MINOR WRITTEN PRELIM EXAM

Spring 2009

February 18, 2009
8:30 a.m. to 12:00 p.m.

Please answer all questions.

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your name and the problem number. Staple together all sheets for a given problem.

Insert your answers and this exam into the manila envelope supplied. The exam questions will be returned to you along with your answers after they have been graded.

The following are some helpful items:

$h = 6.625 \times 10^{-34}\ \mathrm{J \cdot s} = 4.134 \times 10^{-15}\ \mathrm{eV \cdot s}$

$e = 1.6 \times 10^{-19}\ \mathrm{C}$

$c = 3.0 \times 10^{8}\ \mathrm{m/s}$

$k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$

$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$2 \cos A \cos B = \cos(A - B) + \cos(A + B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$

$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

$\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

$\nabla(\mathbf{F} \cdot \mathbf{G}) = (\mathbf{F} \cdot \nabla)\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + \mathbf{F} \times (\nabla \times \mathbf{G}) + \mathbf{G} \times (\nabla \times \mathbf{F})$

$\nabla \cdot (\phi\mathbf{F}) = \phi(\nabla \cdot \mathbf{F}) + \mathbf{F} \cdot \nabla\phi$

$\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \nabla \times \mathbf{F} - \mathbf{F} \cdot \nabla \times \mathbf{G}$

$\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$\nabla \times (\phi\mathbf{F}) = \phi(\nabla \times \mathbf{F}) + \nabla\phi \times \mathbf{F}$

$\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - \mathbf{G}(\nabla \cdot \mathbf{F}) + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F}$

$\nabla \times \nabla\phi = 0$

$\oint_S (\mathbf{F} \cdot \mathbf{n})\, da = \int_V (\nabla \cdot \mathbf{F})\, d^3x$

$\oint_C \mathbf{F} \cdot d\ell = \int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, da$

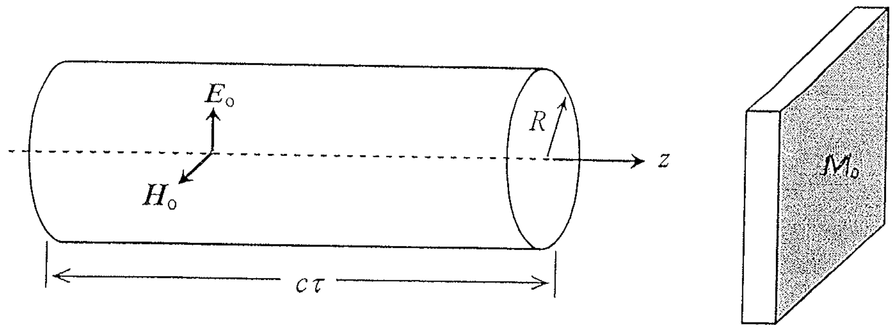
$\oint_S \phi\mathbf{n}\, da = \int_V \nabla\phi\, d^3x$

$\oint_S \mathbf{F}(\mathbf{G} \cdot \mathbf{n})\, da = \int_V [\mathbf{F}(\nabla \cdot \mathbf{G}) + (\mathbf{G} \cdot \nabla)\mathbf{F}]\, d^3x$

$\oint_S (\mathbf{n} \times \mathbf{F})\, da = \int_V (\nabla \times \mathbf{F})\, d^3x$

521/MINOR

A pulse of light, having duration τ and a circular cross-section with radius R, propagates in the free space along the z-axis, as shown. The pulse is long and wide, so that diffraction effects can be ignored. (The beam may be treated effectively as a monochromatic plane-wave albeit one that has a finite duration and a finite cross-sectional area.) For simplicity, assume the beam is linearly polarized, with E- and H-field amplitudes specified as E_o (volt/meter) and H_o (ampere/meter). The impedance of the free-space is $Z_0 \approx 377\,\Omega$.



(4 pts) a) Find the pulse's time-averaged Poynting vector $\langle S \rangle$, the total energy content of the pulse, and its total momentum.

