WRITTEN PRELIM EXAM – FIRST DAY TRKI

Spring 2006

February 14, 2006
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

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$2\cos A \cos B = \cos(A - B) + \cos(A + B)$

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$\cos 2A = 1 - 2\sin^2 A$

$\sin^2(\frac{A}{2}) = \frac{1}{2}(1 - \cos A)$

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$\sinh x = \frac{1}{2}(e^x - e^{-x})$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

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Spring 2006 Written Comprehensive Exam

Question #1 Day 1 Track I

The formulae for the reflection coefficients for s-polarized and p-polarized incident fields at a planar dielectric interface of two media are

$$r_s(\theta) = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \quad r_p(\theta) = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}},$$

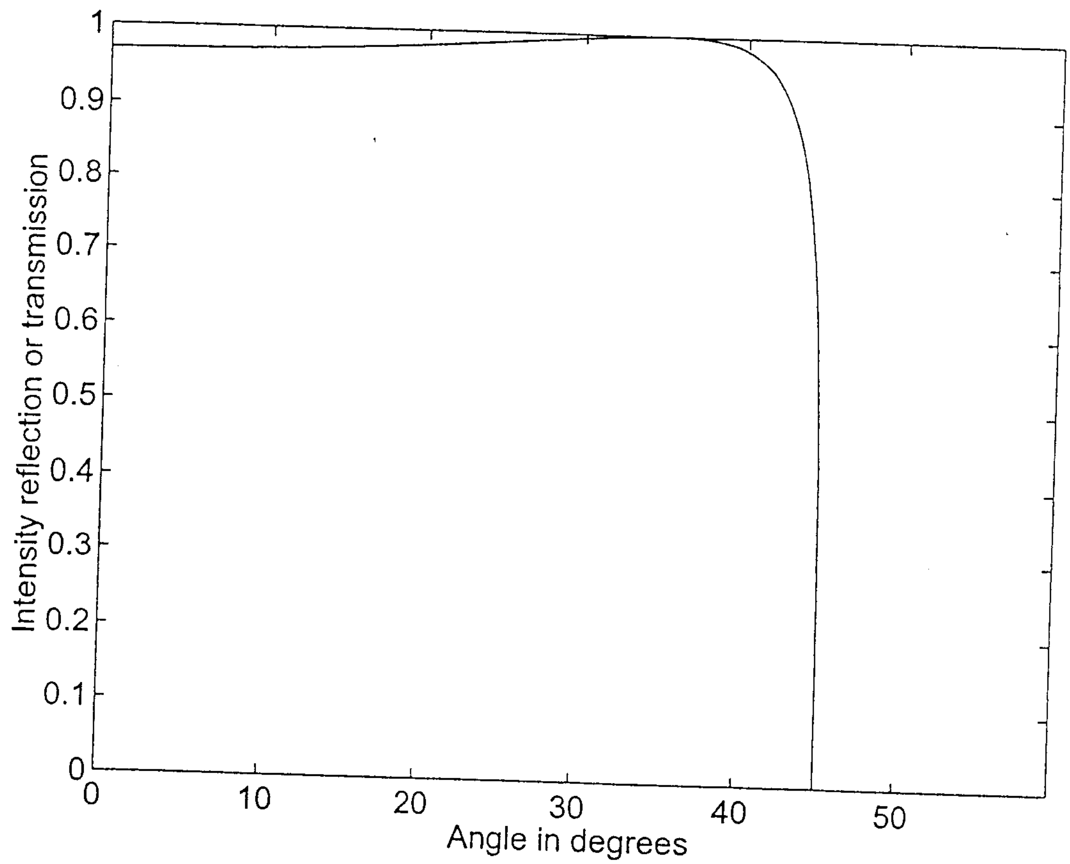
where $n = n_2/n_1$ is the relative refractive-index, n_1 being the refractive-index of the medium the field is incident from and n_2 the refractive-index of the second medium, and θ measures the angle of incidence.

(a) Define what is meant by s-polarized and p-polarized incident fields. (2)

(b) What is meant by total internal reflection (1), and using the information given above obtain an expression for the critical angle for total internal reflection (1).

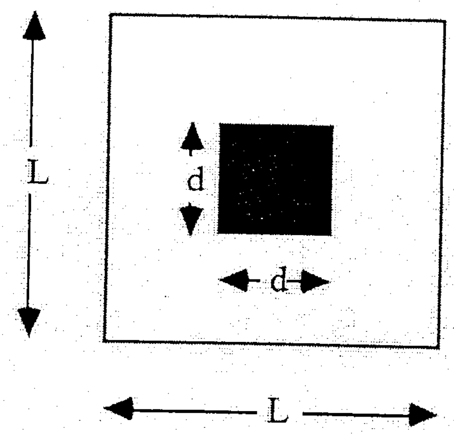
(c) Consider the case that a plane-wave electromagnetic field of randomly varying polarization state is incident on the interface such that the *incident intensity* I_0 is on average equally split between s-polarization and p-polarization. Obtain an expression for the magnitude of the average reflected Poynting vector relative to the input for a field incident at the Brewster angle θ_B. (2)

(d) The figure on the next page shows either the intensity reflection or intensity transmission for either internal or external reflection at a dielectric interface as a function of incident angle θ, and for either an s-polarized or p-polarized incident field. By inspecting the curve explain which combination of the above options the plotted curve corresponds to (Note: It is not sufficient to state the combination of options you choose, you also need to give a brief explanation) (4).


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Intensity reflection or transmission
0
10
20
30
40
50
Angle in degrees

Spring 2006 Written
Comprehensive Exam
Question #2 Day 1 Track I

Find an expression for the intensity distribution in the Fraunhofer diffraction pattern of the aperture shown below. Assume unit-amplitude, normally incident plane-wave illumination.



opaque outside
of square L x L

Spring 2006 Written Comprehensive Exam Question #3 Day 1 Track I

Let $\Lambda(x)$ be the unit triangular function. Find the Fourier transforms of the following functions:

(5 Points) a) $f(x) = \Lambda(x+1) + \Lambda(x-1)$

(5 Points) b) $f(x) = \text{sinc}^2(x)\cos(x)$

Spring 2006 Written
Comprehensive Exam
Question #4 Day 1 Track I

Use Parseval's theorem to compute the following definite integrals:

(3 Points) a) $\int_{-\infty}^{\infty} \text{sinc}^3(x)\,dx$

(3 Points) b) $\int_{-\infty}^{\infty} \text{sinc}^4(x)\,dx$

(4 Points) c) $\int_{-\infty}^{\infty} \cos(2\pi\sigma_0 x)\,\text{sinc}(x)\,dx$ ← (Note: σ_0 is a constant.)

