WRITTEN PRELIM EXAM – FIRST DAY

Spring 2004

February 17, 2003
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

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$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

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$\sin 2A = 2\sin A \cos A$

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$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

$\cos^2(\frac{A}{2}) = \frac{1}{2}(1 + \cos A)$

$\sinh x = \frac{1}{2}(e^x - e^{-x})$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

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$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

In configuration space $(\vec{r}, t)$ the macroscopic Maxwell's equations are

$$\begin{aligned}
\nabla \times \vec{E}(\vec{r}, t) &= -\frac{\partial \vec{B}(\vec{r}, t)}{\partial t}, \\
\nabla \times \vec{H}(\vec{r}, t) &= \vec{J}(\vec{r}, t) + \frac{\partial \vec{D}(\vec{r}, t)}{\partial t}, \\
\nabla \cdot \vec{D}(\vec{r}, t) = \rho(\vec{r}, t) \quad &, \quad \nabla \cdot \vec{B}(\vec{r}, t) = 0.
\end{aligned} \tag{1}$$

(a) Convert the above macroscopic Maxwell equations to $(\vec{k}, \omega)$ space making clear how you made the conversion. (2 points)

For the remainder of this question consider a plane-wave electric field $\vec{E}(\vec{r}, t)$ of frequency ω that is propagating along the z-axis and is normally incident from vacuum onto a medium of complex refractive-index $\mathcal{N}(\omega) = n(\omega) + i\kappa(\omega)$ occupying $z > 0$, $n(\omega)$ and $\kappa(\omega)$ being the refractive-index and extinction index of the medium, respectively. To the left $(z < 0)$ and right $(z > 0)$ of the interface write the positive frequency component of the electric field $\vec{E}(\vec{r}, \omega)$ as

$$\begin{aligned}
\vec{E}(\vec{r}, \omega) &= \vec{x}\left[\mathcal{E}_i e^{i(k_v z - \omega t)} + \mathcal{E}_r e^{i(-k_v z - \omega t)}\right], \quad z < 0, \\
\vec{E}(\vec{r}, \omega) &= \vec{x}\mathcal{E}_t e^{i(k_v \mathcal{N}(\omega) z - \omega t)}, \quad z > 0,
\end{aligned} \tag{2}$$

with $k_v = \omega/c$, and $\mathcal{E}_i, \mathcal{E}_r, \mathcal{E}_t$ being the incident, reflected, and transmitted electric field amplitudes.

(b) Using Faraday's law calculate the positive frequency component of the magnetic induction $\vec{B}(\vec{r}, \omega)$ corresponding to the electric field in Eq. (2). (3 points)

(c) By enforcing continuity of the tangential components of the electric field and magnetic induction at the interface obtain an expression for the reflectivity of the interface $r(\omega) = \mathcal{E}_r/\mathcal{E}_i$. (3 points)

(d) For the case of a transparent dielectric medium with $\mathcal{N}(\omega) = n_0$ demonstrate that the reflected field suffers a π phase-shift for the case of external reflection. (2 points)

Question #2

Spring 2004

Derive and plot the classical expression that describes the anomalous dispersion of a dielectric material composed of non-polar molecules, following the following steps:

1. Discuss the effect of an electromagnetic wave incident on the solid and write the expression for the Lorentz force on an electron. (15%)
2. Explain and arrive at the expression that gives the stationary solution to the equation of motion of the electrons subject to the electric force. (20%)
3. Explain and derive the equation that describes the electric polarization P of the material. (15%)
4. Explain and introduce damping to the equation of motion. (15%)
5. Draw and explain a generic figure that shows the polarization as a function of the frequency of the electromagnetic wave around the resonance frequency. (20%)
6. Explain the effect the damping has on the resonance frequency. (15%)

Use Gaussian reduction to determine the back focal distance of the following three surface optical system:

$n = n_0 = 1.33$	
	$R_1 = 25.0$
$n_1 = 1.50$	
$t_1 = 5.0$	
	$R_2 = -40.0$
$n_2 = 1.60$	
$t_2 = 5.0$	
	$R_3 = -60.0$
$n' = n_3 = 1.33$	

Note: Solutions obtained using raytrace methods will receive zero credit.

