

WRITTEN PRELIM EXAM – FIRST DAY

Spring 2008

February 12, 2008
8:30 a.m. to 12:00 p.m.

Answer all six questions. Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your code number (NO NAMES PLEASE) and the problem number. Staple together all sheets for a given problem.

Insert your answers into the manila envelope supplied and note which problems you have answered on the outside of the envelope.

The following are some helpful items:

$$h = 6.625 \times 10^{-34} \text{ J} \cdot \text{s} = 4.134 \times 10^{-15} \text{ eV} \cdot \text{s}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$c = 3.0 \times 10^8 \text{ m/s}$$

$$k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W/K}^4 \cdot \text{m}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2 \cos^2 A - 1$$

$$\cos 2A = 1 - 2 \sin^2 A$$

$$\sin^2\left(\frac{A}{2}\right) = \frac{1}{2}(1 - \cos A)$$

$$\cos^2\left(\frac{A}{2}\right) = \frac{1}{2}(1 + \cos A)$$

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$$

$$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$$

$$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$$

$$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$$

$$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$$

$$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$$

$$\nabla \cdot (\nabla \times \mathbf{F}) = 0$$

$$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$$

$$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$$

$$\nabla \times \nabla\phi = 0$$

$$\oint_S (\mathbf{F} \cdot \mathbf{n}) da = \int_V (\nabla \cdot \mathbf{F}) d^3x$$

$$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} da$$

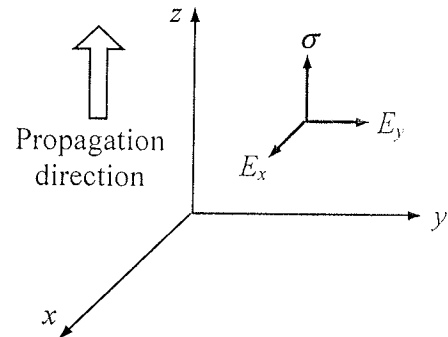
$$\oint_S \phi \mathbf{n} da = \int_V \nabla\phi d^3x$$

$$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n}) da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}] d^3x$$

$$\oint_S (\mathbf{n} \times \mathbf{F}) da = \int_V (\nabla \times \mathbf{F}) d^3x$$

Question 1 - day 1

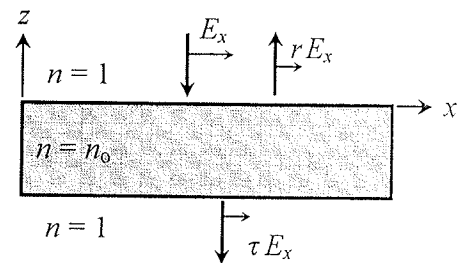
1) A homogeneous plane wave propagates in free space along the z -axis. The oscillation frequency is $\omega = 2\pi f$, the wavelength is $\lambda_0 = c/f$, the propagation constant is $k_0 = 2\pi/\lambda_0$, the speed of light is c , and the impedance of the free space is Z_0 . The only restrictions on the fields are those imposed by Maxwell's equations.



- (2 pts) a) Write expressions for the propagation vector σ , the E -field amplitude $\mathbf{E}_0 = E_{x0}\hat{x} + E_{y0}\hat{y} + E_{z0}\hat{z}$, and the H -field amplitude $\mathbf{H}_0 = H_{x0}\hat{x} + H_{y0}\hat{y} + H_{z0}\hat{z}$, consistent with Maxwell's equations.
- (2 pts) b) What conditions should E_{x0} and E_{y0} satisfy for the plane-wave to be linearly polarized?
- (2 pts) c) What conditions should E_{x0} and E_{y0} satisfy for the plane-wave to be circularly polarized?
- (2 pts) d) Let $E_{x0} = |E_{x0}| \exp(i\phi_{x0})$ and $E_{y0} = |E_{y0}| \exp(i\phi_{y0})$. Assuming $|E_{x0}| > |E_{y0}|$ and $\phi_{x0} - \phi_{y0} = 90^\circ$, what is the polarization ellipticity η of the plane-wave?
- (2 pts) e) Starting from the formula $\langle \mathbf{S}(\mathbf{r}, t) \rangle = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*)$ and showing every step of the calculation, derive an expression for the time-averaged Poynting vector $\langle \mathbf{S}(\mathbf{r}, t) \rangle$ in terms of E_{x0} and E_{y0} .
-

Question 2 - day 1

2) A monochromatic plane-wave is normally incident upon a transparent dielectric slab (i.e., real-valued refractive index n_0). The incident beam is linearly polarized, with E -field along the x -axis, as shown. The slab's reflection and transmission coefficients are r and τ , respectively.



