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WRITTEN PRELIM EXAM – FIRST DAY

Fall 2008

September 23, 2008
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2\cos A \cos B = \cos(A-B) + \cos(A+B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_{\mathrm{S}} (\mathbf{F} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_{\mathrm{C}} \mathbf{F} \cdot d\ell = \int_{\mathrm{S}} (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_{\mathrm{S}} \phi\mathbf{n}\, da = \int_{\mathrm{V}} \nabla\phi\, d^3x$

$\oint_{\mathrm{S}} \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_{\mathrm{V}} [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_{\mathrm{S}} (\mathbf{n} \times \mathbf{F})\, da = \int_{\mathrm{V}} (\nabla \times \mathbf{F})\, d^3x$

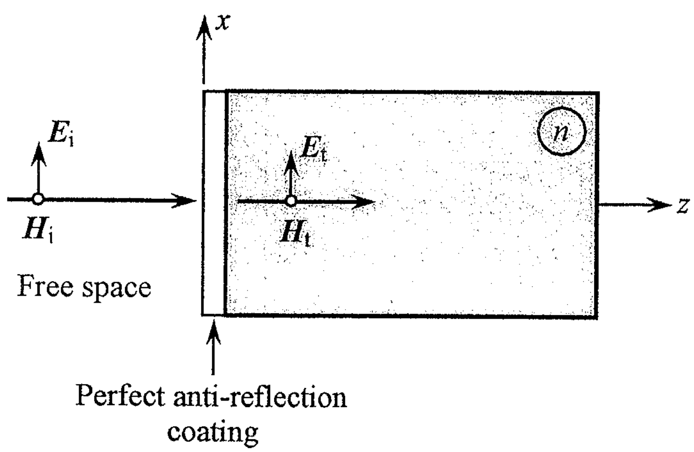
Fall 2008 Comprehensive EXam
Day 1, Question 1

A semi-infinite slab of transparent glass (refractive index = n) is coated with a *perfect* anti-reflection coating on its entrance facet. A monochromatic, linearly-polarized plane-wave arrives at the slab at normal incidence, as shown below. The incidence medium is free space, the vacuum wavelength of the light is λ_o, and the incident E-field is along the x-axis.

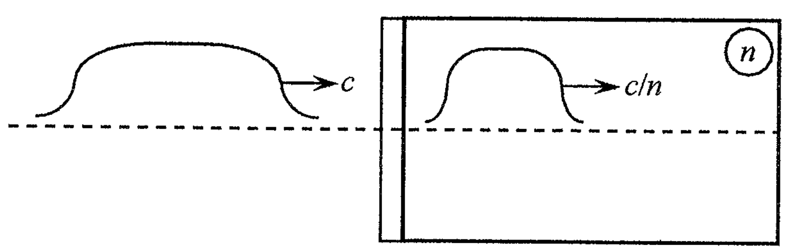
2 pts a) What is the relation between the incident E- and H-fields, E_i and H_i, in terms of the impedance of the free-space, $Z_o = \sqrt{\mu_o/\varepsilon_o}$?

2 pts b) What is the relation between the fields E_t, H_t transmitted into the slab in terms of Z_o and n?

3 pts c) Without making any assumptions about the structure of the anti-reflection coating, simply knowing that the optical energy of the beam passes entirely from the free space into the slab, determine the relation between the incident and transmitted E-fields E_i and E_t.



3 pts d) Assume now that, instead of a plane-wave, the incident beam is a pulse of light having the same central wavelength λ_o as before. Moreover, the front-facet coating is effective as a perfect anti-reflection coating for the entire pulse, and the semi-infinite slab is free from dispersion, so that, inside the slab, the pulse propagates with velocity c/n, as shown below. What are the E- and H-field energies inside the slab? Is the total E-field energy of the pulse equal to its total H-field energy? Is the pulse energy conserved before and after incidence?



Fall 2008 Comprehensive Exam
Day 1, Question 2

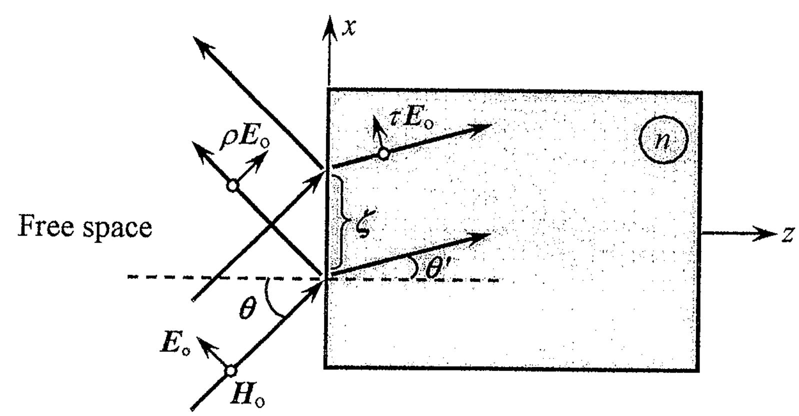
Shown below is a collimated, monochromatic beam with a large, uniform cross-sectional area (essentially a finite-diameter plane-wave), incident on a semi-infinite, transparent medium of refractive index n. The angle of incidence is θ, the medium of incidence is free space, the footprint of the beam along the x-axis is ζ, and the incident beam is p-polarized (with the E- and H-field amplitudes being E_o and H_o, respectively). Denote the E-field amplitude of the reflected beam by ρE_o, that of the transmitted beam by τE_o.