Question #4

Spring 2004

A micro-lens operating at a wavelength of 500 nm has an exit pupil diameter, XPD = 0.8 mm and an image space numerical aperture of na = 0.2. The transverse ray aberration plots are shown below. On the left is plotted E_y, the y-component vs the y-normalized pupil coordinate. In the plot on the right, $E_x \approx 0$, for the x-component of the ray aberration in the x-crosssection in the pupil. The object point is 1° off-axis in the y-meridional plane. The lens is radially symmetric.

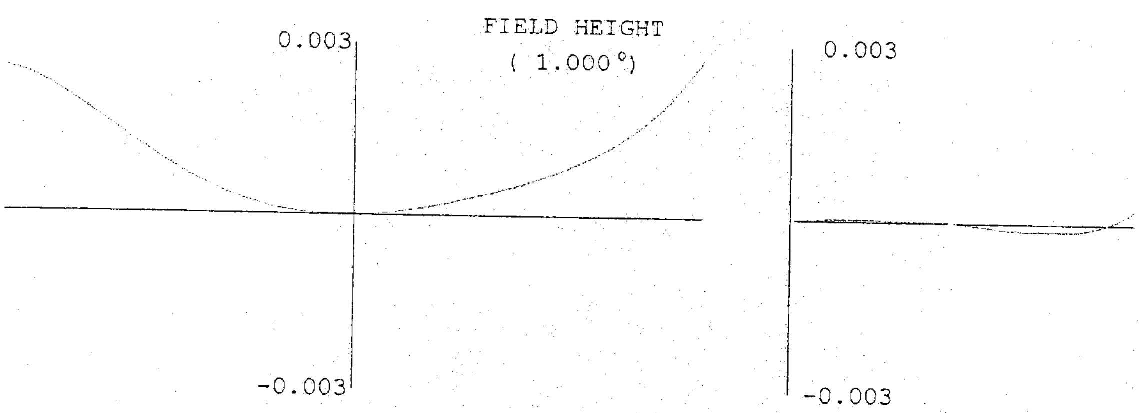
a. In the paraxial approximation what is the distance from the exit pupil to the image plane?

b. Which third order Seidel aberration dominates the ray aberration?

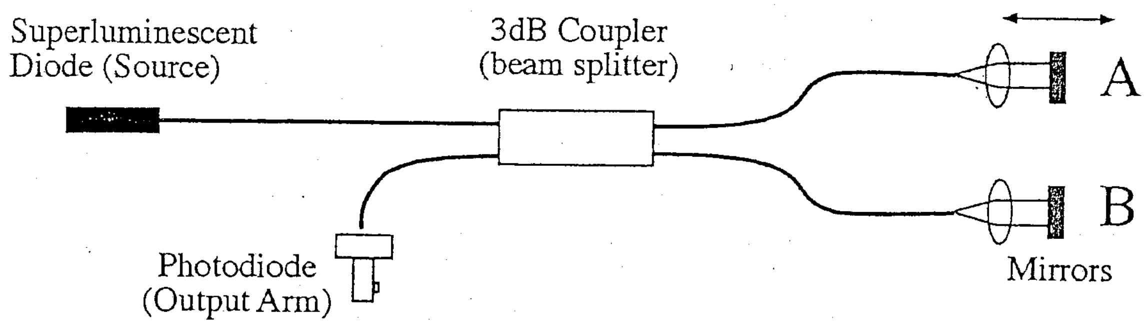
c. Calculate the magnitude of the wave front aberration and provide its functional form in terms of normalized polar pupil coordinates ρ and ϕ and normalized Cartesian pupil coordinates x and y.

d. How would you calculate the RMS wavefront aberration? Give a specific equation but do not evaluate the equation.

