUESTION #9

SPRING 2003

a. The index ellipsoid for an optical material is given by:

$$\frac{x^2}{n_{xx}^{\ 2}}+\frac{y^2}{n_{yy}^{\ 2}}+\frac{z^2}{n_{zz}^{\ 2}}+\frac{2yz}{n_{yz}^{\ 2}}+\frac{2xz}{n_{xz}^{\ 2}}+\frac{2xy}{n_{xy}^{\ 2}}=1.$$

What is the index of refraction for light polarized in the x-direction?

b. In the absence of an applied electric field, GaAs is isotropic and the index ellipsoid is given by:

$$\frac{x^2}{n^2}+\frac{y^2}{n^2}+\frac{z^2}{n^2}=1,$$

where x, y, and z coincide with crystal axes.

What is the index of refraction for an ordinary ray?

c. A linear electro-optic tensor $\vec{r}$ can be defined such that it relates an applied electric field $\vec{E}=(E_x,E_y,E_z)$ to the changes in the index ellipsoid with:

$$\begin{bmatrix} \Delta\frac{1}{n_{xx}^{\ 2}} \\ \Delta\frac{1}{n_{yy}^{\ 2}} \\ \Delta\frac{1}{n_{zz}^{\ 2}} \\ \Delta\frac{1}{n_{yz}^{\ 2}} \\ \Delta\frac{1}{n_{xz}^{\ 2}} \\ \Delta\frac{1}{n_{xy}^{\ 2}} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \bullet \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}, \text{ where } \vec{r} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix} \text{ for GaAs.}$$

Sketch the projection of the index ellipsoid in the x-y plane for GaAs when an electric field is applied in the z-direction.

d. Show that a new set of axes x', y', and z' defined by:

$$x = x'\cos(45^\circ) + y'\sin(45^\circ)$$
$$y = -x'\sin(45^\circ) + y'\cos(45^\circ)$$
$$z = z'$$

are the principal axes for GaAs with an electric field applied along the z-axis.

QUESTION #10 SPRING 2003

In many applications it is necessary to generate normally distributed random numbers, but most computer programs offer only random numbers uniformly distributed from 0 to 1. This question concerns generating the desired normal random variables from the available uniform ones and probes your knowledge of these two distributions along the way.

(a) State the probability density function for a random variable uniformly distributed from 0 to 1.

(b) Use the density in part (a) to compute the mean and variance of a uniformly distributed random variable.

(c) State the probability density function for a normal random variable with mean m and standard deviation σ.

(d) Now suppose that you call the uniform random-number generator N times to produce N samples $\{u_n, n = 1,\ldots, N\}$ from the density in part (a). Form the sum of these samples, which you can call y, and compute the mean and variance of y. State any assumptions you use in this part.

(e) Assume N is large and give an approximate expression for the probability density function of y. State what theorem you have used in making this approximation.

(f) Find a transformation of the form $z = ay + b$ such that z is approximately a random variable drawn from the density given in part (b).

(All parts have equal weight.) Consider a collection of identical two-level atoms in **thermal equilibrium**. The distribution of atoms between these two states is given by the Maxwell-Boltzmann distribution:

$$N_2 = N_1 \exp[-(\hbar\omega_{21})/(k_B T)],$$

where N_1 and N_2 are population densities corresponding to states 1 and 2 (respectively), T is the temperature of the system, and ω_{12} is the angular frequency for a transition between states 1 and 2.

(a) Calculate N_2/N_1 for an optical transition of wavelength $\lambda = 500$nm and a temperature of 300 K. (Use $\hbar = 1.1 \times 10^{-34}$ J s, and $k_B = 1.4 \times 10^{-23}$ J/K).

(b) At what temperature will $N_2 = 0.01\ N_1$? Is there *any* temperature for which N_2 can exceed N_1 ?

(c) Assume atoms are promoted from the ground state to the excited state at a rate W_{12} and that atoms fall from the excited state to the ground state at a rate W_{21}. Write an explicit relationship between W_{12} and W_{21} (this is the principle of detailed balance).

Now assume that the atoms are in thermal equilibrium with a blackbody radiation field. The Planck distribution for blackbody radiation is:

$$I(\omega) = \left(\frac{\hbar\omega^3}{\pi^2 c^2}\right)\frac{1}{\exp[-(\hbar\omega)/(k_B T)] - 1}.$$

(d) On a single set of axes, make **qualitative** sketches of $I(\omega)$ vs. ω for two clearly different values of T. Indicate which sketch corresponds to the higher value of T.

(e) The rate W_{12} induced by blackbody radiation can be written as $W_{12} = N_1 B_{12} I(\omega_{21})$. What is the usual name given to B_{12}?

(f) Similarly to part (e), a total rate W_{21} can be defined for transitions from state 2 to state 1. Write such an equation defining W_{21}. Pay attention to subscripts and the order in which they appear in your expression (maintain the convention used so far in this problem). Define in words any new quantities that you introduce.

(g) Specify three different physical mechanisms that can cause an atom to undergo a transition from the excited state to the ground state.

(h) Could this collection of two-level atoms be used as a gain medium to amplify light via stimulated emission of radiation?