(3 pts) b) Let the pulse be reflected from a massive mirror (i.e., mass $M_o \rightarrow \infty$) whose reflectivity is 100%. What is the mechanical momentum acquired by the mirror after the entire pulse has been reflected? How much kinetic energy does the mirror acquire in this process? If the mirror happens to have a *finite* mass M_o, where does its kinetic energy come from?

(3 pts) c) Instead of a perfect reflector, assume now that the object of mass M_o is a perfect *absorber*. Once the pulse has been fully absorbed, what will be the mechanical momentum of the absorber? What will be its kinetic energy? Where does this kinetic energy come from? What happens to the remaining energy of the light pulse?

MINOR

Comprehensive Question

OPTI-502 S'09

An object is located 75 mm to the left of the first element of a doubly-telecentric system. The conjugate image is located 10 mm to the right of the second element of the system. The image size is half the object size.

Using thin lenses in air, determine the system layout by providing the focal lengths, spacings and stop position.

MINOR

Comprehensive Examination Spring 2009

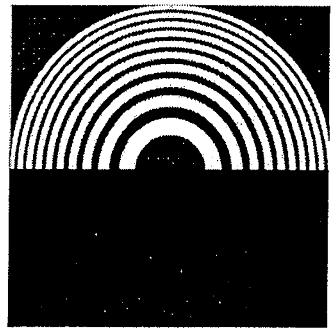
OPTI 503 Mathematics for Optics

(Each of the five parts is worth two points.)

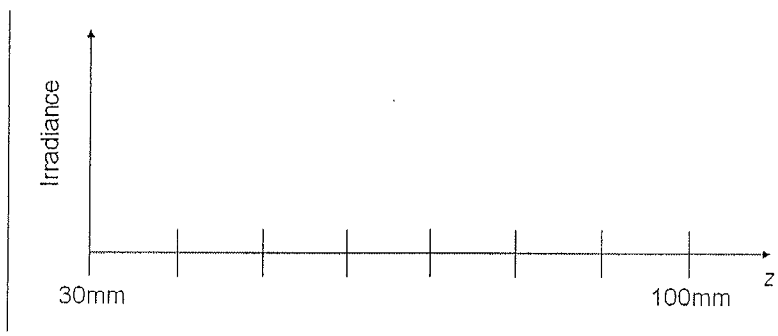
Given the matrices $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

a. What is the rank of $\mathbf{A}$? The trace of $\mathbf{A}$?
b. What is the value of $\mathbf{B} = \mathbf{IA}$?
c. Is $\mathbf{C} = \mathbf{I} + \mathbf{A}$ singular? (Explain your reasoning.)
d. What is the determinant of $\mathbf{A}$?
e. What are the eigenvalues and eigenvectors of $\mathbf{A}$?

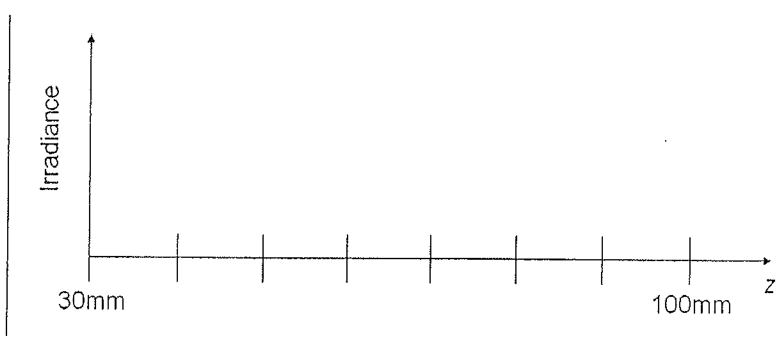
A simple binary amplitude (0 or 1 transmission) Fresnel zone plate is fabricated as shown below, with one-half of the zone plate covered by an opaque mask. White areas indicate unity transmission. The plate and mask are illuminated by an on-axis plane wave with $\lambda = 500$nm and irradiance 100W/m^2. The zone plate is designed with ten open zones, and its primary focus (without the half-plane opaque mask) is designed for a primary focus at a distance of 100mm behind the plate. The optical axis is defined normal to the plate at the center of the rings.