- (2.5 pts) a) Express the average rate of flow of optical energy $\langle S_z \rangle$ (i.e., energy per unit area per unit time) in the incident beam in terms of E_x .
- (2.5 pts) b) Show that the fraction of reflected optical energy is $R = |r|^2$, while the fraction of transmitted optical energy is $T = |\tau|^2$.
- (2.5 pts) c) Use the conservation of energy to derive a relationship between R and T .
- (2.5 pts) d) Use the conservation of momentum to find the radiation pressure (i.e., time-averaged force per unit area) on the slab in terms of the incident beam's $\langle S_z \rangle$ and the slab's R and T .
-

Question 3 - day 1

- (a). Write out the time-dependent Schrödinger equation for the one-dimensional wavefunction $\Psi(x,t)$ of a particle of mass m moving along the x -axis under the action of a simple harmonic oscillator potential having an oscillator angular frequency ω . (1)
- (b). Let the variable n label the eigenstates of the harmonic oscillator, $n \geq 0$ being an integer. Write down the energy eigenvalues E_n of this potential. (1)
- (c). Let $\psi_n(x)$ label the n^{th} bound normalized eigenstate of this potential. What can be inferred about the spatial structure of the eigenstate from the value of n ? (1)
- (d) Provide a sketch of $|\psi_3(x)|^2$ as a function of x . Make sure your sketch clearly shows any trends in the wavelength (or sharpness of curvature) and the amplitude of $|\psi_3(x)|^2$ with position, or describe in words these trends if they are not obvious in your sketch. (1)
- (e). Write down an expression for a general time-dependent wavefunction $\Psi(x,t)$ of the quantum harmonic oscillator as an expansion involving the normalized eigenstates $\psi_n(x)$, the energy eigenvalues E_n , and any other variables you see fit. (1)
- (f). Define a normalized initial wavefunction for the quantum harmonic oscillator $\Psi(x,t=0) = A[\psi_0(x) + \psi_3(x)]$. Evaluate A and determine the values of the expansion coefficients in your general solution in part (e) for $t=0$. (Hint: you may assume A is real and positive.) (2)
- (g). If a particle is initially in a state with the wavefunction $\Psi(x,t=0)$ as defined above, what are the possible results of a measurement of the particle's energy, and the probability of occurrence for each possible result? (2)
- (h). Will the probabilities in part (g) vary for times $t > 0$? (1)

Question 4 - day 1

(10 pts) Suppose you are building a laser that will emit light at $\lambda=500$ nm. The gain medium and pump source is a **30-cm long gas tube** sealed with transparent windows, which is connected to a power supply to excite the gas. When turned on, the gas emits light at 500 nm. The laser will be built by inserting the 30-cm long gas tube into a stable optical resonator built from two mirrors:

- Mirror A, which is flat;
- Mirror B, which is concave and has a radius of curvature $R_B = 50$ cm.

Answer the following:

- (a) (2 pts) Choose and write down a number for an optical cavity length L that is appropriate for this laser, and allows for the construction of a stable optical cavity. You will use this value of L in the following questions.
- (b) (1 pt) What is the free spectral range of your laser (give a number)?
- (c) (1 pt) Make a simple qualitative sketch of your laser cavity, assuming the cavity axis lies along the z axis. Label the mirrors A and B. On the cavity axis, label the point where $z = 0$. Remember, $z = 0$ coincides with the location of the Gaussian waist, where the spatially varying mode radius $w(z)$ is equal to the Gaussian waist w_0 of the cavity modes.
- (d) (1 pt) Write the expression for the Gaussian wavefront radius of curvature $R_{wf}(z)$ as a function of z and the Rayleigh range z_0 .
- (e) (1 pt) What is the value of $R_{wf}(z)$ at the location of Mirror B for an optical field occupying the $TEM_{0,0}$ cavity mode? Give a number.
- (f) (2 pts) Determine the numerical value of w_0 for light of wavelength 500 nm occupying the $TEM_{0,0}$ cavity mode of your laser. HINT: you should use your answers from parts (d) and (e).
Caution: you may need to be careful with sign conventions, depending on how you set up the problem. If you don't know how to answer this, make a reasonable guess at the answer, and use this guess in the next problem.
- (g) (2 pts) What is the **maximum** value that the Gaussian radius $w(z)$ reaches **within the cavity**? Give a number. (HINT: if you don't immediately see how to answer this, first visualize or sketch $w(z)$ vs z .)