Spring 2006 Written Comprehensive Exam
Question #5 Day 1 Track I

Consider a collimated, coherent, light beam of wavelength λ incident on a piece of ground glass. Some distance away from this ground glass is an imaging screen. By analyzing the physics of this problem, we find that the square of the real component of the complex wavefield at location (x_0, y_0) has the following characteristic function[1],

$$\phi_{U_{\text{re}}^2}(\omega) = (1 - j\omega a^2)^{-1/2}$$

where a is a constant. Similarly, the square of the imaginary component of the complex wavefield at this same location follows the same exact distribution. That is, $\phi_{U_{\text{im}}^2}(\omega) = \phi_{U_{\text{re}}^2}(\omega)$. In addition, we were able to show that U_{re}^2 and U_{im}^2 are independent from one another.

1. (25%) Using the information above, derive the characteristic function for the beam intensity I measured at location (x_0, y_0).
2. (25%) What is the signal-to-noise ratio (S/N) for this intensity? Signal to noise ratio is a ratio of the mean of I divided by the standard deviation of I.

Now say that we use a scanning aperture to try to improve our S/N. That is, we use an aperture that sums the intensities around the location (x_0, y_0). Let us say that the new measured intensity is

$$I = \sum_{m=1}^{M} I_m,$$

where M is the total number of intensity measurements performed by the scanning aperture and I_m are the individual intensity measurements. Assume that the I_m are independent from one another.

1. (25%) What is the characteristic function for our new intensity measurement? (Do not assume that M is large!)
2. (25%) What is the S/N for this new measurement? Did using an aperture help?

[1] The definition of characteristic function that I use is that $\phi_x(\omega) = \langle \exp(j\omega x) \rangle_x$.

Spring 2006 Written Comprehensive Exam Question #6 Day 1 Track I

A 5X Galilean telescope is comprised of two thin lenses separated by 80 mm. The objective lens is 50 mm in diameter, and the eye lens is 10 mm in diameter. This telescope is to be used with a human eye with a 4 mm diameter pupil. The separation between the eye lens and the eye is 10 mm. For distant objects, what is the unvignetted object field of view (in degrees) of this system?

A blank raytrace sheet is attached.

Surface	0	1	2	3	4	5	6
f							
−ϕ							
t							
y							
u							
y							
u							
y							
u							
y							
u							

WRITTEN PRELIM EXAM – SECOND DAY

Spring 2006

February 15, 2006
8:30 a.m. to 12:00 p.m.

TRKI

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Spring 2006 Written Comprehensive Exam Question #7 Day 2 Track I *

a) (2 Points) Aluminum is a metal with plasma frequency at $\hbar\omega_p = 15.3$ eV ($\lambda_p = 810$ A°). Describe whether aluminum is reflective or transmissive at 500 A° and at 5000 A°.

b) (2 Points) Plot the dispersion relation for surface plasmons. Is the surface plasmon frequency smaller or larger than the volume plasmon frequency?

c) (2 Points) Plot the dielectric function versus frequency in the vicinity of lattice vibration frequencies for an insulator like NaCl. Using this plot, describe at what frequency NaCl is absorptive.

d) (2 Points) Plot dispersion relations for acoustic phonons, optical phonons and photons on the same graph. Using the plot describe whether acoustic phonons can absorb single photons. How about optical phonons?

e) (2 Points) Describe gain clamping in semiconductor lasers by plotting carrier density versus pumping rate.

Spring 2006 Written Comprehensive Exam
Question #8 Day 2 Track I

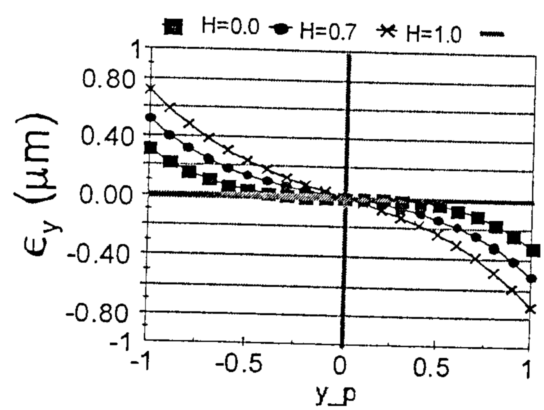
A single lens imaging system is being designed to take images of people entering Meinel. The camera system is mounted just above each door and must be able to provide clear images of facial features. The system is diffraction-limited in the absence of aberrations. Modeling of the system using a lens design package produces the ray fan plots below as well as the spot diagrams shown (the system is operated at f/1 and the plots are for an 0.5 micrometer wavelength).

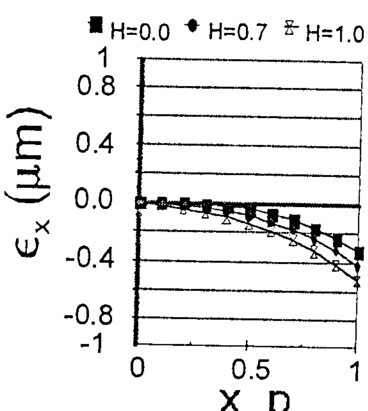
20% a) What third order aberrations are present and which ones dominate the system shown? Explain how you arrived at your answer.