1.) (5pts) On the graph below, sketch the on-axis irradiance from a distance of 30mm behind the plate to 100mm behind the plate. Indicate positions of maxima and minima (if any), and indicate irradiance values at those locations. State any assumptions that you make.



2.) (5pts) Replace the opaque half-plane mask with a phase plate that alters the phase of transmitted light relative to the uncovered section of the zone plate by π radians in transmission, but does not affect amplitude. On the graph below, sketch the on-axis irradiance from a distance of 30mm behind the plate to 100mm behind the plate. Indicate positions of maxima and minima (if any), and indicate irradiance values at those locations. State any assumptions that you make.



MINOR

Prelim Question for Optics Minor Student
OPTI 508
Matthew Kupinski

Suppose a light source is randomly emitting photons at an average rate of ν photons/time. The photons are emitted independently, and in *each* time interval dt either 0 or 1 photons are emitted. The probability p of emitting a photon during interval dt is $p = \nu dt$.

1. (10%) What is the probability that two photons are emitted in two consecutive time intervals?
2. (10%) What is the probability that only one photon is emitted in two consecutive time intervals?
3. (10%) What is the probability that at least one photon is emitted in two consecutive time intervals?
4. (10%) What is the probability that at least one photon is emitted in 10 consecutive time intervals?

Now consider a total exposure time T. (Also see the aside below about binomial random variables.)

5. (15%) Write a probability for m, the total number of photons emitted in time T.
6. (15%) Now consider dt to be very small compared to T. Because dt is small, the actual probability of a photon being emitted in the interval dt is also small. Under these conditions, what probability function describes m, the total number of photons emitted in time T. (Do not derive the probability law; simply state the name of the law and give an equation for it.)
7. (15%) Using the probability law given in part 5, what is the mean and variance of the random variable m? What is the mean and variance assuming the probability law in part 6? Explain why both answers are consistent with one another.
8. (15%) We now add a second source with the same average rate ν in photons/time. We label the number of photons emitted from the first source and the second source as m_1 and m_2, respectively. What probability law describes the total number of photons emitted from both of these sources in time T (i.e., the probability law on m_1+m_2)? What is the mean and variance of m_1+m_2?

Aside on the binomial distribution: Consider an experiment where a coin is flipped a total of N times. The probability of obtaining a heads in each flip is given by p and each flip is independent from one another. The probability law governing the total number of heads m obtained in N trails is given by the binomial distribution,

$$P_N(m) = \left(\frac{N!}{m!(N-m)!}\right) p^m (1-p)^{N-m}$$

MINOR

Prelim Question for Optics Minor Student
OPTI 508
Matthew Kupinski

Suppose a light source is randomly emitting photons at an average rate of ν photons/time. The photons are emitted independently, and in *each* time interval dt either 0 or 1 photons are emitted. The probability p of emitting a photon during interval dt is $p = \nu dt$.

1. (10%) What is the probability that two photons are emitted in two consecutive time intervals?
2. (10%) What is the probability that only one photon is emitted in two consecutive time intervals?
3. (10%) What is the probability that at least one photon is emitted in two consecutive time intervals?
4. (10%) What is the probability that at least one photon is emitted in 10 consecutive time intervals?

Now consider a total exposure time T. (Also see the aside below about binomial random variables.)