2 pts a) In terms of ρ, τ, n, E_o, and the free-space impedance $Z_o = \sqrt{\mu_o/\varepsilon_o}$, what are the H-field amplitudes of the reflected and transmitted beams?

3 pts b) Determine the rate of flow of optical energy (per unit area per unit time) for the incident, reflected, and transmitted beams?

3 pts c) Use conservation of energy to find a relationship between ρ and τ in terms of n and θ.

2 pts d) How is the relation between ρ and τ obtained in part (c) affected if the incident beam happens to be s-polarized?



Fall 2008 Comprehensive Exam
Day 1, Question 3

This problem has four parts each being 2.5 points for a total of 10 points.

Part 1

Define depth of focus DOF and derive the formula for DOF in terms of the blur circle diameter "B" and the lens focal number f/#.

Part 2

Assume that a lens is working at a magnification of $m = 0$. The DOF for this lens is +/- 4 mm and its working f/# is 16. What is the blur diameter B?

Part 3

Define the hyperfocal distance and make a drawing to explain it.

Part 4

A lens is working at a magnification of 1/10. The object plane is tilted 45 degrees with respect to a plane perpendicular to the optical axis. What is the tilt angle of the image plane where images are well in focus? Make a sketch of the situation.

An air-spaced triplet is comprised of three thin lenses in air:

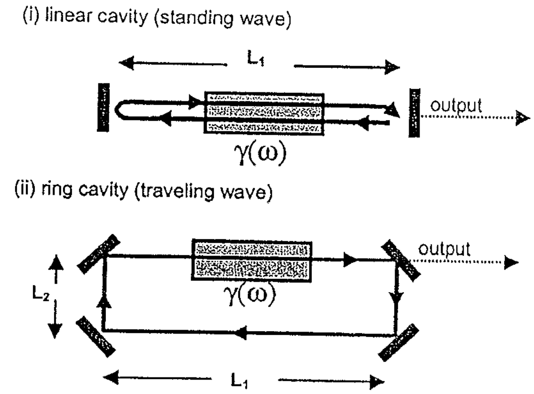
f1 = 25 mm	t1 = 10 mm
f2 = -50 mm	t2 = 10 mm
f3 = 25 mm	

Part A (60%) Use Gaussian methods to determine the system Focal Length and Back Focal Distance.

Part B (40%) The system stop is located halfway between the first two thin lenses. Use Gaussian methods to determine the locations of the system Entrance and Exit Pupils.

Note: No raytrace analysis is permitted for either portion of this problem.

The gain in each CW laser shown to the right is characterized by gain coefficient $\gamma(\omega)$. Cavity (i) is a standard 2-mirror standing wave cavity with round-trip cavity length $2L_1$. Cavity (ii) is a *uni-directional* ring laser in which the laser field propagates in a single clockwise direction as shown (due to additional optical elements we will not consider). This cavity is made up of 4 mirrors with round-trip cavity length $P = 2L_1 + 2L_2$. For each question below, consider only the lowest order mode of the cavity.



(a) For the linear cavity shown in (i), write the expression for the resonant frequencies, ν_q, indexed by longitudinal mode number "q". What is the free-spectral range of this cavity? For simplicity, assume all mirrors are flat (i.e. consider plane waves only). **(10%)**

(b) Under the same assumptions as in (a), write an expression for the resonant longitudinal mode frequencies and free spectral range for the *ring* cavity shown in (ii). Recall that the resonance condition for an optical cavity requires an integer number of wavelengths fit within one round-trip cavity length. **(10%)**

(c) If one mirror of the *linear* cavity is flat and the other has a radius of curvature of R= 20 cm, is this cavity considered stable if $L_1 = 15$ cm? Show your work. **(15%)**

(d) If *both* mirrors of the linear cavity are flat, the cavity can be considered "conditionally stable". To reach lasing threshold with this cavity geometry and finite diameter mirrors, would $\gamma(\omega)$ need to be greater or less than that required for the same laser with the cavity described in part (c)? Justify your answer in two or three sentences along with specifying what is meant by "conditional stability". **(20%)**

Assume the laser gain profile $\gamma(\omega)$, centered at ω_o, is *inhomogeneously* broadened (such as in an atomic gas at room temperature) and both lasers in (i) and (ii) operate on a *single* longitudinal mode, at frequency ω_q, located slightly below line center (i.e. $\omega_q < \omega_o$). The minimum round-trip gain coefficient required to reach lasing threshold in each case is γ_T.