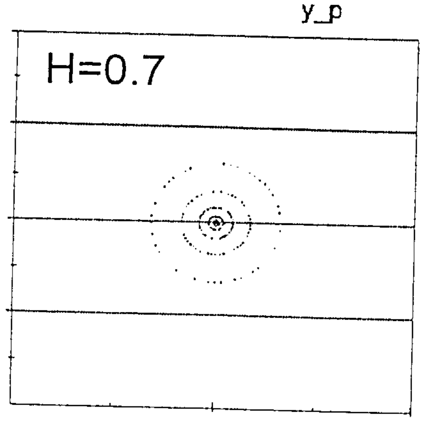
40% b) Sketch ,z as a function of the pupil position in the y-direction for the 0.5 micrometer wavelength. Also sketch on the same plot a similar curve for a wavelength of 0.6 micrometers. Explain your sketch (note that quantitative values are not critical, but shape and relative position are more important).

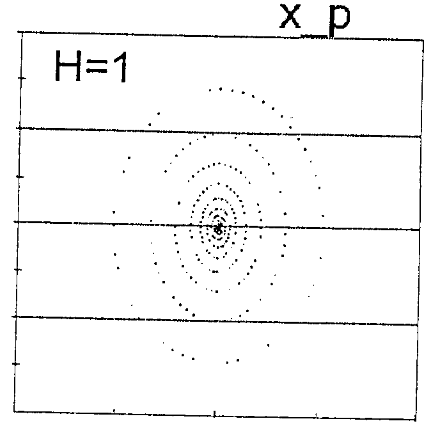
20% c) Aberrations currently limit the usefulness of the camera.. What aberration would you attempt to correct and how would you do this (note that the f/# of the system is fixed and this is not an option). Be specific in your response and give quantitative values if possible.

20% d) You are now allowed to alter the f/#. Would this be useful in improving the image quality? Explain.









Spring 2006 Written Comprehensive Exam
Question #9 Day 2 Track I

A satellite sensor is viewing the ground from space. The ground is illuminated by a solar irradiance that has a value of 1369 W/m^2 normal to the earth's surface. The incident solar irradiance is at a solar angle of 60 degrees from normal. The surface reflectance for this illumination angle is 0.0938 and the temperature of the ground is 300 K.

The sensor has a single lens that is circular with a diameter of 0.391 m with a 0.782 m focal length. The sensor uses a square detector that is 25 micrometers on a side and sees an area of the ground that is 500 m^2 from an altitude of 700 km.

Ignore all atmospheric effects for the questions A, B, and C.

40% A) Compute the radiant flux through the detector for the sensor described above.

30% B) The purpose of the sensor is to detect hot spots due to fire or lava flows. The design is such that a 10% change in radiant flux is sufficient to detect the fires/lava. What temperature is needed for the surface to have a 10% change in radiant flux at the sensor if the ground area of the hot spot is 1 m^2?

20% C) The sensor designers are still not sure that the 10% change is sufficient. Give three modifications to the sensor or its operation that will allow it to more easily determine that a hot spot is present.

10% D) You must now consider atmospheric effects. Will this make it easier or harder to see the hot spots? Explain

Spring 2006 Written Comprehensive Exam
Question #10 Day 2 Track I

Solve all of the following problems. 10 points total. If you don't know an answer, make a good guess and justify it; partial credit is available. You may find the following information useful:
$\hbar = 10^{-34}$ J·s $\qquad$ 1 eV $= 1.6 \times 10^{-19}$ J

1. Recall that the mode frequencies of a two-mirror optical resonator are given by

$$\nu_{mnq} = \Delta\nu_F[q + \frac{1}{\pi}(1+m+n)\cdot\cos^{-1}(\sqrt{g_1 g_2})],$$

where $\Delta\nu_F$ is the free spectral range of the resonator and g_1 and g_2 are the usual cavity stability parameters. Suppose that a laser resonator is made with two identical concave mirrors, each having a radius of curvature of $R = 100$ cm. Laser output occurs for two cavity modes, each having the same longitudinal mode number, but different transverse mode numbers. The two transverse modes are $TEM_{0,0}$ and $TEM_{1,0}$.

(a) *(1 pt.)* If the frequency separation between the two lasing cavity modes is 1/3 of the free spectral range, what is the length L of the cavity?

(b) *(1 pt.)* For this value of L, is this cavity stable, unstable, or conditionally stable?

(c) *(1 pt.)* What is the free spectral range of this laser? (Give a number)

2. Let ψ_n and E_n represent the normalized eigenfunctions and energy eigenvalues for a 1-D potential well $V(x) = (1/2)m\omega^2x^2$. The quantum numbers n are the integers $n \geq 0$.

(a) *(1 pt.)* Write an expression for E_0 in terms of known constants and the given parameters.

(b) *(1 pt.)* Let $\Psi_a(x,t)$ be a superposition state that is defined such that at time $t = 0$, $\Psi_a(x,0) = A\cdot(\psi_0 - \psi_1)$. What is the value of $|A|^2$?

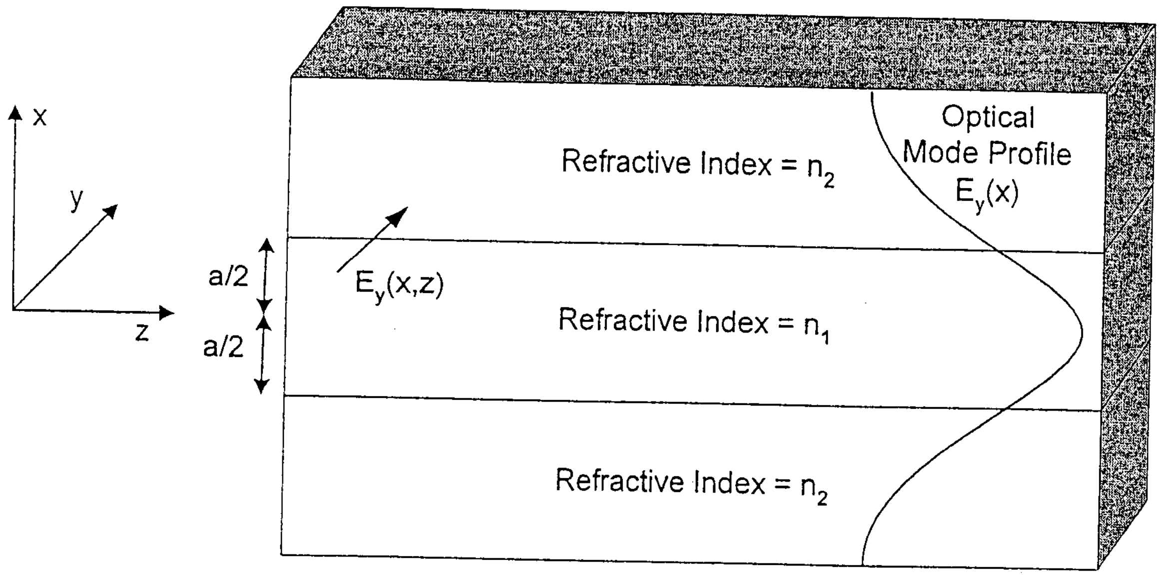
(c) *(2 pts.)* Write a complete time-dependent expression for the **probability density** of the state $\Psi_a(x,t)$ defined above. Simplify as much as possible so that any time-dependent functions are real functions of ωt.