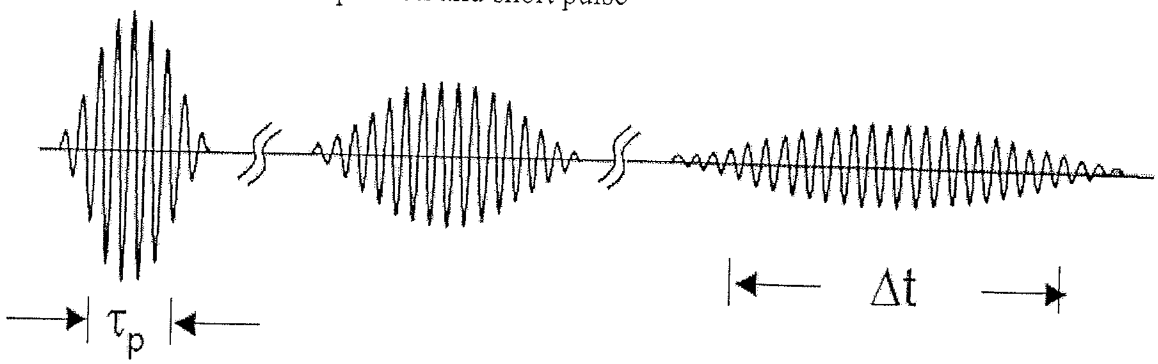
5. (15%) Write a probability for m, the total number of photons emitted in time T.
6. (15%) Now consider dt to be very small compared to T. Because dt is small, the actual probability of a photon being emitted in the interval dt is also small. Under these conditions, what probability function describes m, the total number of photons emitted in time T. (Do not derive the probability law; simply state the name of the law and give an equation for it.)
7. (15%) Using the probability law given in part 5, what is the mean and variance of the random variable m? What is the mean and variance assuming the probability law in part 6? Explain why both answers are consistent with one another.
8. (15%) We now add a second source with the same average rate ν in photons/time. We label the number of photons emitted from the first source and the second source as m_1 and m_2, respectively. What probability law describes the total number of photons emitted from both of these sources in time T (i.e., the probability law on m_1+m_2)? What is the mean and variance of m_1+m_2?

Aside on the binomial distribution: Consider an experiment where a coin is flipped a total of N times. The probability of obtaining a heads in each flip is given by p and each flip is independent from one another. The probability law governing the total number of heads m obtained in N trails is given by the binomial distribution,

$$P_N(m) = \left(\frac{N!}{m!(N-m)!}\right) p^m (1-p)^{N-m}$$

4 MINOR

Problem 1: Chromatic dispersion and short pulse



A Gaussian pulse has a time-bandwidth product given by $\Delta f_p \tau_p \approx 0.44$, where Δf_p is the pulse bandwidth in Hz and $\tau_p = 10$ ps (10^{-12}s) is the pulse width in time. The central wavelength of the pulse is 1.5 micron. The pulse propagates inside a type of glass with the Sellmeier equation given by

$$n^2 = 1 + 0.7\lambda^2 .$$

Here λ is in unit of micron.

(a) What is the phase velocity of the pulse (m/s)?
(b) What is the group velocity of the pulse (m/s)?
(c) What is the dispersion coefficient in (s/m-nm)?
(d) At what distance does it take for the pulse to spread to $\Delta t = 20$ ps?

509
MINOR

A radiometer is being designed to monitor the temperature of an oven used in the fabrication of ceramics. The oven operates at 1000 K but must be shut down if the temperature differs from this temperature by more than 1 K. The goal is to be able to determine this 1-K change with a confidence of 99%. The requirement means that an SNR of 100 is needed on the change in temperature.

60% a) Determine the NEP (noise equivalent power) needed to achieve the 1-K determination for the detector that is to be used as part of the radiometer.

Ignore all losses in the optical system and assume the radiometer is overfilled. Space limitations mean that the sensor has an entrance aperture area of 1 cm^2 when viewing through the window of the oven. The allowed detector size and focal length leads to a full-field FOV that gives a sensor solid angle of $1H10^{-5}$ sr. The radiometer collects light over the entire spectral range of the oven's emission.

20% b) The radiometer is also used to measure an oven operating at 500 K. Will the radiometer designed in part (a) satisfy the same SNR requirements in this case? Explain.

20% c) The oven is found to have an emissivity that is 0.8. Will the radiometer designed in part (a) satisfy the same SNR requirements? Explain.

MINOR

OPTI512R

A plane wave is normally-incident on an aperture that is described by the transmission function

$$t_o(x_o, y_o) = \sum_{n=-N/2}^{N/2} \sum_{m=-N/2}^{N/2} \mathrm{rect}\left(\frac{x_o - nd}{a}, \frac{y - md}{a} \right)$$

A. Approximately where do the Fresnel and Fraunhofer regions begin for this aperture? Your answer should be expressed in terms of the parameters of the problem, with appropriate assumptions about the observation locations. If you don't remember the equations, sketch the picture that shows how you can deduce the regions.