(e) For the linear cavity in (i) under steady-state lasing conditions, plot the gain coefficient $\gamma(\omega)$ vs. ω. On the axes, indicate γ_T, ω_q, and ω_o and the value of the *small signal* gain coefficient at line center, $\gamma_o(\omega_o)$ **(25%)**

(f) In practice, a gain profile $\gamma(\omega)$ is measured by sending a weak probe beam through the medium and measuring the transmission (gain) vs. ω. As in saturation absorption spectroscopy, the measurement can depend on the direction of the probe beam. To measure $\gamma(\omega)$ for laser (ii), assume we send a probe beam through the gain medium in the *counter-clockwise* direction (i.e. *opposite* direction of lasing mode). Plot the measured $\gamma(\omega)$ vs. ω, labeling the critical points as was done in part (e). **(20%)**

Assume that an atom of mass m is confined to a three-dimensional harmonic oscillator potential well centered about the position $(x,y,z) = (0,0,0)$. The harmonic oscillator is characterized in the usual way by the oscillation frequencies ω_x, ω_y, and ω_z in the x, y, and z directions, respectively. For this problem, let $\omega_y = 100\omega_x$ and $\omega_z = 100\omega_x$.

(a) (1.5 pt) Write down the Hamiltonian associated with this problem.

(b) (1.5 pt) Write a complete expression for the eigenvalues of the Hamiltonian.

(c) (1 pt) What is the energy difference between the **ground state** and the **second excited state** (ie, the third bound state) of the potential?

For the remaining questions, let $\Psi(x,y,z,t)$ be defined as the wavefunction corresponding to the **second excited state** of the potential well. We will write $\Psi(x,y,z,t)$ as a product of functions, that is:

$$\Psi(x,y,z,t) = \psi_x(x)\, \psi_y(y)\, \psi_z(z)\, F(t),$$

where $\psi_x(x)$, $\psi_y(y)$, and $\psi_z(z)$ are each separately normalized real functions.

(d) (1.5 pt) Sketch $\psi_x(x)$ vs. x.

(e) (1.5 pt) Write out the mathematical expression for $F(t)$, defining any new parameters (if you use any) in terms of parameters already defined above.

(f) (0.5 pt) Is $\Psi(x,y,z,t)$ a stationary state?

(g) (2.5 pts) Write the **normalized** mathematical expression for $\psi_y(y)$. (Note that the parameter is y, not x!)

HINT #1: $\sqrt{\pi} = \int_{-\infty}^{\infty} e^{-u^2} du$.

HINT #2: If you are left with a length parameter in your expression, and you do not remember how to write it in terms of the parameters already given, then write out the **one-dimensional** Time-Independent Schrödinger Equation associated with your expression for $\psi_y(y)$. (This equation is obtained using the separation of variables technique on the full three-dimensional Time-Independent Schrödinger Equation, but you shouldn't need to go through that procedure!) If you solve the equation and compare terms appropriately, you will be able to determine an expression for the length parameter in your problem.

Fall 2008 Comprehensive Exam
Day 1, Question 5

Consider a two-level atom with excited state 2 and ground state 1 and with transition energy $\hbar\omega_0 = (E_2 - E_1)$ interacting with a coherent optical field $\mathbf{E}(t) = \mathbf{k}|E_0(t)| \cos[\omega t - \varphi(t)]$.

1. Write down:

   the time dependent Schrödinger equation;

   the total time-dependent wave function for a two level atom in the interaction representation in terms of the slowly varying amplitudes $c_1(t)$ and $c_2(t)$ and the radial wave functions $\Psi_1(\mathbf{r})$ and $\Psi_2(\mathbf{r})$;

   the interaction Hamiltonian in the dipole approximation. (15% total – 5% each)

2. Now assume that the applied light field is in resonance with the atom [$\delta = \omega_0 - \omega = 0$], that it is zero for $t < 0$ and a constant value for $t > 0$ [so $\chi = \Omega_0/2$, where $\Omega_0 = -(\mu_z)_{12}E_0/\hbar$], and that $\varphi(t) = 0$. Then the coupled equations for the amplitudes in the rotating wave approximation limit in the interaction representation are:

$$dc_1(t)/dt = (-i\Omega_0/2)\, c_2(t)$$
$$dc_2(t)/dt = (-i\Omega_0/2)\, c_1(t)$$

Obtain the solutions for $c_2(t)$ and $c_1(t)$ for the case that the atom is initially in the ground state. (Hint: First obtain a differential equation for $c_2(t)$ that is independent of $c_1(t)$, then write down a general solution for $c_2(t)$, and apply initial conditions.) (35%)

3. What is the equation for the probability $P_2(t)$ for finding the electron in the upper state? Plot $P_2(t)$ making sure to put values on both axes. What is the numerical value of $P_2(t)$ after a time corresponding to a π pulse and to a 2π pulse? What is the name of this phenomenon? Describe this phenomenon using the motion of the Bloch vector (in words or sketch). (25% total - 5% each)

