MINOR WRITTEN PRELIM EXAM

Spring 2010

March 10, 2010
8:30 a.m. to 12:00 p.m.

Please answer all questions.

Start each answer on a new page.

In the upper right hand corner of each sheet you hand in, put your name and the problem number. Staple together all sheets for a given problem.

Insert your answers and this exam into the manila envelope supplied. The exam questions will be returned to you along with your answers after they have been graded.

The following are some helpful items:

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$e = 1.6 \times 10^{-19}\ \mathrm{C}$

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$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/K^4 \cdot m^2}$

$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$

$\mu_0 = 1.26 \times 10^{-6}\ \mathrm{H/m}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

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$\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$

$\nabla\phi\psi = \phi\nabla\psi + \psi\nabla\phi$

$\nabla\cdot(\mathbf{F} + \mathbf{G}) = \nabla\cdot\mathbf{F} + \nabla\cdot\mathbf{G}$

$\nabla\times(\mathbf{F} + \mathbf{G}) = \nabla\times\mathbf{F} + \nabla\times\mathbf{G}$

$\nabla(\mathbf{F}\cdot\mathbf{G}) = (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F})$

$\nabla\cdot(\phi\mathbf{F}) = \phi(\nabla\cdot\mathbf{F}) + \mathbf{F}\cdot\nabla\phi$

$\nabla\cdot(\mathbf{F}\times\mathbf{G}) = \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G}$

$\nabla\cdot(\nabla\times\mathbf{F}) = 0$

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$\nabla\times\nabla\phi = 0$

$\oint_S(\mathbf{F}\cdot\mathbf{n})\,da = \int_V(\nabla\cdot\mathbf{F})\,d^3x$

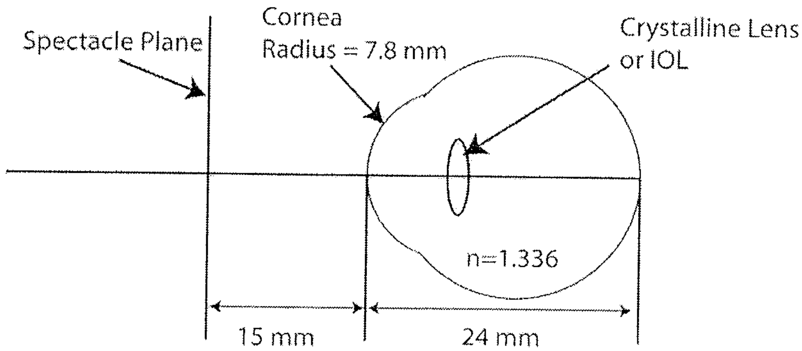
$\oint_C \mathbf{F}\cdot d\ell = \int_S(\nabla\times\mathbf{F})\cdot\mathbf{n}\,da$

$\oint_S \phi\mathbf{n}\,da = \int_V \nabla\phi\,d^3x$

$\oint_S \mathbf{F}(\mathbf{G}\cdot\mathbf{n})\,da = \int_V[\mathbf{F}(\nabla\cdot\mathbf{G}) + (\mathbf{G}\cdot\nabla)\mathbf{F}]\,d^3x$

$\oint_S(\mathbf{n}\times\mathbf{F})\,da = \int_V(\nabla\times\mathbf{F})\,d^3x$

OPTI 535 Spring Prelim Exam 2010



Cataracts are an opacification of the crystalline lens that obscures vision. Today, cataractous crystalline lenses are surgically removed and replaced with an artificial intraocular lens (IOL). This problem investigates some of the imaging properties of the eye and IOLs. For simplicity, assume that the cornea is a single refractive surface with radius 7.8 mm, the refractive index of the eye is 1.336 and the length of the eye is 24 mm from the cornea to the retina. The figure above illustrates the simplified eye model.

A. (20%) Prior to the invention of IOLs, the crystalline lens was removed in cataract surgery and nothing was inserted in its place. This procedure left the cornea as the only refracting element in the eye. Based on the eye model above, what power spectacle lens would be needed to bring a distant object into focus on the retina? The spectacle lens is placed 15mm in front of the eye and you can assume it is a thin lens.
B. (20%) A contact lens (i.e. a lens placed in contact with the cornea) can also be used to correct this eye. What power contact lens is needed to bring the distant object into focus?
C. (20%) These spectacle and contact lenses are bulky and uncomfortable. Suppose a 20 diopter IOL is used instead. Where inside the eye does the lens need to sit to bring distant objects into focus. Again, assume a thin lens for the IOL.
D. (20%) In the young crystalline lens, then power can be changed to bring near objects into focus. One downside of conventional IOLs is that they have fixed power, leaving no ability to focus up close. Much research has been done on "accommodating" IOLs which can change power. For the IOL in part C, how much would its power need to change by to bring an object 330 mm from the cornea into focus?
E. (20%) An alternative accommodating IOL is a lens with a fixed power, but one which can move axially within the eye. How far and in what direction would the lens in part C need to move to bring an object at 330 mm into focus?