Question 5 - day 1

A lamp-diffuser combination is used to illuminate the ground. The configuration of the system is illustrated in the accompanying figure.

A small lamp (0.5 cm^2 area) that operates at 3000 K has a radiometric output of 200 W . The lamp is mounted in a perfectly-absorbing hemisphere. The lamp illuminates a **hemispheric** diffuser that can be assumed to be a perfect lambertian surface. The diffuser is spherical in shape with radius of 8 cm and the lamp is at the center of curvature of the diffuser. The distance between the lamp and ground is 3 m .

60%

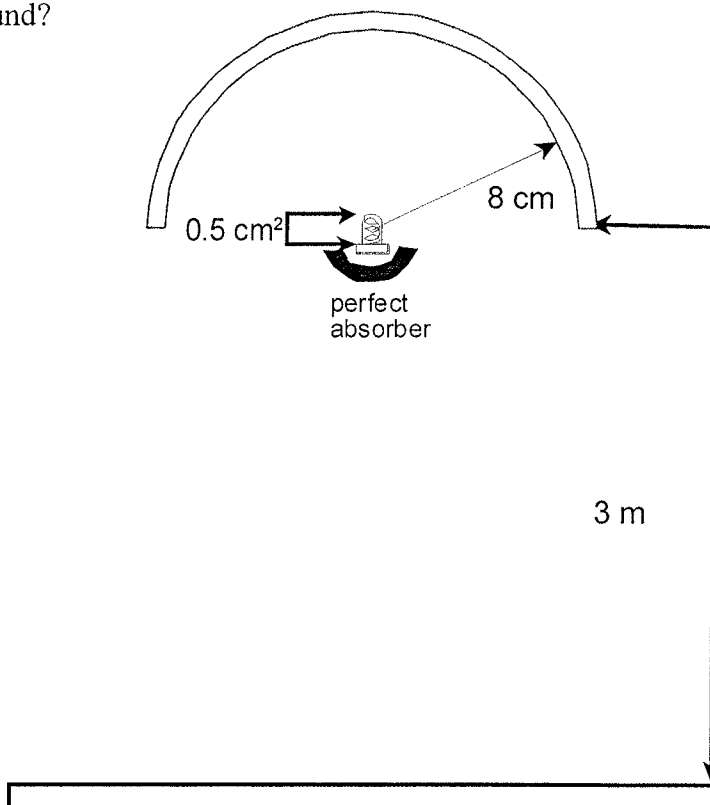
A) Determine the irradiance for a small area on the ground directly under this illumination system (ignore any effects due to obstructions caused by the absorber).

20%

B) Qualitatively describe what happens to the irradiance on the ground if the size of the diffuser is increased to a 16-cm radius?

20%

C) What happens to the irradiance on the ground if the original system is tilted 30 degrees relative to the ground?



Question 6 - day 1

A single, biconvex lens system has been developed from a lens with a focal length of 40 mm (at a wavelength of 500 nm) and diameter of 10 mm. The image at the focus of the lens for a point object that is on axis and at infinity is found to be circular with a diameter of 128 micrometers.

- a) Determine the location of the marginal focus with respect to paraxial focus.
- b) What are the values at marginal focus for all of the 3rd order aberration coefficients that can be determined from the information given? Give all values in waves.
- c) Sketch the meridional ray fan for $H=0$ and for paraxial focus. Label the values of the graph for the center and edge of the pupil. Sketch the meridional ray fan for the marginal focus case on the same graph (precise values are not critical for the marginal focus sketch).
- d) Sketch the spot diagram for marginal focus. Explain your sketch.