3. (a) *(1 pt.)* A hydrogen atom in the $|n=2, l=1, m=0\rangle$ state can be excited to an $n = 4$ state via absorption of 246 nm light. What are the possible values of l for the upper state in this transition?

(b) *(1 pt.)* Into what state(s) can the $|n=3, l=2, m=0\rangle$ state decay by spontaneous emission of a single optical photon? Using the form $|n,l,m\rangle$, list all possible results.

(c) *(1 pt.)* What is the approximate wavelength of radiation associated with the hydrogen transition $|n=2, l=0, m=0\rangle \leftrightarrow |n=3, l=1, m=1\rangle$?

Spring 2006 Written Comprehensive Exam Question #11 Day 2 Track I



Consider the guided modes, in the slab waveguide picture above, that have an electric field parallel to the slab interfaces (along the y-direction), have a z-dependence that is given by a simple, plane wave-like, phase factor, and have no y-dependence.

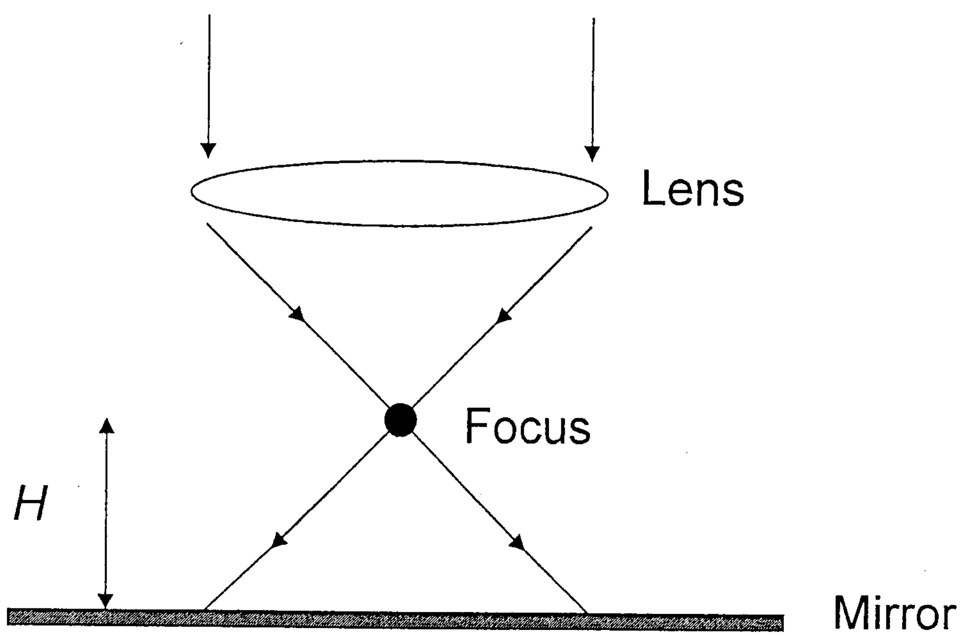
A. Starting with the general form of the Helmholtz equation (monochromatic wave equation) $\nabla^2 E_y(x,y) + k^2 E_y(x,y) = 0$, find the Helmholtz equations for the mode profile E_y (x) in the central slab and in the surrounding cladding regions. **(3 Points)**

B. Find the solutions for the Helmholtz equations in the central slab that are symmetric about the x=0 plane, and the physically reasonable solutions in the cladding regions. **(4 points)**

C. Use the boundary conditions on the electric field to eliminate two of the unknown constants in the solutions. **(3 points)**

Spring 2006 Written
Comprehensive Exam
Question #12 Day 2 Track I

A lens is used to focus a $\lambda = 1\mu m$ laser beam so that the focus point is $H = 2.5$ μm above a perfect mirror. The full cone angle of the focus beam is 90°. Make an accurate sketch of the interference pattern generated between the incident laser beam and the reflected beam between the focus and the mirror. The sketch should include the positions, shapes and order numbers of the fringes only in the region where interference occurs. Do not include any interference that may occur above the focus toward the lens. Ignore any effects due to diffraction or phase shift from the mirror surface. State any assumptions that you make.



WRITTEN PRELIM EXAM – FIRST DAY TRK II

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Spring 2006 Written Comprehensive Exam Question #1 Day 1 Track II

The formulae for the reflection coefficients for s-polarized and p-polarized incident fields at a planar dielectric interface of two media are

$$r_s(\theta) = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \quad r_p(\theta) = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}},$$