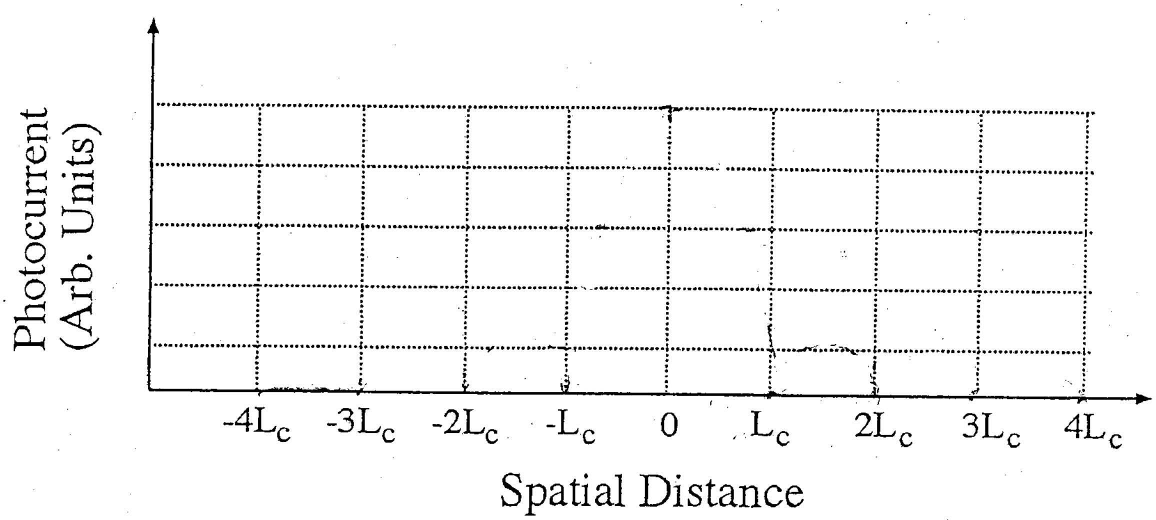
e. This same lens is used at a field angle of 2° off-axis. Assuming this is the only aberration, plot the transverse ray aberration and specify its magnitude.



1. The following figure shows a balanced 3 dB optical coupler connected to a single mode optical fiber. This instrument can be modeled as a Tyman-Green interferometer, with the equivalent components as shown below in parentheses. A superluminescent diode (λ~1310 nm) is used as a low coherence source with a coherence length of 15 μm. The optical paths of A and B are designed to be equal and coupling losses into and out of the optical fibers are considered to be negligible.

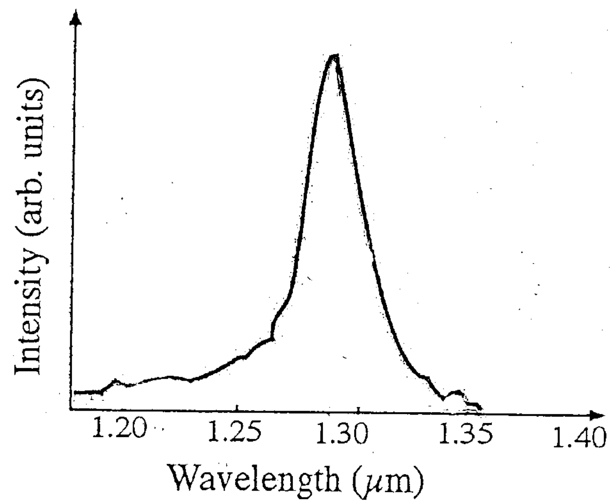


a) Mirror A is moved longitudinally +/- three coherence lengths. Plot the output current from the photodiode on the axis below. Assume that the power spectrum is a rectangle centered at the mean frequency.



continued on next page

b) The spectra from the superluminescent diode is shown in the figure below. Indicate how the spectra would change if the coherence length of the superluminescent diode were doubled. You may want to draw on the spectra. Also, explain your reasoning in words.



Question #6

Spring 2004

a. A radiometer must detect a 4000 K lambertian disk 2 cm in diameter with an emissivity of 0.9. Assume an ideal photon detector (quantum efficiency of 1) 10 mm in diameter and a bandpass filter with a 20 nm FWHM centered at 660 nm with transmission = 0.8. Ignoring the atmosphere, at what distance, L, can the disk be detected with an uncertainty of 0.1 (at 1 standard deviation confidence) when measuring with an effective noise bandwidth of 1000 Hz?

b. Describe three distinct ways to increase the range of this system and indicate the relative effectiveness of each method.

c. How can the equation for the distance L be changed to include the attenuations effects for horizontal propagation through the atmosphere?