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Question 5 - day 1

This problem deals with the 2D Radon transform. The parts are weighted equally

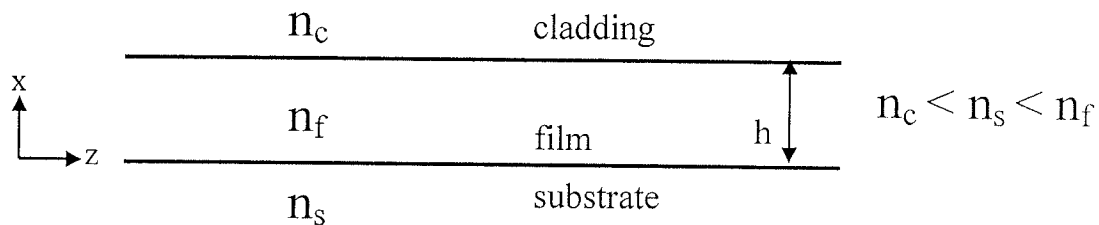
- (a) Describe and sketch an imaging system of your choice in which the data are at least approximately described by the 2D Radon transform.
- (b) Discuss any approximations or assumptions that are implicit in using the 2D Radon transform for the system chosen in part (a)
- (c) Give a mathematical expression for the 2D Radon transform, clearly explaining all symbols used. Use a diagram if needed.
- (d) State the central slice theorem
- (e) Explain one way in which the central slice theorem can be used to invert the 2D Radon transform.

Question 6 - day 1

Radar, medical ultrasound (US) imaging, and time-domain optical coherence tomography (OCT) are three very different imaging techniques. However, they have in common that they employ propagating waves and they each produce an image that is a 2D cross-section through a 3D object, where one of the image dimensions is the distance (depth) of the object point from the radiation source and the other image dimension is perpendicular to the distance direction.

For each of the three imaging techniques:

- a) (10%) State the type of radiation used to probe the object.
- b) (10%) What is the wavelength of the radiation employed? (There is a range of wavelengths utilized in these techniques, so an answer within the range is acceptable).
- c) (10%) What is the source device that produces the radiation utilized?
- d) (10%) What is the object property measured in each technique?
- e) (30%) Describe the mechanism by which the information in the depth dimension is measured in each technique. In other words, if there is a point object at some distance z , how does the imaging system determine where that point object is located?
- f) (10%) What determines the axial or depth resolution? An equation can be useful here to describe the axial resolution in a concise manner.
- g) (20%) Assume that all three imaging systems are designed to "focus" at some particular depth (distance) away from the source. What property of wave propagation sets the limit on the lateral resolution achievable, and what factors determine this limiting resolution? An equation can be a succinct way to answer the last part of this question.



Derive the *characteristic equation* for TM (Transverse Magnetic) modes for an asymmetric 3-layer slab waveguide with the geometry shown above. In your derivation, start from the field solutions below (i.e. after the continuity of H_y at the boundaries has already been applied): (10 points)

$$H_y = A e^{\gamma_s x}, \quad x < 0 \quad \text{substrate}$$

$$H_y = A \cos \kappa_f x + B \sin \kappa_f x, \quad 0 < x < h \quad \text{film}$$

$$H_y = (A \cos \kappa_f h + B \sin \kappa_f h) e^{-\gamma_c (x-h)}, \quad x > h \quad \text{cladding}$$

Hint: For TM modes:

$$\frac{\partial H_y}{\partial x} = i\omega\epsilon E_z, \quad i\beta H_y = i\omega\epsilon E_x, \quad \frac{\partial E_z}{\partial x} + i\beta E_x = i\omega\mu_0 H_y$$

Question 6 - Day 1

- a) Sketch the attenuation (in dB/km) for a typical single-mode silica optical fiber (used in modern Telecommunications) as a function of wavelength within the wavelength range of 800 nm - 1800 nm. The sketch should show the distinct features of the attenuation spectrum and give an approximate value for the minimum attenuation. **(6 points)**
- b) Briefly discuss the two main intrinsic attenuation mechanisms fundamentally limiting the minimum attenuation in a typical single-mode silica optical fiber. **(2 points)**
- c) Briefly discuss and identify in your graph the major extrinsic cause for attenuation and how it has recently been eliminated. **(2 points)**