4. Now suppose the light field is detuned from the two-level resonance by δ. Describe the motion of the Bloch vector in this case (in words or sketch). What is the equation for the rate of rotation of the Bloch vector? What is the rate of rotation for an atom with $\delta = \sqrt{3}\Omega_0$; does its Bloch vector stay in phase with that for $\delta = 0$? Describe in words the effect of inhomogeneous broadening on this phenomenon. Describe in words one experiment to observe this phenomenon. (25% total – 5% each)

Fall 2008 Comprehensive Exam
Day 1, Question 6

Consider a two-level atom (ground state $|g\rangle$, excited state $|e\rangle$) interacting with a single mode of a quantized electromagnetic field in a cavity. When the cavity mode is exactly resonant with the atomic transition frequency the Hamiltonian has the form

$$H = \hbar\omega|e\rangle\langle e| + \hbar\omega\, a^{+}a + \hbar\kappa\left(a|e\rangle\langle g| + a^{+}|g\rangle\langle e|\right)$$

In the absence of atom-field coupling ($\kappa = 0$), this Hamiltonian has eigenstates of the form $|\varphi, n\rangle$, where the atomic state $\varphi = g,e$ and n is the number of photons in the cavity mode.

(a) Now let $\kappa \neq 0$. Write out a corner of the matrix for H in the basis of uncoupled states corresponding to $n \leq 3$. Arrange the basis states in order of increasing energy. (20%)

(b) Find the eigenenergies of the coupled atom-field system. (20%)

(c) Find the eigenstates $|\pm, n\rangle$ corresponding to these eigenenergies, expressed in terms of the uncoupled states. (20%)

(d) For both $\kappa = 0$ and $\kappa \neq 0$ draw an energy level diagram of the eigenstates of the atom-field system, including all levels with $n \leq 3$. The diagram must clearly indicate the relative magnitude of the level splittings. (20%)

WRITTEN PRELIM EXAM – SECOND DAY

Fall 2008

September 24, 2008
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

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$2\sin A \sin B = \cos(A - B) - \cos(A + B)$

$2\sin A \cos B = \sin(A + B) + \sin(A - B)$

$2\cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2\sin A \cos A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

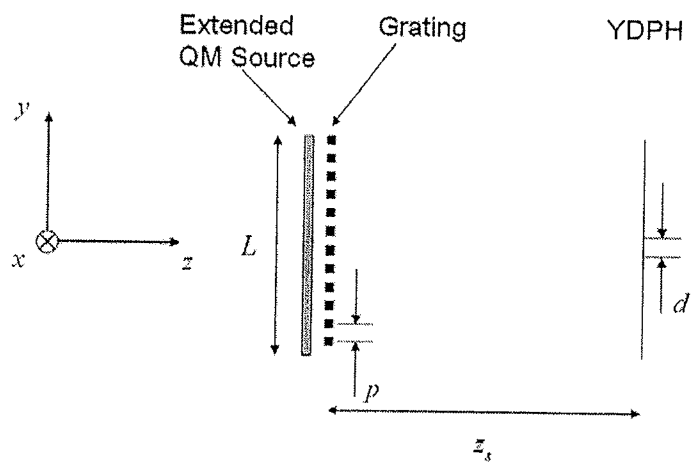
$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

Fall 2008 Comprehensive Exam
Day 2, Question #7

A grating is placed in front of a quasimonochromatic and spatially incoherent extended source, as shown below. The grating is a 50% duty cycle transmission grating with a transmission function given by

$$t(x_s, y_s) = \left[\mathrm{rect}\left(\frac{y_s}{p/2}\right) * \frac{1}{p}\mathrm{comb}\left(\frac{y_s}{p}\right)\right] \mathrm{rect}\left(\frac{y_s}{L}\right) \mathrm{rect}\left(\frac{x_s}{5L}\right) ,$$

where $L = 5$ mm and $p = 50$ μm. The source wavelength is $\lambda = 500$ nm.



a.) (5 pts) A young's double pinhole (YDPH) interferometer is placed $z_s = 1$ m from the grating. Derive a mathematical expression for the fringe visibility as measured through the YDPH as a function of d, the spacing between the pinholes. State any assumptions that you make. Leave the expression in terms of L, p, z_s, λ and d.

b.) (3 pts) Make a reasonably accurate sketch of the visibility function from part (a) as a function of d, including relative spacing between visibility maxima and relative widths of features. Label the horizontal axis in terms of physical distances, like mm.

c.) (2 pts) If the YDPH is replaced with a 50% duty cycle transmissive grating, where the period of the grating is equal to the pinhole separation corresponding to the first non-zero maximum of visibility from part (a), will the transmitted light experience the Tablot effect? If so, what differences might be present in the reconstruction planes compared to a grating illuminated with a collimated laser beam?