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You are evaluating the quality of a simple radiometer (basic tube with no optics) that is used to view a 3000 K lamp source. The system parameters are listed in the table below and relate to the normal operation of the system.

5 points
a) The requirement is to measure the source with and SNR of 10,000 or greater. Does the system satisfy the requirement? Support your conclusion and state any assumptions.

2 points
b) Regardless of the answer in part (a), there is a need to double the SNR of the system. Recommend a course of action and explain why you are making this recommendation

1 points
c) Tests of the amplifier show that the it actually produces a noise that is three times larger than that given in the table. Are you concerned? Explain.

2 points
d) Compute the noise equivalent power (NEP) of the detector in this case.

Parameter	Value
Detector material	Silicon
Center wavelength	550 nm
Bandwidth	30 nm
Johnson noise	2.4×10^{-13} Amps
Photon noise	6.2×10^{-12} Amps
Noise from amplifier electronics	5.2×10^{-14} Amps
Responsivity	0.35 A/W
Receiver System Electronics Bandpass	Δf=800 Hz
Detector Normalized Spectral Detectivity, D^*	2×10^{12} cmHz$^{1/2}$w^{-1}
Detector area	1 mm^2
Irradiance from a laboratory source at center wavelength at detector	28.0 W/(m^2 μm)

Question #8

Spring 2004

Consider the conduction band in a graded AsAs/$Al_xGa_{1-x}As$ quantum-well structure. The composition parameter x varies smoothly as a function of growth direction z in such a way that the quantum well confinement potential $V_c(z)$ is of the form $\frac{1}{2}fz^2$ where f is a real constant.

Using the Schroedinger equation for the envelope function $\xi(z)$, determine the confinement shift ε^{conf} of the lowest electronic subband (i.e. the difference between the lowest confinement energy and the bottom of the quantum well). To do this, write down the Schroedinger equation for $\xi(z)$ and use the fact that the wave function of the lowest state is of the form $\xi(z) = Nexp(-az^2)$ where N and a are real constants. Present your result in terms of the parabolic quantum well constant f, the effective electron mass m_e, and $\hbar$.

Question #9 Spring 2004

Let x be a random variable characterized by the probability density function (PDF) $p(x)$.

1. (10%) Define the characteristic function $\psi(\omega)$ of the random variable x. (NOTE) Different books may have slightly different definitions of the characteristic function. Be consistent with your definition throughout this problem.

2. (10%) What is the value $\psi(0)$? Show your work.

3. (20%) Relate $\frac{d^n}{d\omega^n}\psi(\omega)\big|_{\omega=0}$ to the nth moment of the random variable x.

4. (30%) Now, let $p(x)$ be

$$p(x) = \frac{a}{2}\exp\left(-a|x|\right) \tag{1}$$

with $a > 0$. Find the characteristic function for the random variable x that has this PDF. Show your work.

5. (20%) Let y be a random variable that is independent from x and distributed the same (*i.e.*, both have PDF given by Eqn. 1). Determine the characteristic function of $z = x + y$.

6. (10%) Now, envision N random variables x_i where $i = 0, 1, \ldots, N$, and N is larger than 10. Each random variable x_i is independent from one another and distributed the same. Approximately, what is the PDF of the random variable z where,

$$z = \frac{1}{N}\sum_{i=1}^{N} x_i$$

and why? (NOTE) No need to derive any properties of the PDF of z. I'm simply looking for its approximate shape and reason for this shape.

An image *i(x)* is repeatedly formed from unknown and randomly selected object intensities $o_1, o_2, \ldots, o_n$, located, respectively, at fixed points $x_1, \ldots, x_N$. The points have a common spacing Δx. The image obeys

$$i(x) = \sum_{n=1}^{N} o_n s(x - x_n),$$

where *s(x)* is a fixed spread function. Thus *i(x;**o**)* is a stochastic process. The associated power spectrum $\tilde{S}_i(\omega)$ is defined to be $\left\langle \left| I(\omega) \right|^2 \right\rangle$ where $I(\omega)$ is the image spectrum. Find $\tilde{S}_i(\omega)$ if

(a) $\langle o_m o_n \rangle = \sigma^2 \delta_{mn}$; the object has zero correlation length.