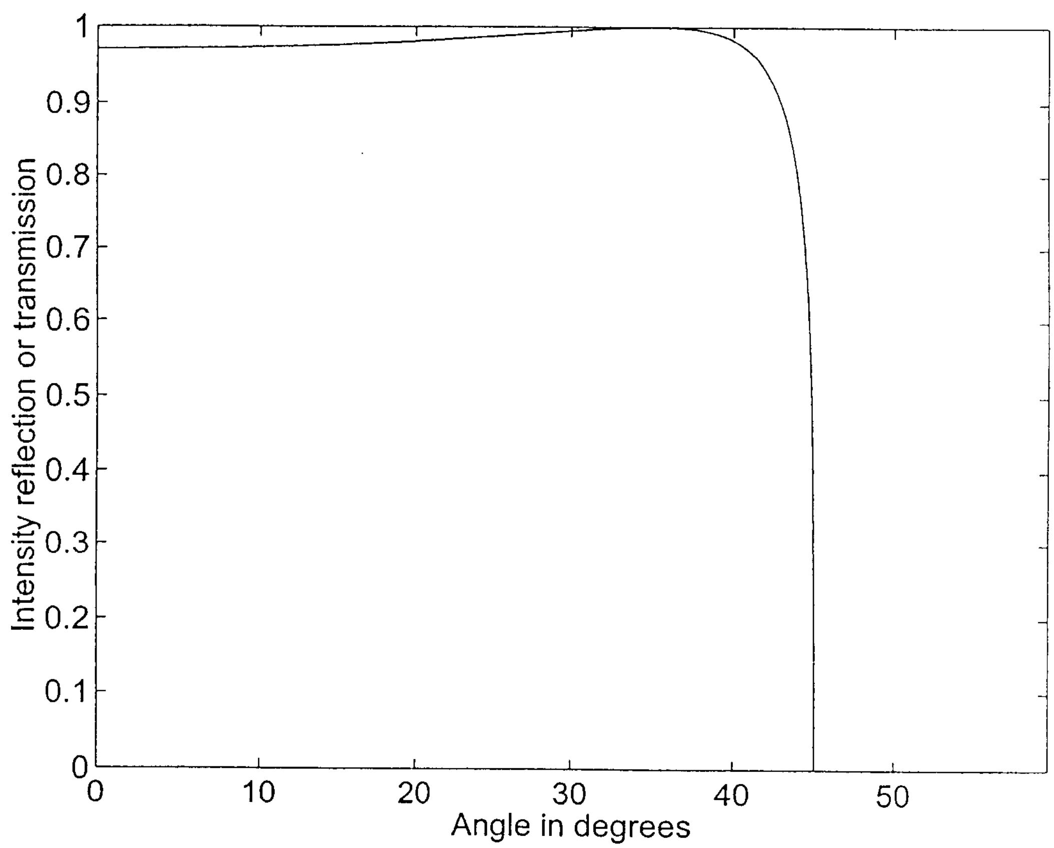
where $n = n_2/n_1$ is the relative refractive-index, n_1 being the refractive-index of the medium the field is incident from and n_2 the refractive-index of the second medium, and θ measures the angle of incidence.

(a) Define what is meant by s-polarized and p-polarized incident fields. (2)

(b) What is meant by total internal reflection (1), and using the information given above obtain an expression for the critical angle for total internal reflection (1).

(c) Consider the case that a plane-wave electromagnetic field of randomly varying polarization state is incident on the interface such that the *incident intensity* I_0 is on average equally split between s-polarization and p-polarization. Obtain an expression for the magnitude of the average reflected Poynting vector relative to the input for a field incident at the Brewster angle θ_B. (2)

(d) The figure on the next page shows either the intensity reflection or intensity transmission for either internal or external reflection at a dielectric interface as a function of incident angle θ, and for either an s-polarized or p-polarized incident field. By inspecting the curve explain which combination of the above options the plotted curve corresponds to (Note: It is not sufficient to state the combination of options you choose, you also need to give a brief explanation) (4).



Intensity reflection or transmission
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
Angle in degrees

Page 2 of 2

Spring 2006 Written Comprehensive Exam Question #2 Day 1 Track II

This question deals with basics in metal and crystal optics.

(a) The complex refractive-index $\mathcal{N}(\omega) = [n(\omega) + i\kappa(\omega)]$ for a field of frequency ω propagating in a metal may be deduced from the dielectric function $\epsilon(\omega) = \epsilon_0(1 - \omega_p^2/\omega^2)$. Obtain expressions for the refractive-index and extinction-index from this model for frequencies above and below the plasma frequency (2), and plot both versus field frequency ω, making sure to indicate key features (1).

(b) Consider the design of a high-reflectivity mirror using a thin metal film of thickness L for an incident field of frequency ω. Give a brief explanation of any design requirements on the metal plasma frequency and skin depth δ in relation to the incident frequency and film thickness, respectively (2).

(c) Consider propagation of a monochromatic field of frequency ω along the z-axis of a crystal such that a x-polarized field experiences absorption α_x and index of refraction n_x, and a y-polarized field experiences absorption α_y and index of refraction n_y. Write down an expression for the positive frequency component of the vector electric field $\vec{E}(z, \omega)$ in the crystal given that it has field components E_x and E_y at the input $z = 0$. (1)

(d) Next we consider the use of the crystal in part (c) to convert an input linear polarized beam into a circularly polarized beam. Obtain expressions for the minimum crystal length to realize this (2), and also the angle θ of the input linear polarization state with respect to the x-axis (2).

Spring 2006 Written
Comprehensive Exam
Question #3 Day 1 Track II

Consider excitons in GaAs. The Schroedinger equation for the relative motion of the electron-hole pair is (in Gauss units)

$$\left\{-\frac{\hbar^2\nabla^2}{2m_r}-\frac{e^2}{\varepsilon_0 r}\right\}\varphi_n(\vec{r})=\left(E_n-E_g\right)\varphi_n(\vec{r}).$$

Discuss all differences of this equation to that of a hydrogen atom. Specify the Rydberg law for the discrete excitonic eigenenergies (without derivation). Give an order-of-magnitude estimate for the binding energy of the lowest exciton and compare it to the energy of the lowest state in a hydrogen atom.

Furthermore, specify the Schroedinger equation of the center-of-mass motion of the exciton (carefully identifying how the mass of the electron, m_e, and the hole, m_h, enter the equation). Specify the wavefunctions that are solutions to this equation.

Finally, sketch the complete excitonic spectrum as function of the center-of-mass wavevector K_c.

(10 points)

Spring 2006 Written
Comprehensive Exam
Question #4 Day 1 Track II

Consider the phonon dispersion relation of a diatomic lattice (two atoms of mass M_1 and M_2, respectively, per unit cell). For simplicity, consider only a simple one-dimensional crystal (see Fig. 1) with harmonic interactions (f denotes the force constant and is the same for all interactions).

Proceed as follows. First, without any derivation, sketch the dispersion relation within the first Brillouin zone, properly labeling the acoustic and optical branches.