Fall 2008 Comprehensive Exam
Day 2, Question 8

Use Jones calculus to show that two half-wave plates at angle θ between them are equivalent to a rotator through angle 2θ. A rotator simply rotates the electric vector of linearly polarized laser light in a plane perpendicular to the axis of propagation. The wave plates are illuminated by an on-axis plane wave. The half-wave plates are simple uniaxial crystals, where the e and o axes are perpendicular to the direction of propagation.

Fall 2008 Comprehensive EXam
Day 2, Question 7

There is no need for excessive formulae to answer the eight parts of this question, you may simply state equations you feel are relevant.

1. What distinguishes the geometric ray path that joins an initial point at the input of an optical system to a final point at the output of the optical system, as opposed to any other arbitrary path that joins the initial and final points? (1 points)

2. Name two exact solutions of the scalar Helmholtz equation for a monochromatic optical field and give formulae for them. (2 points)

3. In the terminology of optics what is the name of the function that provides the wave optical disturbance produced at the output of an optical system due to a point source at the input? (1 point)

4. Sketch the variation of the on-axis intensity $I(\rho = 0, z)$ versus propagation z distance past a uniformly illuminated circular aperture, and indicate the propagation ranges corresponding to the Fresnel and Fraunhofer regions. (3 points)

5. Name and sketch an example of an interferometer based on division of amplitude. (3 points)

Fall 2008 Comprehensive Exam
Day 2, Question 8

This question is related to the linear electro-optic effect, or Pockel's effect, in a uniaxial crystal with point group $\bar{4}2m$ with a DC electric field E_{DC} applied along the Z-axis, for which the index ellipsoid may be written as

$$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} + 2r_{63}E_{DC}XY = 1, \tag{1}$$

(X, Y, Z) being the principal axes system defined with respect to the orthonormal basis vectors $\vec{X}, \vec{Y}$, and $\vec{Z}$.

(a) Go through the explicit calculation to transform the index ellipsoid to the new axes system (x, y, z) defined with respect to the orthonormal basis vectors $\vec{x}, \vec{y}$, and $\vec{z}$, where $Z = z, X = (x-y)/\sqrt{2}, Y = (x+y)/\sqrt{2}$. (4 points)

(b) Argue that with respect to the new axes system (x, y, z) the crystal is effectively biaxial in the presence of the DC electric field, and evaluate the principal refractive-indices n_x, n_y, and n_z (you may assume $|r_{63}E_{DC}| << 1$). (3 points)

(c) For propagation along the z-axis obtain an expression for the minimum value of the magnitude of the DC electric field required to create a quarter-wave plate for a medium of length L and a free-space wavelength λ. (3 points)

Fall 2008 Comprehensive Exam
Day 2, Question ¯9

For a hydrogen atom, the potential energy of the electron in MKS units is given by

$$V(\mathbf{r}) = -\frac{e^2}{4\pi\epsilon_0 r},$$

where, as usual, $r = |\mathbf{r}|$. When this potential is used in the Schrödinger equation, it is found that the energy levels of hyrdogen are given by

$$\mathcal{E}_n^H = -\frac{e^4 m}{2\hbar^2 (4\pi\epsilon_0)^2 n^2} \equiv \frac{1}{n^2}\mathcal{E}_1^H,$$

where n is the principal quantum number. The ground-state energy $\mathcal{E}_1^H$ is -13.5 eV or -1 Rydberg.

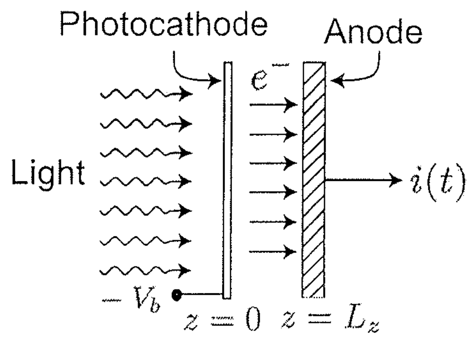
These formulas apply also to a good approximation to donor impurities in a semiconductor, except that the electron mass m must be replaced by the effective mass and the permittivity of free space must be replaced by the permittivity of the medium.

(a) (3 points) Use these considerations to compute the ground-state energy $\mathcal{E}_1^D$ of a donor in silicon, where electrons in the conduction band have an effective mass of about $0.26m$ and holes in the valence band have an effective mass of $0.37m$. The permittivity of Si is $\epsilon = 11.7\epsilon_0$.

(b) (2 points) Plot the donor energy levels to scale in the bandgap of silicon.

(c) (5 points) Give a qualitative plot showing how the conductivity of N-type Si varies with temperature. Discuss the results and relate them to the plot in part (b). Indicate on the plot the position of the Fermi level at low temperature and high temperature.

Fall 2008 Comprehensive Exam
Day 2, Question 10



The figure above depicts a simple vacuum photodiode.

(a) Explain how it works.

(b) How does the output current vary with the photon flux at fixed bias voltage and wavelength.

(c) How does the output current vary with wavelength at fixed photon flux and bias voltage?

(d) How does the current vary with bias voltage at fixed photon flux and wavelength? Consider both polarities of the bias in your answer to this question.