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$e = 1.6 \times 10^{-19} \text{ C}$	$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$
$c = 3.0 \times 10^8 \text{ m/s}$	$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$
$k_B = 1.38 \times 10^{-23} \text{ J/K}$	$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$
$\sigma = 5.67 \times 10^{-8} \text{ W/K}^4 \cdot \text{m}^2$	$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$
$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$	$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$
$\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$	$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$
$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\nabla \cdot (\nabla \times \mathbf{F}) = 0$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$
$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$	$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$
$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$	$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$
$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$	$\nabla \times \nabla\phi = 0$
$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$	$\oint_S (\mathbf{F} \cdot \mathbf{n}) da = \int_V (\nabla \cdot \mathbf{F}) d^3x$
$\sin 2A = 2 \sin A \cos A$	$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} da$
$\cos 2A = 2 \cos^2 A - 1$	$\oint_S \phi \mathbf{n} da = \int_V \nabla\phi d^3x$
$\cos 2A = 1 - 2 \sin^2 A$	$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n}) da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}] d^3x$
$\sinh x = \frac{1}{2} (e^x - e^{-x})$	$\oint_S (\mathbf{n} \times \mathbf{F}) da = \int_V (\nabla \times \mathbf{F}) d^3x$
$\cosh x = \frac{1}{2} (e^x + e^{-x})$	

Question 7 - day 2

A Petzval objective consists of two separated positive thin lenses. The separation of the two lenses must be less than the focal length of the first element.

A zoom lens is constructed using two thin lenses in a Petzval configuration. Both thin lenses have a focal length of 200 mm. The object is at infinity.

Plot the system focal length and the back focal distance as a function of the element separation over the entire available zoom range. Be sure to note the maximum and minimum focal lengths that are possible with this two-element zoom lens.

Question 8 - day 2

Problem: There are four parts to this problem. The first is related to a singlet thin lens and the others are related to a doublet thin lens. Both lenses, singlet and doublet, have a focal length of 100 mm and work at $F/2$. The object is at infinity unless otherwise stated.

- a) (30%) For the singlet lens, what is the difference in longitudinal focal length for the F and C wavelengths? The lens is made out of N-BK7(517642) glass.
- b) (40%) Design a thin lens achromatic doublet from N-BK7(517642) and F2(620364). What is the power of each element?
- c) (15%) An object is located 200 mm in front of the thin achromat lens, what is the image location and transverse magnification?
- d) (15%) At this object location (part c) what is the difference in longitudinal image location between the F and C wavelengths?

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Question 9 - day 2

a.) (2.5 pts) Draw and label the essential elements of a Michelson interferometer.

b.) Where are the fringes localized in a Michelson interferometer if

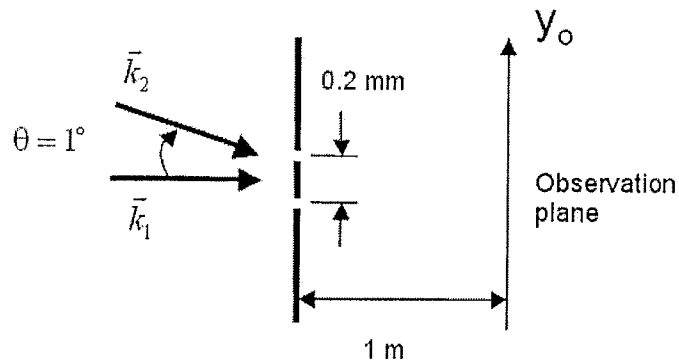
i) (2.5pts) I have circular fringes?

ii) (2.5pts) I have essentially straight fringes?

c.) (2.5pts) I adjust a Michelson interferometer to have white light fringes. Are the fringes circular or straight? Explain.

Question 10 - day 2

Two plane waves illuminate a Young's double pinhole interferometer as shown. $\lambda_1 = 532$ nm and $\lambda_2 = 632.8$ nm. The angle between the plane waves is 1 degree. \vec{k}_1 is normally incident onto the aperture plane. The pinhole spacing is 0.2 mm, and the distance from aperture plane to observation plane is 1 m. Ignore finite size of the pinholes, and assume that the observation region is limited to a small area around the axis. You may assume that irradiances reaching the observation plane from both pinholes and both wavelengths are equal and constant over the observation area. State any additional assumptions that you make.