Furthermore, specify Newton's equations for the atomic displacements u_j and u_{j+1}.
Use a plane-wave-like ansatz (trial function) to describe the eigenmodes of the system (i.e., the modes where all atoms oscillate at the same frequency Ω). Denote the wavevector by k. Requiring non-trivial solutions, solve for the square of the dispersion relation, $\Omega_{\pm}^2(k)$.

(10 points)

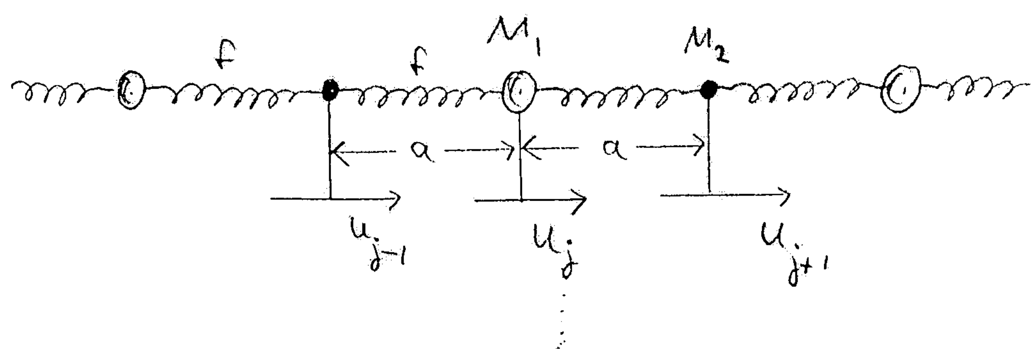


Fig. 1: Schematic of a one-dimensional phonon model.

Spring 2006 Written Comprehensive Exam
Question #5 Day 1 Track II

In the following we consider a bosonic particle whose internal degrees of freedom corresponds to a 3-dimensional state space. We can measure two physical quantities that depend on these degrees of freedom, and the corresponding quantum observables have the form

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

in the representation $\{|u_1\rangle, |u_2\rangle, |u_3\rangle\}$.

(a) What are the possible outcomes of a measurement of A? Same question for B. (20%)

(b) Is it possible to prepare the system in a state for which the joint outcome of measuring A and B can be predicted with certainty? Explain! (20%)

In the following, let α, β be the largest possible values resulting from measurements of A, B, and let $|\alpha\rangle$, $|\beta\rangle$ be the corresponding post-measurement states.

Alice has two identical particles of this type, and prepares them in the joint state $|\Psi\rangle = |\alpha(1)\rangle|\beta(2)\rangle + |\beta(1)\rangle|\alpha(2)\rangle$.

(c) Is the state $|\Psi\rangle$ entangled? Explain! (20%)

Alice retains one particle for herself, and gives the other to Bob. Bob measures the observable A on his particle, gets the outcome α and communicates the result to Alice.

(d) Find the two-particle state following Bob's measurement. (20%)

(e) Alice now measures B on her particle. Find the probabilities of the possible outcomes. (20%)

Spring 2006 Written Comprehensive Exam
Question #6 Day 1 Track II

Consider in the following an ^{85}Rb atom in its electronic ground state, with electon orbital- and spin-angular momenta $L = 0$ and $S = 1/2$. The nuclear spin of this isotope is $I = 5/2$.

(a) Using the theory of addition of angular momentum, $\mathbf{F} = \mathbf{L} + \mathbf{S} + \mathbf{I}$, determine the number of magnetic sublevels $|F, m_F\rangle$ in the electronic ground state. (20%)

The ground state contains manifolds of magnetic sublevels split by the hyperfine interaction. In alkali atoms the upper manifold has the largest value of the total angular momentum, $F_{\max}$. Also, the Landé g_F factors for the upper/lower manifolds are $1/F_{\max}$, and $-1/F_{\max}$ respectively.

We now place the atom in a magnetic field $\mathbf{B}(x) = B(x^2 + x^3)\mathbf{z}$, $B > 0$. The field is weak enough that the Zeeman interaction energy is always much less than the hyperfine interaction energy.

(b) Find the Zeeman interaction energy $V_{F,m_F}(x)$ for each hyperfine state $|F, m_F\rangle$. (15%)

The Zeeman interaction traps some of the magnetic sublevels in a potential minimum centered on $x = 0$. In the following we consider an atom in the magnetic sublevel that is most strongly bound.

(c) Find the trap oscillation frequency in the harmonic approximation. (15%)

(d) The trapping potential is not harmonic. Using perturbation theory, find the first order correction to the energy of the trap ground and first excited state. Then find the first order correction to the state vector for the trap ground state. (50%)

Hints: $X = \sqrt{\frac{\hbar}{2M\omega}}(a^+ + a)$, $[a, a^+] = 1$, $[N, a] = -a$, $[N, a^+] = a^+$.

WRITTEN PRELIM EXAM – SECOND DAY

TRK II

Spring 2006

February 15, 2006
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla\cdot\mathbf{F}) + \mathbf{F}\cdot\nabla\phi$

$\nabla \cdot (\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla\times\mathbf{F}) + \nabla\phi\times\mathbf{F}$

$\nabla \times (\mathbf{F}\times\mathbf{G}) = \mathbf{F}(\nabla\cdot\mathbf{G}) - \mathbf{G}(\nabla\cdot\mathbf{F}) + (\mathbf{G}\cdot\nabla)\mathbf{F} - (\mathbf{F}\cdot\nabla)\mathbf{G}$

$\nabla \times (\nabla\times\mathbf{F}) = \nabla(\nabla\cdot\mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F}\cdot\mathbf{n})\, da = \int_V (\nabla\cdot\mathbf{F})\, d^3x$

$\oint_C \mathbf{F}\cdot d\boldsymbol{\ell} = \int_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G}\cdot\mathbf{n})\, da = \int_V [\mathbf{F}(\nabla\cdot\mathbf{G}) + (\mathbf{G}\cdot\nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n}\times\mathbf{F})\, da = \int_V (\nabla\times\mathbf{F})\, d^3x$

Spring 2006 Written Comprehensive Exam Question #7 Day 2 Track II

A) Sketch a configuration, including the components needed, for a simple Erbium Doped Fiber Amplifier (EDFA) using co-propagating 980 nm pump laser. Assume that in addition to the pump laser and the Er-doped fiber, you have only two other optical components available. **(2 points)**

B) Figure 1 below shows the calculated population inversion along a typical EDFA with a single pump laser. The parameters used in modeling were:

- Fiber length: 20 m.
- One input signal at 1550 nm with input power of 0.001 mW.
- Co-propagating pump of 100 mW at 980 nm.