OPTI 541 Comprehensive Exam Problem. Spring 2010

1. Assume a HeNe laser transition at 632.8 nm, 0.5 Torr pressure, room temperature, and with 150 nsec radiative lifetime. Calculate and show what the dominant linewidth broadening mechanism would be (natural, collisional, or doppler broadening).

2. Explain the figure shown on the next page of absorption coefficient in CO_2 as a function of pressure: which particular broadening mechanism dominates in which region, and describe why the absorption coefficient doesn't continue to increase with pressure even though number density of CO_2 is increased at higher pressures?

3. A CO_2 laser has a TEM_{00} mode output. The laser is 1 m long, has a flat output mirror and a 5 m totally reflecting mirror.

a. What is the axial (longitudinal) mode spacing?
b. Where is this cavity on a stability diagram?
c. What is the spot size at the output mirror?
d. What is the far field spot size, 1 km distant from the laser?

$$2\alpha(\omega) = N\sigma(\omega) \qquad \sigma(\omega) = A_{21}\frac{\lambda^2}{4}g(\omega) \qquad w^2(z) = w_0^2\left(1+\left(\frac{z}{z_0}\right)^2\right) \qquad R(z) = z\left(1+\left(\frac{z_0}{z}\right)^2\right)$$

$$z_0 = \frac{\pi w_0^2}{\lambda} \qquad \theta = \frac{\lambda}{\pi w_0} \qquad \Delta\nu = \nu_0\sqrt{\frac{8kT\ln 2}{Mc^2}} \qquad \Delta\nu = N\sigma\sqrt{\frac{8kT}{\pi M}} \qquad \sigma = \frac{\pi}{4}D^2$$

$$\Delta\nu = \frac{214}{\lambda(\text{microns})}\sqrt{\frac{T}{M}}\cdot 10^6\ \text{Hz} \qquad \Delta\nu = \frac{A_{21}}{2\pi}\ \text{Hz} \qquad \Delta\nu = \frac{5}{4}\cdot 10^8\frac{D^2P}{\sqrt{MT}}\ \text{Hz}$$

Where D ($\mathring{A}$); P (Torr); T (K); M (AMU)

$k = 1.38\cdot 10^{-23}$J/K 1 AMU $= 1.66\cdot 10^{-27}$kg He=4 AMU Ne=20 AMU

$M_{proton}c^2 \simeq 1\ GeV$ $M_{electron}c^2 \simeq 0.5\ MeV$

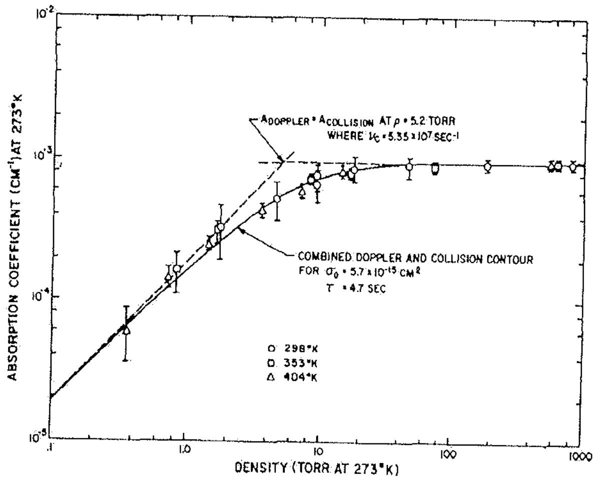


Fig. 1. **Absorption coefficient of CO_2 gas for the 10.6-μ**

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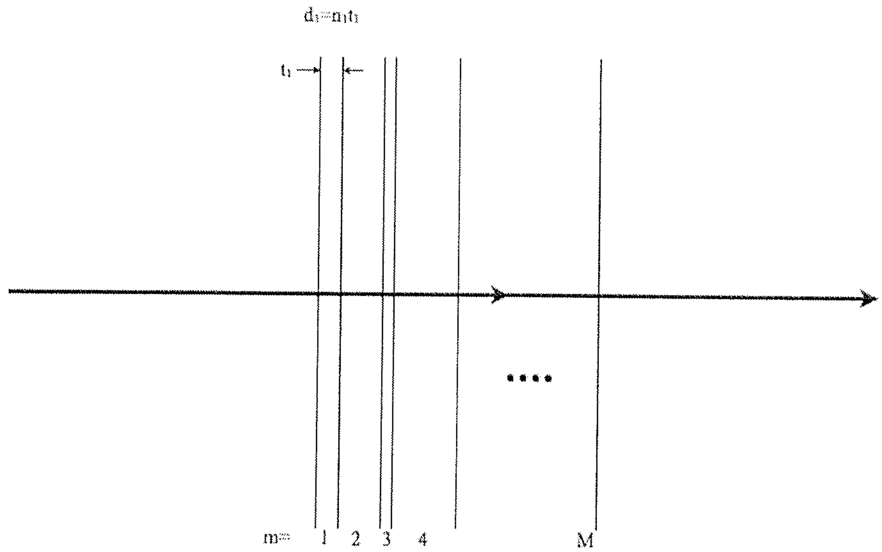
OPTI 508: Probability and Statistics in Optics