- a.) (2.5pts) Derive an expression for the observation-plane irradiance pattern resulting from \vec{k}_1 by itself. What is the minimum fringe period?
- b.) (2.5pts) Derive an expression for the observation-plane irradiance pattern resulting from \vec{k}_2 by itself. What is the minimum fringe period?
- c.) (2.5pts) Derive an expression for the total irradiance.
- d.) (2.5pts) What is the period of the modulation in fringe irradiance due to the combination of the two waves?

Question 11 - day 2

- (a) (3 points) schematically plot the absorption and reflection spectra of NaCl, an insulator, as a function of frequency near the phonon resonance.
- (b) (2 points) Write down the expressions for conservations of energy and momentum for Stokes and anti-Stokes scattering processes, with ω_i and ω_s being the frequencies of the incident and scattered photons, \mathbf{q}_i and \mathbf{q}_s being the wavevectors of incident and scattered photon; and Ω and \mathbf{K} being the phonon frequency and wavevector, respectively.
- (c) (2 points) Use these conservation expressions and obtain an equation relating the magnitude of the phonon wavevector K as a function of q_i and the angle θ between \mathbf{q}_i and \mathbf{q}_s .
- (d) (2 points) If the phonon is acoustic, use the result in part (c) and obtain a relation for the acoustic phonon frequency as a function of speed of light c , speed of sound v_s , and ω_i .
- (e) (1 point) What is the typical value of the acoustic phonon energy? You may use the relation in part (d) to get this typical; value.

Question 11 - day 2

Parts are equally weighted

- (a) Define and explain the term *radiance* and give its units.
- (b) What equation describes the propagation of the radiance in material media? What physical processes are described by the various terms in this equation
- (c) What is a Lambertian radiator?
- (d) Suppose a square Lambertian radiator of side L is placed in the plane $z = 0$ and imaged to the plane $z = 4f$ by an ideal thin lens of focal length f and diameter D . Ignoring diffraction, explain how you would compute the radiance in the image plane and illustrate your discussion with appropriate sketches. You should explain what basic principles you are invoking, but little or no math is required.
- (e) Given the radiance in the image plane of any optical imaging system, how would you compute the irradiance? How would you compute the output of a detector in that plane?

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Question 12 - day 2

Parts are equally weighted

A beam of light with irradiance $1 \mu\text{W}/\text{cm}^2$ is normally incident on the large face of a slab of undoped silicon of dimension $1 \text{ mm} \times 1 \text{ mm} \times 0.1 \text{ mm}$. Answer each of the questions below *twice*, once assuming that the wavelength of the light is 500 nm and once assuming it is $5 \mu\text{m}$.

- (a) How many photons per second are incident on the silicon?
- (b) What fraction of the photons is reflected?
- (c) What fraction is absorbed in the silicon?
- (d) What properties of the silicon determine the equilibrium concentration of holes and electrons when the material is illuminated?
- (e) What properties of the silicon determine its conductivity when the material is illuminated?

(For parts d and e, define or explain all terms you use.)

Potentially useful information:

The charge on an electron is 1.6×10^{-19} Coulomb.

The bandgap of silicon at 300 K is 1.12 eV.

The index of refraction of silicon in the visible and infrared is approximately 3.5

The intrinsic carrier concentration of silicon at 300 K is about 10^{10} cm^{-3} .

Planck's constant times the speed of light is 1.2398 in units of eV- μm .

Question 12 - day 2

- (a) (2 points) Schematically plot the absorption spectrum of a semiconductor near its bandgap as a function of photon energy in the linear regime.
- (b) (2 points) Schematically plot the absorption spectrum as a function of photon energy for an electrically pumped semiconductor near its bandgap and assume that the pumping level is large enough that the semiconductor experiences gain.
- (c) (4 points) Plot the carrier density, n and the number of photons, N in the lasing mode of a semiconductor laser, as a function of pumping rate, resulting from the rate equations.
- (d) (2 points) What is gain clamping in semiconductor lasers?