(b) $\langle o_m o_n \rangle = \sigma^2 \left(\delta_{mn} + 2^{-1} \delta_{m,n+1} + 2^{-1} \delta_{m,n-1} \right)$; triangular correlation.

(c) $\langle o_m o_n \rangle = \sigma^2 \left(\delta_{mn} + \delta_{m,n+1} + \delta_{m,n-1} \right)$; boxcar correlation.

The procession from (a) to (b) to (c) has stronger and stronger correlation. What is the trend in the power spectrum $S_i(\omega)$?

(a). Write out the full (time-dependent) Schrödinger equation for a one-dimensional simple harmonic oscillator potential having an oscillator angular frequency ω. Assume a particle mass of m.

(b). Let the variable n label the eigenstates of the harmonic oscillator, where n is an integer equal to or greater than 0. Specify the energy eigenvalues of this potential.

(c). Let $\psi_n(x)$ label the n^{th} bound normalized eigenstate of this potential. Provide a sketch of $|\psi_4(x)|^2$ vs. x. Make sure your sketch clearly shows any trends in the wavelength (or sharpness of curvature) and the amplitude of $|\psi_4(x)|^2$ with position, or describe in words these trends if they are not obvious in your sketch.

(d). Write the full time-dependent wavefunction $\Psi_4(x,t)$ in terms of $\psi_4(x)$ and other variables already used in this problem.

(e). If a particle is in the eigenstate $\psi_4(x)$ at time $t = 0$, what is the probability that the particle will be found in state $\psi_0(x)$ at a later point in time $t = t_1$?

(f). Evaluate this integral: $\int_{-\infty}^{\infty} \psi_0^*(x)\psi_4(x)dx$

(g). Define a new normalized wavefunction $\Phi(x,t) = A[\Psi_0(x,t) + \Psi_4(x,t)]$. Evaluate A.

(h). If a particle is in a state with the wavefunction $\Phi(x,t)$ as defined above, what are the possible results of a measurement of the particle's energy, and the probability of occurrence for each possible result?

(i). What is the expectation value of energy for the state $\Phi(x,t)$?

Question #12

SPRING 2004

(a) Briefly explain how the technique of laser Q-switching works, making clear why Q-switching can produce very high energy laser pulses. (20%)

(b) What is meant by the term "saturation" when discussing the behavior of an optical transition in a gain medium (with no resonator present)? Explain physically (in terms of level populations, transition rates, and pumping and decay rates) what is happening when saturation occurs. (20%)

(c) Draw an energy level diagram for an optically-pumped 3-level laser (such as a ruby laser), and discuss the requirements on the various excitation and decay rates between the levels needed to achieve lasing. (20%)

(d) Sketch the behavior of a lasing medium's gain coefficient as the optical pumping rate for a CW optically-pumped 3-level laser is increased from zero to a rate well above threshold. On the same plot, sketch the behavior of the output power of the laser. Finally, explain why your curves have the shapes that they do. (20%)

(e) Consider a simplified laser consisting of a gain medium of length L with gain coefficient γ_g sandwiched between two plane parallel mirrors having reflectivities R_1 and R_2. Derive an expression for the gain needed to reach the lasing threshold. (20%)

2

In configuration space $(\vec{r}, t)$ the macroscopic Maxwell's equations are

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For the remainder of this question consider a plane-wave electric field $\vec{E}(\vec{r}, t)$ of frequency ω that is propagating along the z-axis and is normally incident from vacuum onto a medium of complex refractive-index $\mathcal{N}(\omega) = n(\omega) + i\kappa(\omega)$ occupying $z > 0$, $n(\omega)$ and $\kappa(\omega)$ being the refractive-index and extinction index of the medium, respectively. To the left $(z < 0)$ and right $(z > 0)$ of the interface write the positive frequency component of the electric field $\vec{E}(\vec{r}, \omega)$ as

$$\begin{aligned} \vec{E}(\vec{r}, \omega) &= \vec{x} \left[\mathcal{E}_i e^{i(k_v z - \omega t)} + \mathcal{E}_r e^{i(-k_v z - \omega t)} \right], \quad z < 0, \\ \vec{E}(\vec{r}, \omega) &= \vec{x} \mathcal{E}_t e^{i(k_v \mathcal{N}(\omega) z - \omega t)}, \quad z > 0, \end{aligned} \tag{2}$$

with $k_v = \omega / c$, and $\mathcal{E}_i, \mathcal{E}_r, \mathcal{E}_t$ being the incident, reflected, and transmitted electric field amplitudes.