Consider a monochromatic beam traveling perpendicularly through the atmosphere. Each layer of the atmosphere has an index of refraction n_m and a thickness t_m associated with it. Thus, the net optical path length between the source and detector caused by the atmosphere is,

$$d_{net} = d_1 + d_2 + \ldots d_M = \sum_{m=1}^{M} n_m t_m.$$

The layers of the atmosphere are changing so let us consider all the indices of refraction n_m's to be random. We also consider all the thicknesses t_m's to be random as well. The n_m's are all identically distributed and independent from one another. Similarly, the t_m's are also independent and identically distributed. Finally, all the n_m's are independent from all the t_m's. Let us further state that the mean index of refraction is $\overline{n}$ and the variance is σ_n^2. The mean thickness of a layer of the atmosphere is $\bar{t}$ and the variance is σ_t^2.

1. (50%) What is the mean and variance of the net optical path length d_{net}? (Show all work)
2. (20%) Assume that M, the number of layers in the atmosphere, is large. What can you say about the distribution of d_{net} and why?
3. (30%) The net phase is $\phi_{net} = kd_{net}$ where $k = 2\pi/\lambda$. Any values greater than 2π are wrapped back to be between 0 and 2π. Assume that the variance you computed in part 1 is large compared to 2π. What is the distribution of the ϕ_{net} and why?

604 Prelim Problem Spring 2010

Part a) 50%

The following integral comes up when we try to solve the Helmholtz equation with the Fourier transform:

$$\wp \int_{-\infty}^{\infty} \frac{\exp(-2\pi i \xi x)}{4\pi^2 \xi^2 - k^2} d\xi$$

Note the Cauchy principal value symbol. Use contour integration to perform the integration.

Part b) 50%

The following integral comes up in tomographic imaging:

$$\wp \int_{-\pi}^{\pi} \frac{\exp(in\theta)}{\cos(\theta)} d\theta$$

Note the Cauchy principal value symbol. The parameter n is an integer. Use contour integration to find the value of the integral. This integral also gives the Fourier series expansion coefficients for the functionsec (θ).

OPTI 636: Noise in Imaging Systems

Let us assume that you make a single measurement of an object that is either of type H_1 or of type H_2. Your task is to classify a given object based on this single measurement. You assume the following: the probability density function (PDF) for the measurement g under the H_1 hypothesis is given by,

$$pr(g|H_1) = \left(\frac{1}{2\pi}\right)^{1/2} \exp\left(-\frac{g^2}{2}\right) \tag{1}$$

and the PDF for the measurement under the H_2 hypothesis is given by

$$pr(g|H_2) = \left(\frac{b^2}{2\pi}\right)^{1/2} \exp\left(-\frac{(bg-a)^2}{2}\right), \tag{2}$$

where $a > 0$ and $b > 0$ are parameters.

a. (20%) Derive the likelihood ratio $\Lambda(g)$ for this problem.

b. (20%) Show that

$$y(g) = \frac{k\Lambda(g)}{1 + k\Lambda(g)}$$

is a monotonic transformation of the likelihood ratio where k is a positive constant.

c. (20%) Describe how you would use the expression in part a) or the equation shown in part b) to classify a given measurement.

d. (40%) Describe an ROC curve and how it is generated. Which test statistic (the one in part a or in part b) will have a better ROC curve?

637 Prelim Problem Spring 2010

Part a) 30%

This problem is about finding an implicitly defined reconstruction for a discrete-to-discrete imaging system. The data is $\mathbf{g}$, the discretized object is $\mathbf{f}$, and the system matrix is $\mathbf{H}$ in the object space basis. Find the equation for the vector $\mathbf{f}$ that minimizes

$$(\mathbf{g} - \mathbf{Hf})^{\dagger} \mathbf{P} (\mathbf{g} - \mathbf{Hf}) + \alpha \mathbf{f}^{\dagger} \mathbf{Qf}$$

for positive-definite symmetric matrices $\mathbf{P}$ and$\mathbf{Q}$.

Part b) 30%

Derive the corresponding Landweber-type algorithm.

Part c) 40%

Find the condition for convergence of this algorithm and discuss what it converges to in the limit as $\alpha \to 0$.