B.1. Roughly sketch the forward and backward propagating Amplified Spontaneous Emission (ASE) powers (arbitrary values) along the fiber. **(3 points)**

B.2. Briefly explain the behavior of the population inversion along the fiber. **(3 points)**

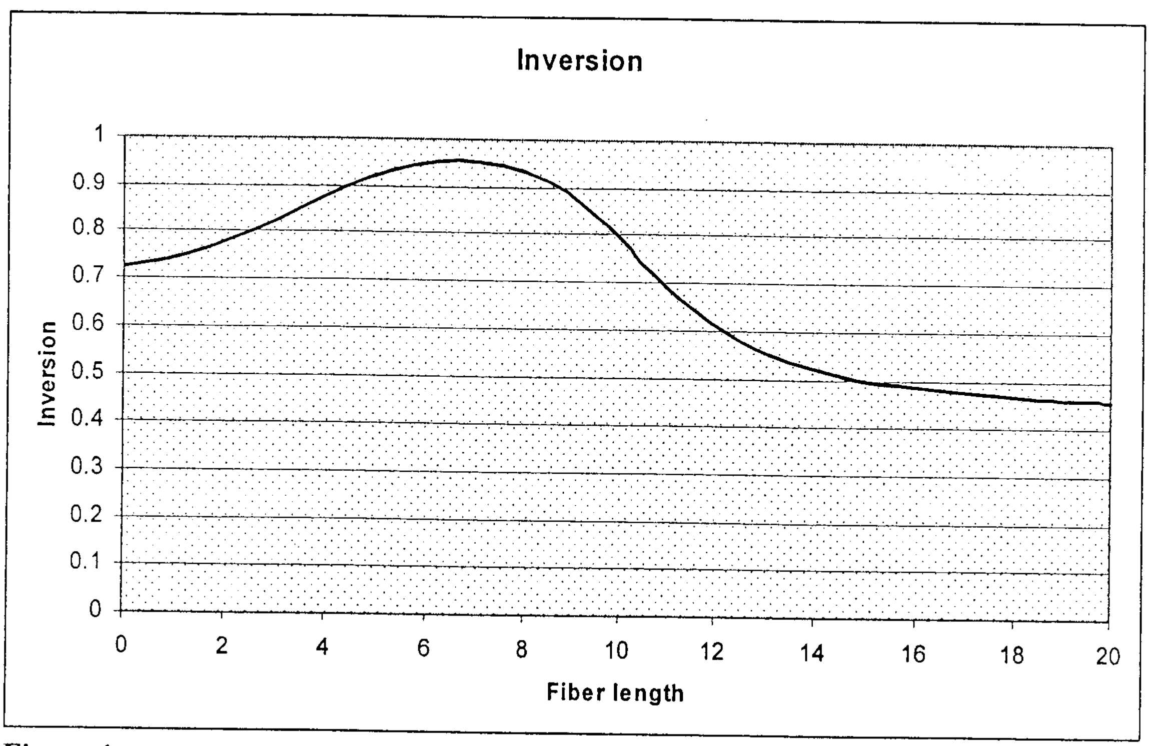
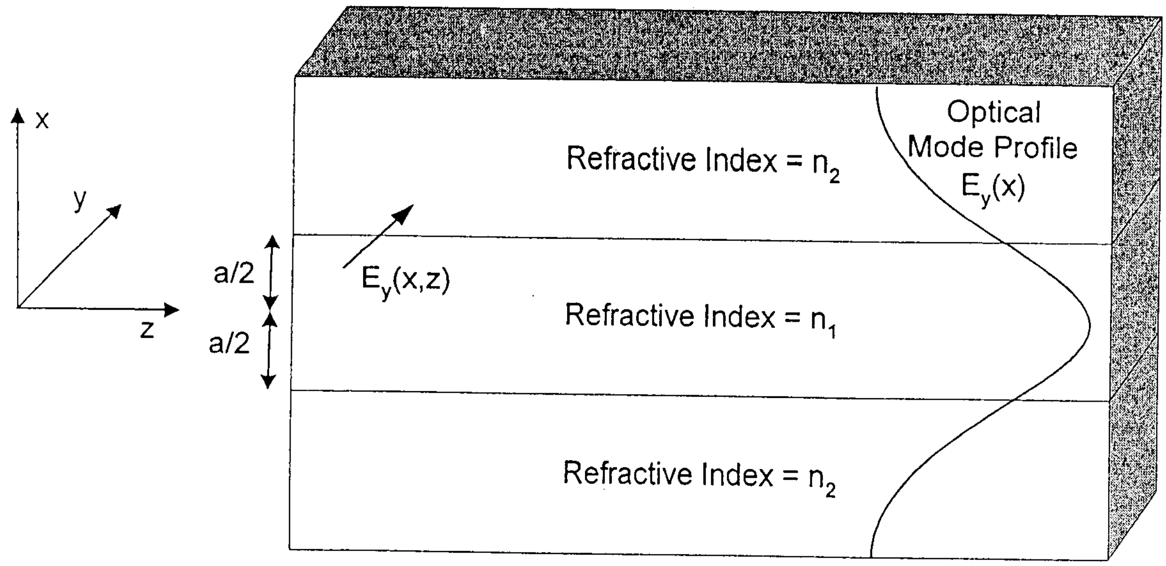


Figure 1.

C) In terms of noise properties of an EDFA, briefly explain why 980 nm pumping is advantageous compared to 1480 nm pumping. **(2 points)**

Spring 2006 Written
Comprehensive Exam
Question #8 Day 2 Track II



Consider the guided modes, in the slab waveguide picture above, that have an electric field parallel to the slab interfaces (along the y-direction), have a z-dependence that is given by a simple, plane wave-like, phase factor, and have no y-dependence.