B. Consider the case where a □ λ and d □ λ. Write the Fresnel and Fraunhofer diffraction patterns and discuss the differences between them.

>> >> $a \ll \lambda$?

Fresnel Diffraction Formula:

$$u(x_2, y_2) = \frac{\exp\{jkz_{12}\}}{j\lambda z_{12}} \exp\left[j\frac{\pi}{\lambda z_{12}}(x_2^2 + y_2^2) \right] \mathcal{F}\left\{ u(x_1, y_1) \exp\left[j\frac{\pi}{\lambda z_{12}}(x_1^2 + y_1^2) \right] \right\} \Bigg|_{\substack{\xi = x_2/\lambda z_{12} \\ \eta = y_2/\lambda z_{12}}}$$

$$= \frac{\exp\{jkz_{12}\}}{j\lambda z_{12}} \iint u(x_1, y_1) \exp\left\{ j\frac{\pi}{\lambda z_{12}} \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 \right] \right\} dx_1 dy_1$$

Fraunhofer Diffraction:

$$u(x_2, y_2) = \frac{\exp\{jkz_{12}\}}{j\lambda z_{12}} \exp\left[j\frac{\pi}{\lambda z_{12}}(x_2^2 + y_2^2) \right] \mathcal{F}\{ u(x_1, y_1) \} \Bigg|_{\substack{\xi = x_2/\lambda z_{12} \\ \eta = y_2/\lambda z_{12}}}$$

MINOR

Comprehensive Question

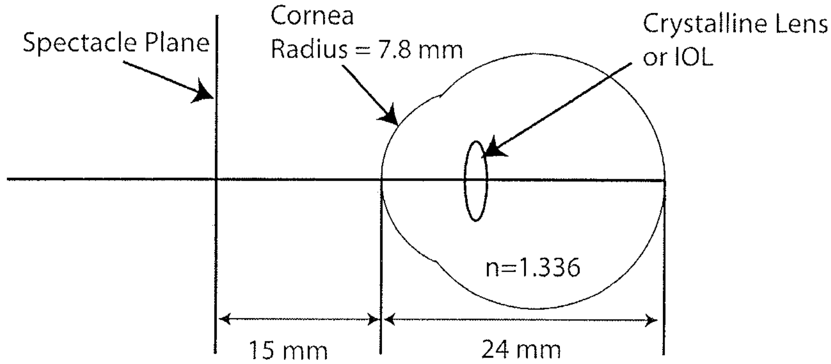
OPTI-517 S'09

A) A concave spherical mirror is illuminated from a point object at infinity and its radius of curvature is 1000 mm. If the aperture diameter is 100 mm, how much spherical aberration is introduced by the mirror in the reflected light beam? (35/100 points)

B) If the aperture stop is located at the center of curvature of the mirror in part a), how much coma and astigmatism are present in the images for a 1 degree semi-field of view? Make a drawing and explain. (15/100 points)

C) Explain how a Schmidt camera works and make a drawing showing all components, the marginal ray, the chief ray and the stop aperture.(15/100 points)

D) If a field flattener lens is used to flatten the field in the Schmidt camera, what should be the optical power of the field lens? (35/100 points)



Cataracts are an opacification of the crystalline lens that obscures vision. Today, cataractous crystalline lenses are surgically removed and replaced with an artificial intraocular lens (IOL). This problem investigates some of the imaging properties of the eye and IOLs. For simplicity, assume that the cornea is a single refractive surface with radius 7.8 mm, the refractive index of the eye is 1.336 and the length of the eye is 24 mm from the cornea to the retina. The figure above illustrates the simplified eye model.