(e) What is meant by the term quantum efficiency for this device?

Parts (b)-(d) should be illustrated with simple sketches. Parts are equally weighted.

Fall 2008 Comprehensive Exam
Day 2, Question 9

(a) (3 points) Consider a transition from a state in the valence band of a semiconductor with wave function $\psi_1(r)$ to a state in the conduction band with wave function $\psi_2,(r)$, with ψ_1 and ψ_2 having well defined parities. Assume that the transition is dipole allowed with the interaction potential $V(r) = -er$. What should be the parities of the wave functions ψ_1 and ψ_2 for the transition to be allowed? Explain your answer.

(b) (4 points) Write down the energy and momentum conservation rules for exciton transitions in a semiconductor when the incident photon energy is $\hbar\omega$ and its momentum is $\hbar q$.

(c) (3 points) Using the angular momentum conservation rule for exciton transition in a semiconductor, determine the angular momentum of the exciton's envelope function.

Fall 2008 Comprehensive Exam
Day 2, Question 10

(a) (5 points) Drive the Beer's law for one-photon absorption in a solid. Identify the symbols you use. Is the one-photon absorption coefficient related to the real or imaginary part of the dielectric function?

(b) (5 points) Write down the propagation equation for the variation of the light intensity, I inside a solid as it propagates through it for a two-photon absorption process. Solve this differential equation for I(z) vs z, where z is the direction of laser propagation. Plot I(z) at a fixed position z as a function of incident lase intensity I_o and show optical limiting behavior.

Fall 2008 Comprehensive Exam
Day 2, Question 9

Consider the matrix element $\langle \nu'\vec{k}\ell' | e\vec{r}\cdot\vec{E} | \nu\vec{k}\ell\rangle$ of the light-matter-coupling Hamiltonian in a semiconductor quantum well, where ν labels the conduction and valence bands and ℓ labels the subbands. Representing the Bloch wave functions as products of envelope (Φ) and lattice-periodic (u) functions, show that

$$\langle \nu'\vec{k}\ell' | e\vec{r}\cdot\vec{E} | \nu\vec{k}\ell\rangle \simeq \sum_n \Phi^*_{\nu'\vec{k}\ell'}(\vec{R}_n)\Phi_{\nu\vec{k}\ell}(\vec{R}_n)\left\{ \mathrm{v}_{cell}\langle \nu' | e\vec{r}\cdot\vec{E} | \nu\rangle_0 + e\vec{R}\cdot\vec{E}\langle \nu' | \nu\rangle_0 \right\} \qquad (1)$$

where the sum extends over all unit cells and v_{cell} is the volume of the unit cell. Here, we assumed $u_{\nu\vec{k}}(\vec{r}) \simeq u_{\nu\vec{0}}(\vec{r})$ and the subscript 0 on the matrix elements indicates integrals involving the $u_{\nu\vec{0}}(\vec{r})$ functions.

Without a detailed derivation, argue briefly on the basis of Eq. (1) that in the case of interband transitions with $\nu' = c$ and $\nu = \mathrm{v}$, the $\Delta\ell = 0$ selection rule holds, i.e. $\langle \nu'\vec{k}\ell' | e\vec{r}\cdot\vec{E} | \nu\vec{k}\ell\rangle \sim \delta_{\ell',\ell}$. Are there any additional assumptions (in addition to those leading to Eq. (1)) you need to make for this selection rule to be valid?

(10 points)

Fall 2008 Comprehensive Exam
Day 2, Question 10

In contrast to vacuum, where the light dispersion is given by

$$\omega(k) = ck \tag{1}$$

the light dispersion in a simple metal is determined by the dielectric function

$$\varepsilon(\omega) = \varepsilon_\infty \left(1 - \frac{\tilde{\omega}_{pl}^2}{\omega^2} \right) \tag{2}$$

where $\tilde{\omega}_{pl}$ is the effective plasma frequency. For simplicity, we are neglecting any damping effects.

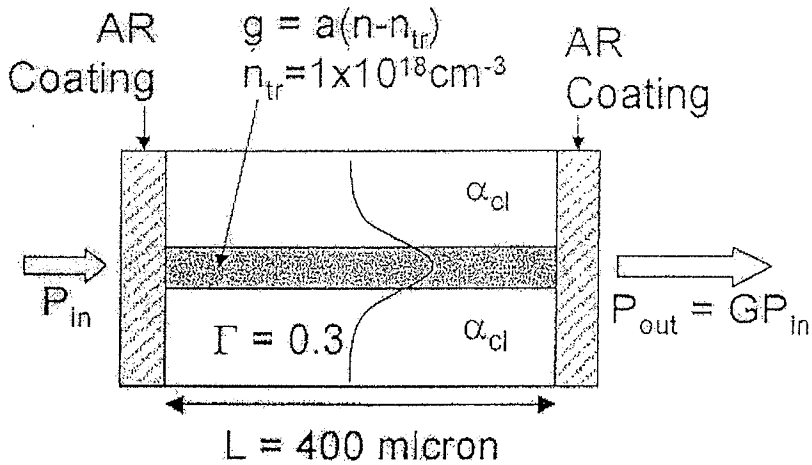
State the general dispersion relation $\omega(k)$ in terms of k and $\varepsilon(\omega)$. Then, using Eq. (2), derive the polariton dispersion $\omega(k)$ for the metal. (Hint: if you don't remember where the $\varepsilon(\omega)$ has to be placed in Eq. (1), you can quickly derive it from Maxwell's propagation equation. This derivation, however, will not give you additional points in this problem.)