(b) Using Faraday's law calculate the positive frequency component of the magnetic induction $\vec{B}(\vec{r}, \omega)$ corresponding to the electric field in Eq. (2). (3 points)

(c) By enforcing continuity of the tangential components of the electric field and magnetic induction at the interface obtain an expression for the reflectivity of the interface $r(\omega) = \mathcal{E}_r / \mathcal{E}_i$. (3 points)

(d) For the case of a transparent dielectric medium with $\mathcal{N}(\omega) = n_0$ demonstrate that the reflected field suffers a π phase-shift for the case of external reflection. (2 points)

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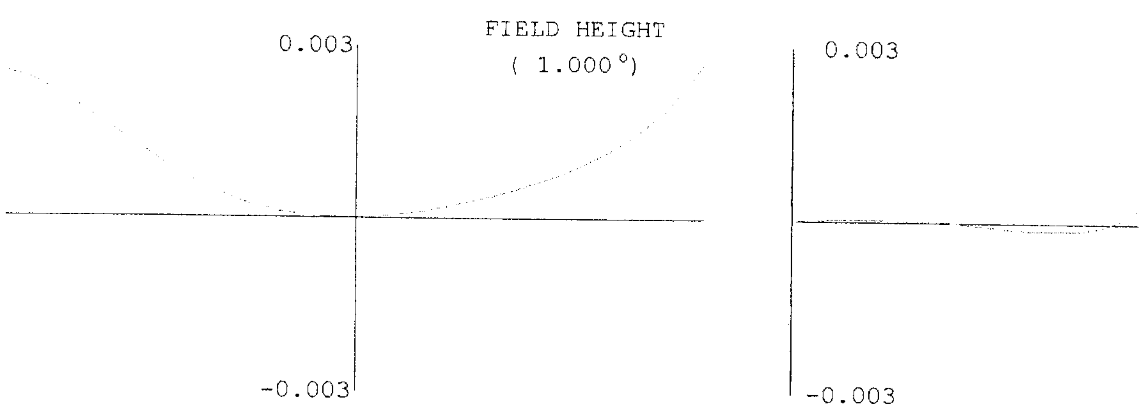
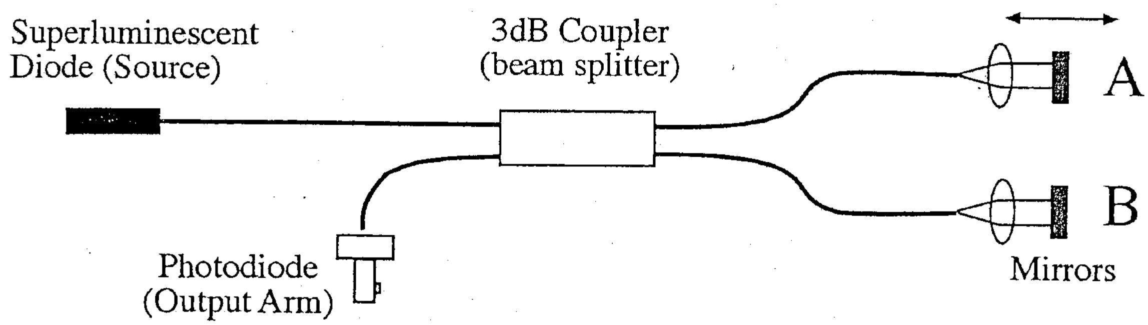
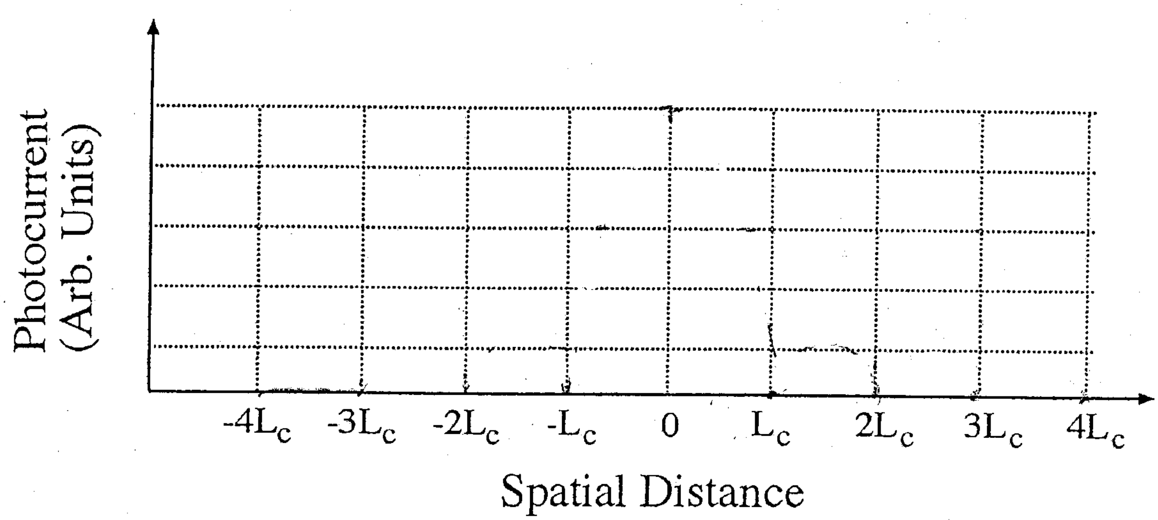