A. Starting with the general form of the Helmholtz equation (monochromatic wave equation) $\nabla^2 E_y(x,y) + k^2 E_y(x,y) = 0$, find the Helmholtz equations for the mode profile $E_y(x)$ in the central slab and in the surrounding cladding regions. **(3 Points)**

B. Find the solutions for the Helmholtz equations in the central slab that are symmetric about the x=0 plane, and the physically reasonable solutions in the cladding regions. **(4 points)**

C. Use the boundary conditions on the electric field to eliminate two of the unknown constants in the solutions. **(3 points)**

Spring 2006 Written Comprehensive Exam Question #9 Dav 2 Track II

We are going to perform repeated measurements of a two-level atom at regular time intervals $n\Delta t$. The initial state of the atom will be the upper level, so the outcome of the measurement at $n = 0$ is described by the probability vector

$$\begin{bmatrix} p_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

which indicates that the probability of finding the atom in the lower energy level E_0 is 0 and the probability of finding the atom in the upper energy level E_1 is 1. If these probabilities are $p_n$and $q_n = 1 - p_n$ at time $n\Delta t$, then we have

$$\begin{bmatrix} p_{n+1} \\ q_{n+1} \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} p_n \\ q_n \end{bmatrix}$$

To keep things simple we will use the numerical values $a = 2/3$ and $b = 1/3$ in the transition matrix.

a) Find the probability vectors for $n = 1, 2, 3$. (20%)

b) To what probability vector do these vectors seem to be converging? Show that this equilibrium probability is an eigenvector of the transition matrix and find the eigenvalue. (20%)

c) Let $X_n$be the binary random variable that describes the outcome of our measurement at time $n\Delta t$. Thus $X_n = 0$ with probability p_n and $X_n = 1$ with probability q_n. Find the entropy $H(X_n)$ for $n = 0, 1, 2, 3$ and for the equilibrium probability distribution. Is entropy increasing or decreasing? (20%)

d) Find the mutual information $I(X_1, X_2)$. (20%)

e) Which will be larger, $I(X_1, X_2)$ or $I(X_1, X_3)$? Why? (20%)

Spring 2006 Written Comprehensive Exam
Question #10 Day 2 Track II

Consider a three-level system with two degenerate lower levels $|b\rangle$ and $|c\rangle$ coupled by electric dipole interaction to an upper level $|a\rangle$. Taking the energy of the ground states $|b\rangle$ and $|c\rangle$ to be zero, the semiclassical Hamiltonian describing this system is

$$H = \hbar\omega|a\rangle\langle a| + \hbar\wp(|a\rangle\langle b| + |a\rangle\langle c| + H.c.).$$

The general form of the state $|\psi(t)\rangle$ at time t is of the form

$$|\psi(t)\rangle = c_a(t)|a\rangle + c_b(t)|b\rangle + c_c(t)|c\rangle.$$

a. (30%) What are the equations of motion for the probability amplitudes $c_i(t)$?

b. (20%) Instead of the ground states $|b\rangle$ and $|c\rangle$, we introduce the new states

$$|g_+\rangle = (|b\rangle + |c\rangle)/\sqrt{2} \text{ and } |g_-\rangle = (|b\rangle - |c\rangle)/\sqrt{2}.$$

Why is it possible to use these states to describe the atoms instead of $|b\rangle$ and $|c\rangle$?

c. (30%) The general state of the system in terms of these states is

$$|\psi(t)\rangle = c_+(t)|g_+\rangle + c_-(t)|g_-\rangle + c_a(t)|a\rangle.$$

What are the equations of motion of the probability amplitudes $c_+(t)$, $c_-(t)$, and $c_a(t)$? (Hint: Express the Hamiltonian H in terms of these states.)

d. (20%) Why is the state $|g_-\rangle$ called a *dark state*? (Hint: What happens if the system is initially in the state $|g_-\rangle$?) What is the physical origin of this behavior?

Spring 2006 Written
Comprehensive Exam
Question #11 Day 2 Track II

Starting from the density matrix equation for a two-level system with upper level 2, ground level 1, transition frequency ω, and incident field E of frequency Ω:

$$\dot{\rho}_{11}(t) = -iX^{*}\rho_{21} + iX\rho_{12} + \gamma_2\rho_{22}$$

$$\dot{\rho}_{22}(t) = iX^{*}\rho_{21} - iX\rho_{12} - \gamma_2\rho_{22}$$

$$\dot{\rho}_{12}(t) = i\delta\rho_{12} - iX\left(\rho_{22} - \rho_{11}\right) - \gamma\rho_{12}$$

$$\dot{\rho}_{21}(t) = -i\delta\rho_{21} + iX\left(\rho_{22} - \rho_{11}\right) - \gamma\rho_{21}$$

where $\delta = \omega - \Omega$ and $X = \wp E / 2\hbar$.

(a) Solve the equation in steady state for ρ_{22} using $\rho_{11} + \rho_{22} = 1$. 30%

(b) What are the values of ρ_{22} and ρ_{11} for strong field $\left(|X| \gg \gamma, \gamma_2\right)$? 15%

(c) Can one obtain population inversion by driving a two-level system? 15%

(d) What is the rotating wave approximation that was used to obtain the density equation you just solved? 20%

(e) Plot ρ_{22} as a function of laser frequency and show that the linewidth is γ for weak field and power broadens for strong field. 20%

Spring 2006 Written Comprehensive Exam Question #12 Day 2 Track II

Each of the following parts is a verbal description of the pupil function in a unit-magnification imaging system. For each part, sketch and discuss the coherent transfer function, the coherent PSF, the incoherent transfer function, and the incoherent PSF. Qualitative answers are desired, not detailed derivations, but all axes should be labeled clearly and critical dimensions (such as cutoff frequencies) should be specified.

(2 Points) (a) A rectangular slit of dimensions $L_x \times L_y$.

(2 Points) (b) Same as (a) but in the limit $L_x \rightarrow 0$.

(3 Points) (c) Two slits of dimensions $L_x \times L_y$, centered at $(\pm x_0, 0)$, $x_0 > L_x$.

(3 Points) (d) A circular aperture of diameter D.