A. Prior to the invention of IOLs, the crystalline lens was removed in cataract surgery and nothing was inserted in its place. This procedure left the cornea as the only refracting element in the eye. Based on the eye model above, what power spectacle lens would be needed to bring a distant object into focus on the retina? The spectacle lens is placed 15mm in front of the eye and you can assume it is a thin lens.
B. A contact lens (i.e. a lens placed in contact with the cornea) can also be used to correct this eye. What power contact lens is needed to bring the distant object into focus?
C. These spectacle and contact lenses are bulky and uncomfortable. Suppose a 20 diopter IOL is used instead. Where inside the eye does the lens need to sit to bring distant objects into focus. Again, assume a thin lens for the IOL.
D. In the young crystalline lens, then power can be changed to bring near objects into focus. One downside of conventional IOLs is that they have fixed power, leaving no ability to focus up close. Much research has been done on "accommodating" IOLs which can change power. For the IOL in part C, how much would its power need to change by to bring an object 330 mm from the cornea into focus?
E. An alternative accommodating IOL is a lens with a fixed power, but one which can move axially within the eye. How far and in what direction would the lens in part C need to move to bring an object at infinity into focus?

536/M100R

The spatial scale (spatial resolution) of imaging systems is determined by a variety of factors relating to the radiation source used, the characteristics of the imaging system itself, and/or the physical properties of the object.

Choose any five imaging systems from the list below. For each, describe at least two factors that affect the spatial resolution for that imaging technique. Also, describe what spatial scale is resolvable by providing an estimate for the separation of two just-resolvable points. If the imaging system provides 3D spatial information, discuss factors or differences in the resolution in the different directions.

a. The human eye viewing this question printed on a piece of paper.
b. A digital camera recording an image of your friend sitting across the dinner table
c. A standard optical microscope with a 10X eyepiece and 10X objective viewing a standard histology slide.
d. A confocal microscope with a 10 X objective viewing a thick piece of tissue.
e. A transmission electron microscope viewing a thin sample?
f. A 1m telescope without adaptive optics viewing features on the moon (400,000 km away).
g. A 1m telescope with adaptive optics viewing features on the moon.
h. An MRI instrument imaging your head.
i. A film based X-ray mammography system.
j. A radar system at your local airport imaging arriving airplanes.

The five parts chosen will carry equal weight.

OPTI 550, Spring 2009 Comprehensive

Use the information in the table below to answer the questions below for a satellite sensor operating at a wavelength of 600 nm.

The incident solar spectral irradiance normal to the atmosphere and at the top of the atmosphere is 150 W/(m^2) in the spectral band of interest for this sensor. All of the light incident on the sensor's telescope is collected by the detector. The radius of the telescope objective is 95 mm and the detector area is 225 :m^2. The sensor sees an area of the ground of 900 m^2 at a height of 705 km.

A preflight, laboratory calibration using a spherical integrating source gives an output of 205 DN for an input spectral radiance of 5.00 W/(m^2 sr). The system is an 8-bit sensor. The system reports 5 DN when covered.

60% A) Estimate the output of the sensor in DN when it is viewing the ground at a nadir view angle of 30 degrees and a solar zenith angle of 60 degrees (yes, the table is for an angle of 50 degrees). State all assumptions.

20% B) What will happen to the SNR of the above sensor when the sensor is moved to a higher orbit? Explain.

20% C) What will happen to the SNR of the sensor if the wavelength of the spectral band is shifted to a longer wavelength? Explain.

Zenith Angle	Trans-mittance	BRF for 50-degree sun angle
30	0.794	0.20
40	0.770	0.25
50	0.733	0.30
60	0.670	0.35

630/Minor

OPTI 630 question

One can divide the most relevant optical properties in biomedical imaging into 4 categories: let's call them Absorption, Fluorescence, Refractive Index and Polarization. Spatial fluctuations of these properties can be exploited to produce contrast in microscopy.
For each of the 4 properties, explain one mechanism that can lead to structural contrast in microscopy images from biological samples. For example, spatial fluctuations in the refractive index can lead to scattering which is a mechanism used in darkfield microscopy. Provide more detail on this example mechanism and describe one mechanism for each of the other 3 properties in the same fashion.
For the examples you used above, describe the optical components starting from light source and ending with the detector or eye that are needed to produce an image. Work on this last part of the question until the allotted time for the OPTI630 question has expired. It probably is easiest to start with Absorption first. You can describe the optical components with a diagram and noting the function of each component.