1. The following figure shows a balanced 3 dB optical coupler connected to a single mode optical fiber. This instrument can be modeled as a Tyman-Green interferometer, with the equivalent components as shown below in parentheses. A superluminescent diode (λ~1310 nm) is used as a low coherence source with a coherence length of 15 μm. The optical paths of A and B are designed to be equal and coupling losses into and out of the optical fibers are considered to be negligible.



a) Mirror A is moved longitudinally +/- three coherence lengths. Plot the output current from the photodiode on the axis below. Assume that the power spectrum is a rectangle centered at the mean frequency.



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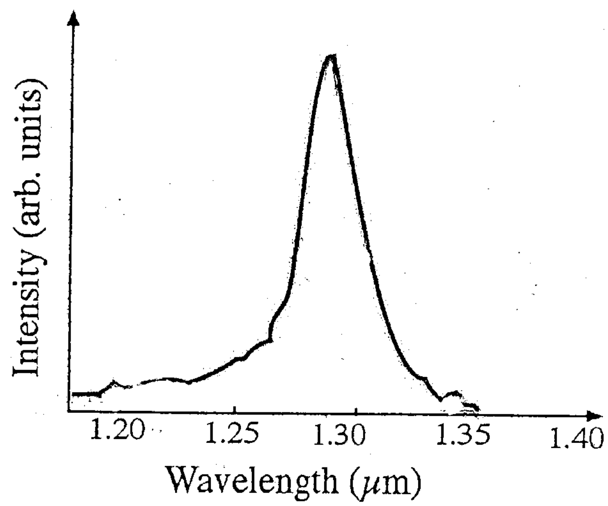
b) The spectra from the superluminescent diode is shown in the figure below. Indicate how the spectra would change if the coherence length of the superluminescent diode were doubled. You may want to draw on the spectra. Also, explain your reasoning in words.



a. A radiometer must detect a 4000 K lambertian disk 2 cm in diameter with an emissivity of 0.9. Assume an ideal photon detector (quantum efficiency of 1) 10 mm in diameter and a bandpass filter with a 20 nm FWHM centered at 660 nm with transmission = 0.8. Ignoring the atmosphere, at what distance, L, can the disk be detected with an uncertainty of 0.1 (at 1 standard deviation confidence) when measuring with an effective noise bandwidth of 1000 Hz?

b. Describe three distinct ways to increase the range of this system and indicate the relative effectiveness of each method.

c. How can the equation for the distance L be changed to include the attenuations effects for horizontal propagation through the atmosphere?