Sketch the dispersion and label all special values and asymptotics properly. Also indicate the region of large reflection.

Finally, derive the group velocity $d\omega / dk$ of the polariton in a simple metal. Specify the group velocity at k=0 and its asymptotic value for $k \to \infty$.

(10 points)

Problem



A semiconductor optical amplifier has the following properties:

- Active region length $L = 400\ \mu m$
- Gain coefficient $g = \Gamma a(n-n_{tr})$, with gain constant $a = 2.5 \times 10^{-16}\ cm^2$
- Electron density in the active layer at transparency $n_{tr} = 1.0 \times 10^{18}\ cm^{-3}$
- Optical confinement factor $\Gamma=0.3$ (The fraction of the waveguide mode overlapping the active layer)
- Absorption coefficient due to free carrier absorption in the doped cladding layers $(1-\Gamma)\alpha_{cl}$, with $\alpha_{cl} = 50\ cm^{-1}$

a) Find an expression for the overall gain G in terms of Γ, a, n, n_{tr}, α_{cl}, and L
b) Use the result of part a to find an expression for the electron density in the active layer as a function of G
c) Determine the electron density in the active layer when $P_{out}/P_{in} = 20$ dB.

Table 1 is a table of *V* number for different LP modes of a step index fiber. For example, the V number for the LP_{23} (l=2 and m=3) mode is 10.173.

Table 1 Cutoff Values of *V* for Some Low-Order LP Modes

V	*m* = 1	*m* = 2	*m* = 3	*m* = 4
l = 0	0	3.832	7.016	10.173
l = 1	2.405	5.520	8.654	11.792
l = 2	3.832	7.016	10.173	13.323
l = 3	5.136	8.417	11.620	14.796
l = 4	6.379	9.760	13.017	16.224

(a) What is the name of the lowest order LP mode? (20%)

(b) How many LP modes are there for a fiber with V=6, including the 2X polarization degeneracy for each LP mode? (40%)

(c) A single step fiber with refractive indexes n_1=1.444 and n_2=1.443 operates at λ=1.55μm. Determine the largest core radius for single mode operation. (40%)

A sensor is being used in the laboratory to calibrate a diffuser. The sensor operates in two modes, radiance and irradiance. Irradiance mode consists of the system operating with a silicon detector that is square with a circular area of 0.0625 mm^2 at the bottom of a tube that is 0.707 m in length and has a diameter of 0.354 m. Radiance mode consists of placing a lens with a focal length of 0.707 m on the front of the tube. A 10-nm wide spectral filter centered at 600 nm is placed directly in front of the detector. The system operates with a 10-bit analog-to-digital converter.

The radiometer while operated in irradiance and radiance mode views a lamp that is 2-cm in diameter from a distance of 2 m. The spectral irradiance from the lamp is reported by the manufacturer as 18.4 W/m^2 μm at a distance of 50 cm.

The sensor also views a circular diffuser from a distance of 2 m. The diffuser is 10 cm in radius and is illuminated by the same lamp above. The distance between the lamp and diffuser is 1 m and the angle of incidence of the lamp on the diffuser is 45 degrees. The radiometer views the diffuser from an angle of 30 degrees.

A third set of measurements are taken with a shutter placed over the radiometer entrance aperture. The outputs from the sensor in the various modes while viewing the lamp, diffuser, and cover:

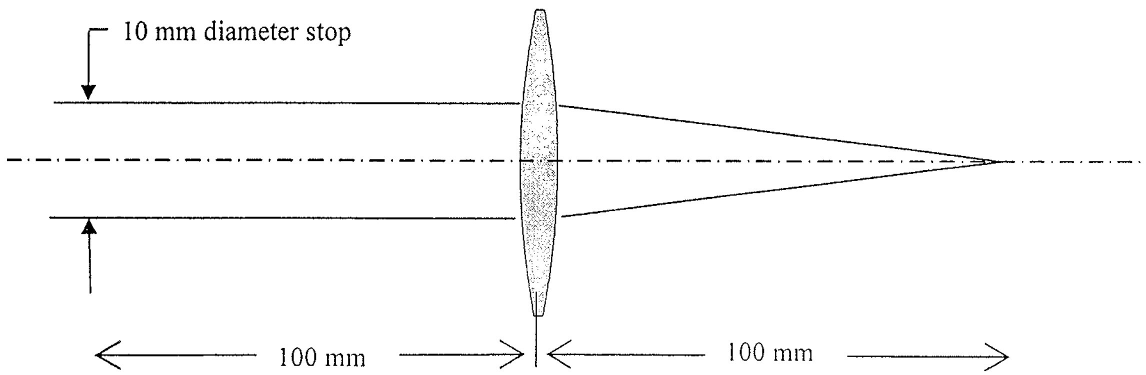
Sensor output (in analog to digital units) for radiance and irradiance modes while viewing lamp, diffusers, and shutter			
Mode	Lamp	Diffuser	Shutter
Irradiance	132	60	60
Radiance	1023	71	60

80% a) Determine an estimate for the reflectance of the diffuser.

20% b) The actual diffuser reflectance is 0.900. Geometry errors are negligible. Explain the most likely cause of your error and how you would correct it.

Consider a single thin lens made of SF57. The focal length of the lens for yellow (d – line) light is 100 mm. Assume an object point at infinity and a 10 mm stop, lens as shown below.

Parameters for the glass and definitions of the wavelengths are given in the tables below.



1. Draw a layout of the lens indicating where along the axis the yellow (d), blue (F), and red (C) light comes to focus. What is the spacing between the red and blue foci (15%)

2. Assume an image plane 100 mm from the lens, as shown. For an on-axis field point, Make sketches a) and b) showing the *chromatic* aberration (assuming other errors are corrected) for F, d, and C light for the case of on-axis imaging. Be sure to label your axes, including values.
   a) Ray fan diagram (transverse ray aberration *vs.* pupil coordinate) (20%)
   b) OPD diagram (wavefront aberration *vs.* pupil coordinate) (20%)

3. Evaluate the chromatic aberration for a field point 0.1 radians off axis. Sketch the following:
   a) Layout of the lens, showing the chief rays for F, d, and C (15%)
   b) Geometric point spread function for white light. Show approximate scale. (10%)
   c) Ray fan diagram (20%)

	SF57
refractive index n_d	1.847
Abbe number ν_d	23.8

n_d is refractive index at the d line, etc.

Abbe number is defined as $\upsilon_d = \dfrac{n_d - 1}{n_F - n_C}$

The F, d, and C lines occur at wavelengths:

$F : \lambda = 486.1$ nm

$d : \lambda = 587.6$ nm

$C : \lambda = 656.3$ nm

Fall 2008 Comprehensive Exam
Day 2, Question 11

This problem deals with task-based assessment of image quality, where quality is defined in terms of the performance of an observer on some specific task of medical or scientific interest. The parts are weighted equally.

(a) Tasks of interest can be categorized as classification or estimation tasks. Give two examples of tasks in each category. What is a binary classification task?

(b) Explain qualitatively how performance of a human observer on a binary classification task can be measured.

(c) Explain the concept of a receiver operating characteristic (ROC) curve and how it can be constructed for a human observer performing a classification task.

(d) Why is area under the ROC curve a useful metric for image quality? What range of numerical values does it take?

(e) What mathematical observer maximizes the area under the ROC curve? What mathematical operations are performed by this observer?

Fall 2008 Comprehensive Exam
Day 2, Question 12

A continuous-to-continuous (CC) imaging system is one maps a function of continuous variables to another function of continuous variables, for example the radiant exitance from a planar object to the irradiance on a detector plane. A continuous-to-discrete (CD) imaging system, on the other hand, is one that maps a function of continuous variables to a discrete array of numbers.

This problem considers a familiar imaging system, a digital camera, which has both CC and CD components.

The parts are weighted equally.

(a) Suppose a planar object is 2.5 m from the camera and that the lens has a focal length of 25 mm. What is the magnification of the system? An approximate answer will suffice if you justify the approximation.

(b) Is the mapping from the object radiant exitance to the irradiance on the image sensor linear and shift-invariant in general? If not, what assumptions and definitions are required to make it linear and shift-invariant?

(c) Now assume that the circular lens is F/10 and diffraction-limited. Give a mathematical expression for the CC mapping from the object radiant exitance, denoted $f(\mathbf{r})$, to the image-plane irradiance, denoted $I(\mathbf{r})$, where $\mathbf{r} = (x, y)$. This is not a radiometry problem, so feel free to introduce an undefined constant of proportionality. Note, however, that the units of $I(\mathbf{r})$ are Watts/m^2. Be explicit about the mathematical form of the point spread function.

(d) Suppose that the camera uses a one-megapixel image sensor of size 5 mm $\times$ 5 mm, and assume that the mean output g_m (in Volts) of the m^{th} detector element is linearly proportional to the energy incident on it during an exposure of time duration τ. With this information, write down an expression for the CD mapping from the image-plane irradiance to the mean detector output g_m. You may assume that the responsivity of each detector element is the same and denote it as R, with units of Volts/Joule. Do not consider noise in this problem.

(e) Write down a general expression for the overall CD mapping from object $f(\mathbf{r})$ to the mean detector output g_m, being explicit about the form of the overall point response function $h_m(\mathbf{r})$.